This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Phys 2700.1

VE RI
TAS

HARVARD
COLLEGE
LIBRARY

AN

EXPERIMENTAL TREATISE

ON

OPTICS,

COMPREHENDING

THE LEADING PRINCIPLES OF THE SCIENCE,

AND

AN EXPLANATION OF THE MORE IMPORTANT

AND CURIOUS OPTICAL INSTRUMENTS AND OPTICAL PHENOMENA,

BEING

THE THIRD PART

OF

A COURSE OF NATURAL PHILOSOPHY,

COMPILED

FOR THE USE OF THE STUDENTS OF THE UNIVERSITY

AT

CAMBRIDGE, NEW ENGLAND.

BY JOHN FARRAR,

PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY.

◆

CAMBRIDGE, N. E.
PRINTED BY HILLIARD AND METCALF,
At the University Press.
SOLD BY W. HILLIARD, CAMBRIDGE, AND BY CUMMINGS, HILLIARD, & CO.
NO. 134 WASHINGTON STREET, BOSTON.
1826.

Phys 2700.1

DISTRICT OF MASSACHUSETTS, TO WIT.

District Clerk's Office.

Be it remembered, that on the seventh day of June, 1826, in the fiftieth year of the Independence of the United States of America, Cummings, Hilliard, & Co. of the said district, have deposited in this office the title of a book, the right whereof they claim as proprietors, in the words following, viz:

" An Experimental Treatise on Optics, comprehending the Leading Principles of the Science, and an Explanation of the more important and curious Optical Instruments and Optical Phenomena, being the Third Part of a Course of Natural Philosophy, compiled for the use of the Students of the University at Cambridge, New England. By John Farrar, Professor of Mathematics and Natural Philosophy."

In conformity to the act of the Congress of the United States, entitled " An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to an act, entitled, " An act supplementary to an act, entitled, ' An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned,' and extending the benefits thereof to the arts of designing, engraving, and etching, historical and other prints."

JNO. W. DAVIS,
Clerk of the District of Massachusetts.

ADVERTISEMENT.

THE body of this volume, like that of the preceding, was selected from Biot's *Precis Elementaire de Physique Experimental.*

In an elementary course of Natural Philosophy the space alotted to Optics is too limited to admit of any thing like a complete treatise. The plan adopted by Biot in his smaller work has been highly approved, and seemed, on the whole, to be best suited to the state of information and wants of most learners. Reference is frequently made to the larger work of this distinguished philosopher for the algebraic formulas and more detailed information relating to the subject under discussion. The part upon the polarisation of light, amounting in the original to about one hundred and fifty pages, is omitted, as not comporting with the design and limits of the work, already perhaps too extended for the time appropriated to these studies. A brief account is given in a note of this new branch of Optics, drawn up by Biot himself, and appended to his translation of Fischer's *Physique Mécanique.*† The other notes are intended to furnish information upon several other topics that have not found a place in the text.

The parts of this course of Natural Philosophy already published are, a treatise upon Mechanics, and a treatise upon Electricity, Magnetism, and Electro-Magnetism. The only remaining part relates to Astronomy, the printing of which is already commenced.

---

† The original not being at hand, the compiler has made use of a translation, contained in Coddington's Optics.

Cambridge, June 7, 1826.

CONTENTS.

OPTICS.

General Remarks 1

CATOPTRICS.

General Laws of Reflection 7
Of the Plane Mirror 11
Of Curved Mirrors 16
Of the Heliostat 21
Of the Forces which produce the Reflection of Light at the Surfaces of Bodies 23

DIOPTRICS.

General Laws of Simple Refraction 33
Exact Determination of the Ratio of Refraction in Solid Substances 37
Determination of the Ratio of Refraction in Liquids . . 40
Determination of the Ratio of Refraction in Aeriform Substances 42
Of Spherical Lenses 46
Determination of the Images produced by Diverging Lenses. Use of these Lenses in correcting shortsightedness . 52
Method of determining the Images produced by Converging Lenses. Use of these Lenses in correcting the defective Vision of longsighted Persons 56
Of Magnifying Glasses and Simple Microscopes . . 60
Physical Theory of Refraction 65
Double Refraction 93
Reflection at the second Surface of Crystals 122
Passage of Light through several Contiguous Bodies possessing the Power of Double Refraction 125
Construction of Double Image Micrometers . . . 128

Of certain Singular Appearances occasioned by Double Refraction 137
Of Double Refraction in Compressed Glass 140

ANALYSIS OF LIGHT.

Of the Dispersion of Light produced by Refraction . . 143
Recomposition of Colours 159
Compound Colours produced by the Mixture of Simple ones 164
Influence of the Unequal Refrangibility of Rays of Light upon Vision through Refracting Surfaces 168
Rainbow 171
Achromatic Combinations 176
Of the Dispersion which accompanies Extraordinary Refraction, and the Separation of the Axes of Double Refraction with respect to the different Simple Rays 184
Dioptric Instruments consisting of several Glasses . . 186
Compound Microscope 187
Amplifying Glass and Achromatic Eye-Glasses . . 195
Dioptric or Refracting Telescope 198
Instruments which consist of a Combination of Mirrors and Spherical Lenses 201
Method of M. Arago for determining the Magnifying Power of Optical Instruments 204
Instruments employed in Optical Experiments . . 206
Camera Obscura . . . . ib.
Megascope 207
Solar Microscope 208
Camera Lucida 209
Organ of Vision . . . . ib.
Reflections, Refractions, and Colours of thin Transparent Bodies 221
Colours produced by thin Laminæ 223
Fits of easy Reflection and easy Transmission . . . 265
Application of the preceding Theory to the Reflection of Rays of Light which have traversed thick Media . . 281
Explanation of the proper and permanent Colours of Bodies 287
Return of Rays reflected interiorly at the second Surface of thick transparent Media 292
Explanation of Coloured Rings and Fits upon the Hypothesis of the Undulations of Light. Principle of Interferences 293
Diffraction or Inflection of Light 302

NOTES.

I. *Of the Spherometer* 313
II. *Polarisation of Light* 314
III. *The Relations of Light and Heat or Caloric* . . 324
IV. *Measure of the Intensities of Light* . . . 331
V. *Description of the Kaleidoscope* 333
VI. *New Optical Experiments and the Improvement of Glass* 335
VII. *On the Magnetising Power of the more refrangible Rays of Light* 345
VII. *Of the Colours of Striated Surfaces* . . . 348
Table of the Refractive and Dispersive Powers of different substances 350

AN

ELEMENTARY TREATISE

ON

OPTICS.

General Remarks.

1. When the sun first appears above the horizon and presents itself suddenly to our view, we feel persuaded that there must be some mode of communication between this luminary and ourselves, which informs us of its existence without the necessity of our coming in contact with it. This mode of communication, which thus takes place at a distance, and through the medium of the sight, constitutes what is called *light.* Bodies, which are capable of exciting it directly, and of thus making themselves known to us, are called self-luminous, as the sun and stars. It would seem, indeed, that all material substances become self-luminous when their temperature is sufficiently raised, and they lose this property by being deprived of their heat. If they receive light, however, from a luminous body, after they have ceased to be self-luminous, they are still capable of sending to us the light thus received, as if it were their own, and in this case they become visible by *reflection.* In this way we perceive the objects about us while the sun is above the horizon, and all becomes obscure and invisible when this light is withdrawn.

In all cases, when an object transmits to us a sensation of its existence by means of light, this transmission takes place in a right line; for if we place fine threads of silk or metal parallel to each other and in the same plane, a luminous point situated at some distance beyond the threads in the same plane will be eclipsed by them; but if we move it a little out of the direction of this plane, it will become visible. Moreover, if we take two metallic plates perfectly plane and bring them by degrees towards each other, by directing the sight to the light of the sky through the space which separates them, we shall always be able to perceive it, however small the distance between the plates; but when one plate is concave and the other convex, vision ceases before they are brought into contact. This fact is confirmed by daily observation; and as it is of continual use, it will be proper to give it a simple enunciation. For this purpose, every right line, drawn from any point of a luminous body to the eye, has a particular designation, and is called a *luminous ray;* and direct vision is supposed to follow the course of these rays. This definition may likewise be employed in a physical sense, to distinguish the component parts of a luminious collection, at least as far as our senses are capable of advancing in the process of dividing. For, however fine a ray of light may be, as long as it is sensible to our organs, we find in it identically the same properties as in the largest collection.

2. It is proved by astronomical observations that the communications established by light between luminous bodies and ourselves is not instantaneous. When the sun is at any point of its orbit, we do not perceive its presence at this point until about 8′ 13″ after it has arrived there. When the satellites of Jupiter, which are small moons illumined by the sun, after having been eclipsed by the body of their planet, disengage themselves from its shadow, a certain time elapses between the absolute instant at which they are thus disengaged, and that at which we begin to perceive them. The interval is longer or shorter, as the earth is more or less distant from Jupiter, and is exactly proportional to this distance. Hence we conclude that the velocity of light is strictly uniform throughout the whole extent embraced by the earth and Jupiter in their annual revolutions.

It follows, moreover, from these phenomena, that after the absolute instant when the satellites of Jupiter enter into the shadow of this planet, we still see them without it, since the sensation which we have of them results from their previous presence at a point of their orbit which they occupied a few minutes before; and also when they disappear, their light has already been interrupted for some time. Thus the communication resulting from the presence of these bodies at any point continues to take place even after they have left it. This communication then must be effected either by means of impulses propagated through an elastic fluid from luminous bodies to us, as sound is transmitted through the air, or by an actual emanation of material particles from these bodies. In all cases, as the sensation of vision takes place even through the substance of certain bodies, called *transparent* or *diaphanous*, it is necessary either that the impulse of the elastic fluid should continue to be propagated through the pores of these substances, or that the luminous particles should continue to move within them and be capable of passing through them.

3. Each of these opinions has had its defenders. Those who are inclined to favour the supposition of an elastic fluid, dwell upon the facility with which this theory adapts itself to rapid and uniform transmission. They insist upon the improbability of an actual emanation of particles having the velocity belonging to the particles of light, and at the same time, of such extreme tenuity as to be able to pass readily through transparent bodies. As to this point we must be guided solely by facts; for *slow* and *rapid* are nothing in themselves, any more than *great* and *small*. The motion of a cannon-ball to us is so rapid that our eyes cannot follow it; this, however, is very slow compared with the revolution of the earth upon its axis; and this in like manner is slow compared with the earth's annual motion; indeed the earth's annual motion falls far short of the rapidity with which light is transmitted. It is certainly more difficult for us to give a large body a great velocity than a small one, since our strength is limited; but what community or what comparison is there between our limited powers, and the extent and kind of power which operates in nature? None, absolutely none. Accordingly, if we divest ourselves of this prejudice, and examine the phenomena by themselves, we shall perceive that a very great part of them take place in a manner exactly conformable to the idea of an emission. When light passes through transparent bodies its motion is precisely such as it ought to be, were it composed of a substance, capable of being attracted by these bodies. If we observe its motion in gaseous or liquid substances of different natures, and then mix these substances, (which we suppose to be such that they can have no chemical action upon each other) the motion of light through the compound may still be calculated by the laws of the attractions of the component substances, pre-

viously known; and the result of this calculation agrees exactly with observation. But who can say in what manner undulations must be formed? And without being able to answer this question, it would seem that they must be formed according to extremely complicated laws. In fine, it is known from other phenomena that luminous rays may be so modified and prepared that their different sides shall present different physical properties; a fact which agrees very well with a series of particles, but which it is much more difficult to conceive and represent, if we suppose a series of impulses. Moreover, these properties, relating to the sides of the rays, are so inherent in them that they manifest themselves in passing through liquids perfectly homogeneous, as the essence of citron, for instance, and that of turpentine; and, if the rays penetrate these liquids in a direction perpendicular to their free surface, which seems to give to pulsations all the conditions of perfect symmetry, they continually exhibit during their transmission properties not symmetrical about the perpendicular to the face of incidence; this it would be very difficult to comprehend on the supposition of transmitted impulses, but is very easily understood if we admit the theory of luminous particles having different properties on their different faces, and being successively submitted to the actions of the particles of a homogeneous medium through which they pass.

In what I shall hereafter offer on the phenomena of light, I shall employ this theory which supposes it material, and which affords in almost all cases great facility of conception and representation; then, when we have studied the phenomena in this manner, I shall present the views that must be substituted according to the system of undulations, and apply them to some phenomena, for which this system offers an easy explanation, while we have never yet been able to deduce them from the hypothesis of the materiality of light.

4. In this exposition we have considered vision as propagated from the object to the eye, but it is not on the external surface of the organ that the sensation takes place; it is in the interior, and according to common opinion, on a nervous membrane which covers the back part of the eye, called the *retina*. Indeed, when the different media, composing the other parts of the organ, happen to be hardened or affected by disease, so as to prevent the light from reaching the retina, the power of vision is destroyed;

but it is again restored when the parts of the organ which have become opaque are removed. Each of these parts may be separately removed without entirely destroying the sight; but if the retina is affected, the power of vision is lost forever. We know, moreover, that two large nerves, proceeding from the brain, spreading over the posterior surface, and branching out into an infinite number of ramifications terminate in the retina, or to speak more correctly, the retina itself is only an expansion of these nerves. Now we know that in all the other organs, sensation is propagated by the nerves. This analogy confirms the supposition that the sensation of vision takes place immediately at the retina, either by impulses propagated through an elastic fluid, if light be transmitted by undulations, or by the direct impression of luminous globules if light is material. I shall treat more particularly of the internal structure of the eye, after having made known the theory of optical instruments; for the eye itself is an optical instrument so perfect and admirable, that the most profound examination fails of comprehending all its wonderful properties, and the most exquisite art has not succeeded in imitating it. Thus far these preliminary observations will suffice; we shall confine ourselves to the consideration of the back of the eye or the retina, as the centre of the organ of sight.

5. When we view an object sufficiently great to be perceptible, the rays coming from its opposite extremities, arrive at the eye in different directions, and consequently cross each other so as to form a certain angle at their point of incidence before the pupil. This is called the *visual angle* or the *apparent diameter* of an object, since, in fact, as we shall see hereafter by many examples, we judge of the actual bulk by the size of the visual angle which it subtends in the eye, together with the idea we have of the distance at which we suppose it to be placed.

When light passes from a luminous body to us, it always comes through different media, such as air, water, or other transparent bodies, whch afford it a passage more or less free. The rays upon entering these bodies, sometimes pursue their course in a straight line; but most commonly they are turned from their original direction, and this phenomenon is called *refraction.* Besides this cause of deviation, it frequently happens that light meets with smooth surfaces which send it off or *reflect* it, by which objects are presented to our sight in an indirect way, when we happen to be in the direction of the rays thus reflected. We shall study the refraction and reflection of light in succession. It would seem proper to begin with the first of these classes of phenomena, since, as we ourselves are immersed in a material fluid, the air, vision cannot take place without the action of the air upon the luminous rays; but as this action is very feeble, and hardly turns them from their natural direction, we shall defer this subject, and begin by investigating the laws of the phenomena of reflection, which are much more simple, and which, taken together, constitute the first branch of the science of optics, called *catoptrics.*

CATOPTRICS.

General Laws of Reflection.

6. In order that the surface of a body may reflect the light regularly and give a distinct image of the luminous points by means of which it is rendered visible, it must be carefully polished, that is, its little inequalities must be removed, as far as possible, by friction. Such is the state to which glass, crystals, and metals, are reduced by art. We shall first consider the phenomena presented by surfaces thus prepared; and afterwards endeavour to show in what manner the polishing contributes to the effect in question.

To proceed methodically in the study of the laws of reflection, we shall begin by determining what these laws are with respect to plane surfaces. It will be easy to extend them, when thus determined, to curved surfaces. For, in all the modifications which light undergoes, by the action of bodies of a sensible extent, the luminous rays may be considered, (at least, as far as they are perceptible to our senses) as mathematical straight lines; so that at each point of a curved surface, reflection takes place in exactly the same manner as if the ray fell upon a plane touching this point; and since we may always calculate the position of the tangent plane for all the points of a given surface, it follows that the reflection of light from any curved surface whatever will be a subject of pure calculation, when the laws according to which it takes place from plane surfaces are once known.

7. For the purposes of this inquiry, and with respect to optical experiments in general, it is indispensable that we should be provided with a room which receives the direct rays of the

sun during a part of the day, at least, and which is furnished with window-shutters made very tight, so that it may at any time be rendered completely dark. In one of these shutters an aperture is supposed to be made of any convenient size, and covered with a metallic plate, pierced with several holes of unequal diameters, and capable of being opened or shut at pleasure. One of these holes being opened when the rays of the sun fall directly upon the shutter, the light will enter the room in the form of a beam or fine collection of rays, which become sensible by being reflected from the particles of dust floating in the atmosphere. We shall hereafter make known a more convenient and ingenious apparatus; the one just described will suffice for the present.

Now if this beam be made to fall obliquely upon a polished, transparent plate of glass, placed horizontally, the following appearances will be exhibited;

(1). The luminous beam is not wholly transmitted through the surface. A part is reflected upward, in a direction depending on its obliquity. If the eye be placed any where in this direction, a lively, brilliant image of the sun will be visible, which seems to come from beneath the glass in the direction of the floor.

(2.) The point where the beam of light meets the plate is visible from every part of the room; but when thus viewed no regular image of the sun is presented, and the light is incomparably less brilliant than when the eye of the observer is placed in the direction of the reflected beam.

(3.) A portion of the incident light escapes reflection at the first surface of the plate and passes to the interior. Arriving at the second surface, another partial reflection takes place, and the remainder passes into the air beneath the glass.

If we now consider the phenomena presented by the first surface, we shall observe that there are three distinct operations. A part of the incident light is reflected regularly in a particular direction; another part is reflected indifferently in all directions, as if the body were not polished; and finally the remainder is transmitted without being reflected. To distinguish these two modes of reflection which take place in the same body, I shall call the first *specular reflection*, since it is this which gives a regular image, whatever be the reflecting surface. I shall call the other mode *radiant reflection*, because it scatters the light in all directions about the point of incidence, as if this point had by itself a radiating power.

If we substitute for the plate of glass one of polished metal, the two first only of these phenomena will be observed. The polished metal reflects regularly one portion of the incident light, disperses another portion, and absorbs or extinguishes the rest; this remainder answers to what the transparent body transmits.

By confining ourselves, therefore, for the present to the two first phenomena which constitute reflection, we shall in the first place inquire into this dissemination of light which renders the point of incidence visible from all parts of the dark room. If we repeat the experiment with different reflecting bodies, and with the same body having different degrees of polish, we shall soon perceive that the imperfection of the polish is the determining cause of the phenomenon. For, the greater this imperfection is, the nature of the reflecting body remaining the same, the more considerable is the portion of light thus dispersed, and the more feeble, on the contrary, is the specular reflection. To be convinced of this, we need only take a plate of glass, having a plane but unpolished surface, and to present it successively, under different angles of obliquity to a solar ray, introduced into a dark room. When the ray meets the plate in a direction nearly perpendicular to its surface, the portion of light reflected specularly will be insensible; the radiant portion, on the contrary, will be very strong, and will render the point of incidence dazzling; but by inclining the reflecting surface to the ray, we shall find that this portion becomes less, and the specular reflection beginning to take place, gives at first a feeble reddish light; soon this light increases, and finally, when the ray becomes nearly parallel to the surface of the plate, it will be almost as strong and as white, as it would be from glass highly polished. Analogous results are observed, if instead of examining the reflection of a solar ray, we undertake to observe, by reflection from an unpolished plate, the images of very bright objects, for example, that of a building illuminated by the sun; for while the incident rays make large angles with the unpolished surface, the images of the objects are not distinctly formed. But soon they will begin to appear when the rays begin to be inclined to the reflecting surface; and in the end they will become perfectly bright and distinct with the smallest inclinations.

Now the direction of the incident ray in relation to the reflecting surface being given, let us inquire what will be the direction g. 1. of that part of the light which is reflected specularly. For this purpose we may make use of the instrument represented in figure 1, which we shall hereafter have occasion to use. It consists, in the first place, of a circular plane *AZB*, placed vertically upon a firm stand capable of being levelled. The circumference *AZB* is graduated, and carries too metallic indices *S*, *O*, turning on the same centre and having two small holes *S′*, *O′*, at equal distances from the plane of the circle. At the centre *C* is placed a plate of polished glass, *CG*, which, by means of screws, is fixed perpendicularly to the plane of the circle; it consequently takes a horizontal position when the circle is vertical. In order to give it this adjustment the plane of the circle is first rendered vertical by being directed to the vertical bars of a window frame or other vertical object; a spirit level is then placed upon the glass plate, and the plate is moved by means of its adjusting screws till this level indicates a horizontal position. Finally, above this glass and before the centre of the circle is fixed a metallic plate, or thin wedge, the rectilineal edge of which *CL* forms a straight line, proceeding from the centre *C* perpendicularly to the plane of the circle; and upon this line is made a light stroke *C′*, at the same distance from this plane with the hole in the indices; so that the three points, *S*, *O*, *C′*, are always in the same plane parallel to the graduated circle. The instrument is now to be placed before an open window, so that the light from abroad may enter the hole *S*, and fall upon the glass under different angles. The index *O* is moved backward or forward till the image of the hole *S* is seen through the opening at *O* exactly upon the edge *CL* of the central plate. It is shown by experiment that this is possible; and the point of incidence is always found to fall precisely upon the stroke *C′*, therefore, *the incident ray and the reflected ray are comprehended in the same plane perpendicular to the reflecting surface.* This is the first fundamental law of reflection. Moreover, the two rays meeting at the axis *CC′* of the graduated circle, their respective inclinations to the reflecting surface are measured by the arcs *BS*, *AO*, which may be read off upon the graduated circle, the divisions commencing at the horizontal diameter *AB*. This being done, it will be found that for all possible positions of the indices, *the incident and reflected rays make the same angle with the reflecting surface.* This is the second general law of reflection, and being taken in connexion with the preceding, it determines all the circumstances of this phenomenon.

8. Through the point of incidence *C′* suppose a line *C′Z′*, drawn perpendicularly to the reflecting surface; the angle *S′C′Z′*, formed with this perpendicular by the incident ray is commonly called the *angle of incidence*, or simply *the incidence*, and *O′C′Z′* is called the *angle of reflection*. The second general law of reflection, therefore, deduced from the preceding observations, is, that *the angle of reflection is always equal to the angle of incidence.*

Of the Plane Mirror.

9. The laws of reflection being known, it is easy to deduce from them the appearances which are observed when the reflecting surface is a plane.

Let *S* be a radiant point, *O* the eye, and *AB* the reflecting plane, which I shall suppose for the present of indefinite extent. Among all the luminous rays which proceed from *S*, there will be one, as *SI*, which, after being reflected, will go to meet the eye at *O*, in the direction *IO*. Then the angles *SIA*, *OIB*, will be equal, according to the second law of reflection. Now draw from the point *S*, a perpendicular *SA*, meeting the reflecting surface in *A*, and produce this line on the opposite side of the mirror to *D*, making *AD* equal to *AS*. From the point *D* draw the straight line *DO* to the eye; *DO*, it is manifest, will be the direction of the reflected ray, and the point *S*, where it cuts the surface of the mirror, will be the point of incidence. Moreover, if the luminous object and the eye are considered as mathematical points, without sensible extent, the ray, thus determined, is the only one which can be reflected to the eye. But the pupil or opening through which the light is admitted into the eye is Fig. 2.

not a mathematical point, but a circle of sensible extent which ig. 3. in man is about 0,08 of an inch in diameter, and which may be represented by *LL*. All the reflected rays which can enter this aperture will reach the retina and contribute to vision. Now each of these rays is determined by the construction just given; whence it is evident that they will form a *pencil* or cone with a circular base, the vertex of which will be *D*, and the base *LL*. It is a fact, that when the eye estimates freely the distances of luminous points, it supposes them placed at the point from which the rays that enter the eye diverge. Thus to the eye situated at *O*, the luminous point, seen by reflection, will appear at the point *D*, that is, as far behind the mirror as it really is before it.

If the radiant object has a definite extent, each of the radiant points, presented by it, will produce its own particular image, according to the laws which we have just explained, and the assemblage of these images will compose the image of the object. ig. 4. Suppose, for example, that this object is an arrow *SS'*; the point *S* of the arrow will have its image at *D*, the image of the other extremity will appear at *D'*, and the intermediate points will be seen in the straight line *DD'*. Thus the entire image will be comprehended between the extreme reflected pencils *DO*, *D'O*; its absolute magnitude *DD'* will be equal to that of the arrow itself; but it will appear inverted from right to left.

According to what we have just now said respecting figure 3, each point of this image is regarded by the eye as situated where it is actually fixed by our geometrical construction. If we substitute, therefore, in the place of this image a real and similar object, that is, an arrow in every respect equal to *SS'*, placing its point at *D*, and its base at *D'*, and if we look at it from the point *O*, the mirror being removed, we shall see it precisely as we see the image *DD'*, having the same magnitude, the same visual angles, and the same degree of brightness, since all the luminous points which compose the real object would transmit to the eye cones of luminous rays exactly similar to those which seem to proceed from the image. It hence follows that objects are seen in the plane mirror with the same forms, the same brightness, and at the same distance, as they would be if viewed through the medium of the air, provided that the quantity of light is the same in the two cases; for we must make allowance for the portion of light which is transmitted when the mirror is transparent, or absorbed when it is opaque.

10. What has now been said is sufficient for the resolution of all problems which may occur respecting reflection and vision by plane mirrors.

As the reflection of light takes place rigorously according to the law we have just demonstrated, it may be employed with great advantage in measuring the angles formed by two plane and polished surfaces. As this measurement is necessary in a great number of physical inquiries, and as no other method has ever been employed sufficiently nice and accurate for this purpose, I shall give some simple examples of the process.

When we operate upon bodies having faces of a sensible extent, for example, on prisms cut and polished by art, like those which are employed in optical experiments, we make use of the instrument which has already been applied to determine the laws of reflection. For this purpose, instead of bringing the glass plate *GG* in contact with the edge *CL*, it should be placed at the distance of about $\frac{1}{16}$ of an inch, being always made perpendicular to the plane of the circle, by means of the adjusting screws with which the instrument is provided. In order to give it this position without other aid than that which the instrument affords, we move the index *O* nearly to the highest part of the circle, and placing the eye behind it, we regulate the glass so that the reflected image of the hole *O′* and that of the eye shall return through this hole from the fixed mark *C′* upon the edge *CL*. When this takes place we are sure that the ray *OC*, parallel to the circle, is at the same time perpendicular to the glass, and reciprocally that the glass is perpendicular to the plane of the circle. We then place upon the glass one of the faces of the prism whose angles we wish to measure. The edge of the prism is now made to slide under *CL*, by turning it in such a manner that the upper surface shall also be perpendicular to the plane of the circle; and this condition is fulfilled when, the index *S* being placed in any point of the circumference, its image, as seen through the other index, moved to a corresponding point, appears on the mark *C′*. Let *S* and *O* be the positions of the two indices answering to this condition. Then it is evident that *N*, the centre of the arc *SO* (a point easily determined by means of the divisions upon the arc) is in the direction of the line rais- Fig. 5.

ed from the centre *C* perpendicularly to the reflecting surface. Moreover, *CZ* is perpendicular to the other face; therefore the angle *ZCN*, measured by the arc *ZN*, is the angle comprehended between the two surfaces of the prism, and which it was proposed to determine. This ingenious method, which was invented by M. Cauchoix, requires but little time and is very accurate in its results.

11. The same principle differently applied, has led to a great number of instruments analogous to this, which are called *goniometers*, that is, instruments for measuring angles. They will be easily understood after what has now been said, for the means of observation and verification are essentially the same in all. I shall nevertheless describe that invented by Dr Wollaston, because it is particularly applicable to mineralogy.

This instrument, represented by figure 7, is composed of a vertical circle of copper, graduated on its edge, and turning about a horizontal axis *AA*. It is itself supported upon a vertical foot *CP*. Within the axis *AA*, made hollow for the purpose, turns the axis *a a*, the projecting extremity of which carries several pieces having rectangular movements, upon which are fixed, with pincers or wax, the crystal whose angles are to be measured. To make use of this goniometer, we stand before a building having several horizontal lines parallel to each other; we then place the base of the instrument on some horizontal plane, so that the limb shall become vertical and perpendicular, or nearly perpendicular, to the lines which are to be employed as sights. The first condition is easily fulfilled by arranging the plane of the limb along some of the vertical lines presented by the building. This being done, we place the eye very near the crystal, and looking at the building by reflection from one of its faces, we turn this in such a manner, that one of the horizontal lines the most elevated, thus seen, shall coincide with one of the inferior lines seen directly. I shall mention soon how we may arrive at this condition. When we have obtained it, we turn the interior axis *a a*, until the same coincidence shall be observed likewise on the other face, whose inclination to the first we wish to measure. We arrive at this by several trials. Now when this coincidence can be thus obtained successively upon the two faces without changing the place of the eye, without touching the crystal, and by the rotation merely of the axis *a a*, we are sure that the intersection of the two surfaces is exactly horizontal, and consequently parallel to the axis *a a*. Then we do not touch the crystal, but setting out from one of its positions, in which the reflection is observed on one of the two surfaces, we turn the limb until the reflexion and the coincidence are observed likewise on the other. This motion takes place by means of the great axis *AA*, which turns with the axis *a a*, the crystal and the limb. The arc by which this has turned, is measured by the division traced upon the limb, and it is evidently equal to the supplement of the angle formed by the two surfaces. But the division traced upon the limb is written in a manner to indicate the angle itself, at least when we first put the index on the point zero.

In order that the application of this method may be easy and sure, it is necessary that the dimensions of the crystal and its distance from the eye should be infinitely small, compared with the distance of the objects used as sights. For if this is observed, the fixed position of the eye is no longer a necessary condition any more than it is in observations made at sea with instruments of reflection. Thus by placing the eye very near the crystal, the approximation of the lines of sight may be considered as indefinite. Dr Wollaston was accustomed to place the instrument in a chamber at some distance from the window, the bars of which were used as sights, and of which the upright parts served to place the instrument in a vertical plane. But without the address of this practised observer, sights so near could not be safely employed; for the liability to error increases with their nearness. We may in general make use of the horizontal and vertical lines of an edifice sixty or eighty yards distant; then, to render the first face of the crystal perpendicular to the limb, we first direct the shaft *t o* parallel to its surface; then, Fig. 8. without taking it from this direction, we turn it upon its axis until the reflected image of one of the horizontal lines becomes parallel to the direct image, and a coincidence may be effected by the mere motion of the axis *a a*. We then see if the same condition is fulfilled for the other face of the crystal, and as in general we find it is not, we effect it by turning the branch *b c* about the point *c* without touching the shaft *t o*; this motion being perpendicular to the plane of the limb cannot alter the perpendicularity of the first face; but for greater security we turn the axis *a a* to effect the coincidence; and as this is almost always necessary, if it have been subjected to any deviation, the correction is applied by the single rotatory motion of the shaft *t o* upon its axis. After one or two verifications of this kind, the coincidence upon the two faces will take place exactly, and their inclination may be observed. One advantage peculiar to this goniometer, is, that it enables us to measure the angles of the smallest fragments of a crystal, to which it is exclusively and exactly applicable; and this property is so much the more valuable, as the small crystals seem to be the only ones in which a perfect regularity of internal construction is to be looked for.

Of Curved Mirrors.

12. In order to discover and determine generally the apparent place, form, and magnitude of images, reflected by curved mirrors, whatever be their figure, it is sufficient to consider the reflection of each luminous ray as taking place upon a plane, tangent to the surface at the point of incidence. We can thus extend to all surfaces by the calculus, the results which we have obtained for the plane mirror. But it is unnecessary to make our calculations so general, for in practice we employ only concave and convex spherical mirrors, these being the only forms which can be accurately made and polished. Indeed, we cannot obtain from these perfect images except when the luminous rays fall upon the surface nearly in a perpendicular direction. This, therefore, will be the only case which we shall need to examine.

In order to determine with exactness the circumstances to be considered, let us imagine, in space, a luminous point sending forth rays upon the different parts of any spherical surface either concave or convex; and considering one of the rays separately, let us endeavour to determine the direction which it takes after reflection. We must remember that reflection takes place always in the plane which passes through the luminous point and the perpendicular erected at the point of incidence. This perpendicular is the radius of the sphere; so that reflection takes place in the plane of the great circle drawn through the luminous point and the point of incidence.

Let *MAM′* be the intersection of the mirror by the plane in question. In this plane at *S* we place the luminous point, and designate by *SI* the incident ray which we are to consider. From the point *I* to the centre of the sphere we draw the perpendicular *IC*, and taking the angle *CIR* equal to *CIS*, we have *IR* for the direction of the reflected ray. Fig. 9.

Since we wish to confine ourselves to the incident rays which are nearly perpendicular, the angles *CIR*, *CIS*, must be very small, even for the rays which fall near the edge of the mirror. This requires too things; (1.) That the surface of the mirror should comprehend but a small number of degrees of the sphere of which it is a part. (2.) That the incident ray *SA*, drawn from the radiant point *S* to the centre of figure of the mirror *A*, should make a very small angle *SAC* with the central perpendicular *AC*, which is called the *axis of the mirror.*

13. These two conditions being fulfilled, if we apply to each ray, proceeding from the point *S*, the construction which we have just given for *SI*, we shall find by the figure, as well as by calculation, that the reflected rays will pass very near each other, within a small space *f*, called the *focus*, which may be considered as a point; so that if it be situated before the mirror, it will give the image of the point *S*. This is in fact confirmed by experiment, as we shall presently see, but we must first determine the situation of *f* for every given situation of the luminous point.

For this purpose, we begin with the case where this point is situated in the axis *AC* produced at so great a distance that all the rays proceeding from it may be considered, for the extent embraced by the mirror, as parallel to this axis. We now find that the focus *F* is situated precisely at the middle point of the axis of curvature *AC* of the mirror, at equal distances from its surface and centre. The interval *AF* thus determined is called the *principal focal distance* of the mirror, and the point *F* is called the *principal focus*. When the mirror is concave toward the radiant point, the principal focus is situated on the same side with this point, before the mirror, and in this focus is a real concentration of light. But if the mirror is convex, the principal focus falls behind its surface, and the reflected rays not being Fig. 10 11 Fig. 10. Fig. 11.

able to pass through the mirror to arrive at it, the focus F only indicates the imaginary point of meeting of these rays produced.

14. This case being resolved in a general manner, nothing is more easy than to find the place of the focus for all other possible situations of the radiant point. For example, let S be this point. Among all the rays proceeding from it, draw SI parallel to the axis AC of the mirror. SI being reflected will pass into the focus F of parallel rays, situated at the middle point of AC; so that the reflected ray will be IF. Drawing now SA to the centre of figure of the mirror, this ray will evidently be reflected on the opposite side of the axis AC, forming with it equal angles on the two sides; and this reflected ray will be Af. Produce IF and Af till they meet; the point f will be their focus, and it will be the focus also of all the other rays which proceed from S. The results obtained by construction being expressed algebraically, we shall have a general formula for determining all the successive values of the focal distances, for all possible curvatures of the mirror, and for all the different distances of the radiant point from its surface. Thence we easily determine the distance, position and form of the images, given by all objects of a sensible extent; for each point of the object sends to the mirror a cone of luminous rays which has its focus at the point indicated by the formula, and the assemblage of these foci forms the image of the object. ig. 12. 13.

In order to verify these observations by experiment, we must take for the object a body of small extent, so that even if it be brought within a small distance of the mirror, the inclinations of the rays which proceed from it may not exceed the limits supposed in our approximations. This object must also be very luminous, and of such a form that it may be easily known if its image is erect or inverted. Nothing better fulfils all these conditions than the flame of a taper, held nearly in the axis of the mirror, and presented successively at different distances from the surface.

ig. 14. 15. We begin with supposing the mirror concave, and place the taper SS' at a considerable distance compared with the diameter of its concavity. Then, since the incident rays may be considered as sensibly parallel, the image ff' will be formed nearly in the principal focus of the mirror at the middle point between its surface and its centre of curvature. The taper is not to be placed precisely in the direction of the axis, but a little to the right or left of it, so that the image may be formed on the opposite side of the axis. If we then place in the focus a plate of ground glass and look at it from behind, we shall see painted upon it a little image of the taper, very bright and moreover inverted, as we are taught to expect both from calcalation and our graphic construction. We may also remove the glass and look simply at the image, placing the eye in the direction of the rays which proceed from it, and at such a distance as would be convenient for seeing a real object distinctly. Now if the object be gradually brought nearer to the mirror, the focal distance increases, and the image recedes towards the centre of curvature, preserving always its inverted position and increasing in size; and when the object reaches the centre, they unite and coincide throughout; if the object be brought still nearer, they separate and the image continues to recede, still increasing in size, and still inverted, of which we may satisfy ourselves by looking at it from behind, either with the naked eye or through a plate of ground glass. These appearances continue till the object reaches the principal focus *F* when the image is removed to an infinite distance from the mirror and is infinitely great, so that it is impossible to place ourselves in a situation to see it. But if the object continue to approach the surface of the mirror, the image reappears behind the surface on the side opposite to the observer. It is now very large and erect; but as the object approaches it diminishes in size, still remaining erect, and finally when the object touches the surface, it becomes equal to the image, and is situated on the same surface. In the second series, when the image appears behind the mirror, it is no longer a real collection of light, but the imaginary place from which the reflected rays would diverge, if produced beyond the mirror, each in the direction determined by reflection, as represented in the figure. Fig. 15. Fig. 16.

16. The appearances produced by convex mirrors have much less variety. The image in this case is always imaginary, and is situated behind the mirror, so that it may be seen with the naked eye, but cannot be received upon a plate of ground glass. When the object is placed at a very great distance from the mirror, the image appears in the principal focus, erect, and much smaller than the object itself would appear at an equal distance; Fig. 17.

it approaches the surface of the mirror as the object approaches, remaining always erect and increasing in size, till finally they coincide upon this surface.

When we wish to make use of spherical mirrors in experiments requiring great exactness, we must be able to determine their foci. If the mirror is concave, it is placed in a room partially darkened, at some distance from a window, from which may be seen different objects at a sufficient distance. Its concave side is then turned towards the window; and it is inclined a little to the right or left, as in figure 15, so that the focus of reflected rays may be thrown into the dark part of the room. We next place in this direction a plate of polished glass, or a piece of white pasteboard *ff*, which may be brought nearer or removed further from the mirror, till an image of the proposed object abroad is distinctly painted upon it. When the point required is found, we measure its distance from the centre of figure of the mirror, and take the double of this for the length of the radius of curvature. But this method requires, agreeably to what we have before said, that the surface of the mirror should comprehend but a small part of the sphere to which it belongs; otherwise the reflected rays would not, strictly speaking, rigorously meet at any single focus, and the approximation which we have made use of would be inapplicable.

17. If the mirror is convex, the operation is more difficult. In this case we paste upon one of its diameters a strip of black
. 18. paper *DO*, and make in this two small circular apertures, at equal distances from the centre of figure. The rays of the sun are then made to fall upon the mirror; and those which are reflected from the two apertures being produced, their imaginary point of diverging is the focus of parallel rays. Measuring their departure from each other at different distances from the surface of the mirror, we easily calculate their point of meeting; and the distance of this point from the surface is half of the radius of the sphere. This method is evidently not capable of much exactness; but if it were necessary to be very accurate in our measurements, which rarely happens with this kind of mirrors, we must make use of the spherometer†. We can still collect by

---

† See note subjoined to this treatise on the spherometer.

a concave mirror the reflected rays, fix the point of their convergence and deduce from them the primitive focus, which will be the focus of the concave mirror proposed.

18. When we wish only to verify by observation the results obtained from calculation, it is of little importance whether we use mirrors of metal or of glass. It is nevertheless necessary to remark, that with the latter substance we have always two images, one reflected from the first surface, the other from the second. The latter of these is usually the brightest, because there is applied to the posterior surface of the glass an amalgam of tin, in order to render the reflection more intense; so that in strictness such a mirror is composed of two others, a first of glass and a second of metal. This doubling of images is not attended with any serious inconvenience when we wish only to observe the general results of reflection; but it would be fatal where much accuracy is required; especially if the images are magnified by lenses placed near the eye, as is the case in astronomical instruments. Therefore, metallic mirrors are the only ones we can make use of. We shall see in one of the following sections, the manner in which they are to be employed in constructing the instruments which are called *reflecting telescopes.*

Of the Heliostat.

19. Most optical experiments, especially those which have for their object to determine the physical properties of light, are performed upon solar rays admitted into a dark room through a very small aperture in the window-shutter. But this method is attended with two inconveniences; the first is the obliquity of the sun's rays to the horizon; the second is the continued motion of the sun.

By the oblique direction of the rays, they are determined towards the floor on entering the room, and we are confined in our experiments to a small space. From this same cause also they can be retained only for a small part of the day, and we are thus limited as to time and the convenience of continuing our experiments in the same place. Finally, on account of the motion of the sun the direction of the rays is perpetually changing, and we are every instant obliged to change the situation of the objects we would present to them.

20. These difficulties are removed by means of an instrument invented by S'Gravesande. It is called a *heliostat*, because it enables us to direct and fix the rays of the sun in any manner we please.

Fig. 19. This instrument is composed of two principal parts, a plane metallic mirror *MM*, and clock-work by which the mirror is continually changing its position so as to keep the reflected beam always in the same direction.

The mirror is made of metal instead of glass, to avoid the inconvenience of a double reflection. That it may take freely every possible position, it is so mounted as to have two motions of rotation perpendicular to each other, one about a horizontal axis *AA*, and the other about a vertical axis *CP*, which serves also as a support.

The clock-work is connected with the mirror by means of a shaft *CQ*, attached perpendicularly to its posterior surface, and of which the extremity *Q* is carried by the hand *C'R* of the dial, by means of the piece *FF*, separately represented in figure 20. This piece, which is made in the form of a fork, has a cylindrical shaft *q q*, which enters freely into a hole made near the extremity *R* of the dial hand, perpendicular to its direction, by which arrangement the fork is capable of turning about its axis *q q*. Between the branches of this fork is a small cylindrical tube *t t*, which turns freely about an axis of rotation *a a*, perpendicular to the direction of the fork. Combining this motion with that of the fork itself about its axis *q q*, it will be seen that the little tube may take in space all imaginable directions. Now if we wish to attach the shaft of the mirror to the clock-work, we take off the fork *FF*, and introduce the extremity *Q* into the tube *t t*, which has precisely the same diameter; the axis of the fork is then replaced in the extremity of the dial-hand, the motion of which is thus communicated to the mirror. But in order that the motion of the mirror may be such as to keep the reflected beam in the same direction, the dial must have a position parallel to the plane of the equator; and there must be a certain relation between the distance and the positions of the mirror which cannot be determined or even explained without the aid of the calculus.†

---

† For a full account of the principle of this adjustment, see Biot's *Traité de Physique*. Tom. iii, p. 176.

A more simple form of the heliostat is sometimes used. Let

Of the Forces which produce the Reflection of Light at the Surfaces of Bodies.

21. When we have found by experiment the actual laws of reflection, we must next endeavour to refer them to mechanical causes, that is, to assign systems of forces capable of producing the same effects. For if we can discover such forces, we are carried at once to the source of the phenomena, we know all their natural and necessary relations; and instead of involving ourselves in details we have only to examine them in their principles, which is far more simple. Let us endeavour then to assign the cause of the phenomena of reflection.

Since we consider light as matter sent forth by the luminous body, we shall seek the conditions to which its particles must be subjected, in order that the phenomena may be such as observation indicates. If we proceed with rigor in this inquiry, the conditions which we shall obtain will be so many properties belonging to light, or to the bodies which act upon it, and which will thus serve to characterize its nature, and the kind of action it experiences from these bodies. If we would afterwards consider light as the effect of pulsations, impressed upon a very elastic medium, these results will not lose any of their value, for they will become so many characteristic conditions, which it is necessary to apply to these pulsations, and the mode of transmission which bodies admit of their possessing. It is thus that physical hypotheses are useful when we regard them simply as the means of connecting; temporarily, phenomena, the cause of

---

the dial plate SS be placed parallel to the plane of the equator, and be made to turn uniformly once in 24 hours by clock-work; and suppose a mirror attached to its under surface, and moveable on an axis coinciding with this surface, and capable of being so adjusted as to reflect the sun's rays perpendicularly through an aperture in the centre of the dial plate. The uniform motion of the dial plate and mirror will preserve the reflected beam constantly in a direction parallel to the axis of the earth. Then, by means of a fixed mirror, this beam may be brought into a horizontal or any other position that may be required.

which is not known to us; and such is the nature of the understanding, that we cannot without this aid, follow the connexion of a series of facts, the principle of which is not discovered. It is thus that the astronomer, who is unable to comprehend generally in his formulas the course of a heavenly body, when it is very complicated, calculates successively the different parts of its path adapted to different orbits, which are not respectively applicable, except through a small extent, and which are changed according as it is found that they become erroneous.

When we consider light as an actual emission from the luminous body, the phenomena of specular reflection seem, at first sight, to be simple results of the elasticity, which causes the luminous particles to be reflected from the surface of polished bodies, as an ivory ball rebounds from a marble table, making the angle of reflection equal to the angle of incidence. But this idea which first presents itself, and which was also first adopted, will not bear examination.

22. Without being acquainted with the absolute dimensions of the particles of light, it will be readily understood that they must be exceedingly small, so small that the most powerful microscopes cannot magnify them to a perceptible size; if it were otherwise, how could they pass, as they do, through large masses of glass, water, and other transparent substances, not only without any retardation, but, as we shall presently show, with an accelerated motion. And finally, when with their incredible velocity, they fall by millions every instant upon the delicate membranes of the eye, how happens it that the organ is not torn in pieces, and that we do not suffer a thousand pangs, unless it be true that the particles are so minute as to render their impulse almost insensible? Now I ask what proportion there can be between the particles of light and the inequalities which still remain upon bodies, polished by processes of art? And considered in relation to light, what difference is there between bodies polished and unpolished? For in polishing bodies we only rub them, as it were, with small and hard particles of dust, which indeed remove their greater inequalities, but leave them furrowed in every direction. But these particles of dust, which we can recognise with a microscope and even discover with the naked eye, are vast masses, and the furrows which they leave upon bodies are of immense depth, compared with the particles of

light. If light, therefore, coming in actual contact with the surface of bodies, were reflected by the mere force of elasticity, the little particles of which it is composed would be dispersed in every direction by the elevations, or lost in the deep cavities, of these bodies; and reflection from the most carefully polished would be hardly more perfect than that from the most rough. But, since reflection from polished bodies is much more abundant, more perfect and regular, we infer that it is not the mere mechanical effect of elasticity, and that the particles of light do not come in actual contact with the polished surface.

23. The force by which the rays are repelled, acts therefore at a distance from the solid surface. It acts, moreover, in general, unequally upon the different particles of the same ray. For in most cases, in which reflection takes place, one part of the incident light is reflected and another transmitted, either because the repulsive force is variable in its action, being at one moment more active and feeble than at another; or, as seems most probable, because all the luminous particles which follow each other successively in the same ray, are not, at the moment of their incidence, in the same physical state, and equally susceptible of being repelled.

24. As to the nature of the reflecting force, we are entirely unacquainted with it. We do not know whether it belongs to the particles of the reflecting body or to those of light; whether it acts by repulsion or attraction; and, considering only its general effects, we might represent it by numberless mechanical causes. But without attempting to determine its nature, we may always compare it to a repulsive force acting at the points of incidence, and tending to repel a certain number of the particles which compose the incident rays.

Let us suppose that the waving line *AB* represents the plane surface of a body covered with natural asperities, or those which art cannot remove, and let us imagine that all the points of this surface, or more generally, that all the particles of the two contiguous media which compose it, exert at a certain distance a repulsive force upon the luminous particles which approach it. This force must be very powerful at the distance where reflection takes place, since it is then sufficient to destroy the prodigious velocity with which the particles of light are impelled, and Fig. 21.

to turn them back in the opposite direction; but it must diminish rapidly as the distance increases. For, if it were otherwise, the direction of the reflected rays would be affected by the parts of the reflecting body, which are at a sensible distance from the point of incidence, and then this direction would depend upon the general form of the reflecting surface, whereas it is determined solely by the direction of the superficial element, which the ray strikes upon; and it really takes place as if all the rest of the reflecting surface did not exist. Moreover, if the thickness of the reflecting body be gradually reduced by grinding away the second surface *A′B′*, without altering the first, the regularity and amount of the reflection are not at all effected, at least till the body is brought to an extreme degree of thinness, scarcely attainable by art. Thus the particles situated at a greater depth than this limit, cannot extend their influence to the reflecting surface, or at least to the distance from it at which reflection takes place; and since their force, which is so great at very small distances, is diminished to such a degree as to become insensible at a little depth, it follows that it decreases as the distance varies with very great rapidity. This will, therefore, be one of the characters which we ought to recognise.

25. Let us now suppose that a beam of parallel rays of light Fig. 22. *SM*, *S′M′*, falls at any angle upon the reflecting surface *AB* of indefinite extent, and let us consider what takes place with respect to the luminous particles *M*, *M′*, when they are near enough to begin to feel the repulsive action of the particles of the body. If the surface is perfectly plane, as *AB*, or which is the same thing, if its inequalities are insensible compared with the distance to which the repulsive force extends, the activity of this force will be the same at every point of the surface, and consequently, the effect upon all the particles of light *M*, *M′*, whose directions, velocities, and dispositions are the same, will be equal. This is the case with polished bodies; but if the Fig. 23. reflecting surface is broken up with large elevations *E*, *E′*, *E″*, separated by deep cavities, *F*, *F′*, the reflecting force cannot be equal at all these points. It is evident, for example, that the luminous particles which enter the cavities will not be reflected in the same direction with those which fall upon the inclined sides of the elevations, nor those which fall upon the sides like those which fall upon the summits. It may even happen that

those which enter the cavities are not reflected from them at all, being driven downward by the repulsion from the elevations. Such a surface can only produce a weak and irregular reflection, like that which takes place at the surface of unpolished bodies; nor is it necessary that the inequalities of the surface should be so large as to be perceptible to the touch or to project sensible shadows from the one to the other, in order to produce such an effect; it is sufficient that they are of sensible dimensions, compared with the distance to which the reflecting force extends. Of this kind are the inequalities of plane glass which has not yet been polished. It is plane if we consider only the general direction of its surface; we can neither measure the height of its asperities, nor obtain points sufficiently small to be introduced into its cavities; but these inequalities are too great for light, and the oppositions resulting from them in the direction of the repelling force, weaken the general repulsion while it makes it irregular. To remedy this inconvenience we endeavour to remove, or at least to diminish, these asperities by rubbing the surface of the glass with some substance whose own asperities are easily overcome, as paper or taffeta, stretched and made smooth by friction; but we should effect the same purpose by diminishing the velocity of the luminous particles, which the repulsive force must overcome before reflection can take place. Now this we do in fact by rendering the direction of the rays more oblique to the reflecting surface, that is, by causing them to form a less angle with its direction. For, if we conceive the velocity of the incident particle of light to be decomposed into two others in rectangular directions, of which one is parallel to the reflecting surface, and the other perpendicular to it, it is evident that only the latter will require to be overcome by the repulsive force of the surface; and it is also evident, that this will diminish as the incident ray becomes more oblique. But besides this, the obliquity is favourable to reflection in another way; for the luminous particles penetrate less directly into the cavities at the surface of the reflecting body, and are more exposed to the action of the summits of its elevations, which form a surface sensibly plane (for to this state we suppose the body reduced,) and produce a uniform repulsive force throughout the whole extent of its superficies. Indeed, we find by experiment that reflection is nearly the same from polished and unpolished glass, when the luminous ray has a great inclination to the surface. In performing this experiment we may make use of the flame of a taper. If the glass is perfectly polished, the specular reflection takes place under all the different angles of incidence; but if it is imperfectly polished the reflection is small, and altogether insensible when the rays fall perpendicularly; but it increases as they become more oblique, and at length becomes as strong as from a polished surface. Finally, if the surface has been simply made smooth but not polished, no regular reflection takes place as we proceed from a perpendicular incidence till we arrive at a certain degree of obliquity, when we begin to perceive a proper image, but very faint and indistinct. The reflection gradually becomes stronger and stronger, and at length it takes place as perfectly as from glass of the highest polish.

26. This explanation of the conditions of reflection, depending on the destruction of the part of the velocity perpendicular to the surface, is attended with a difficulty which it may be well in this place to consider. For this perpendicular velocity being nothing in all those luminous particles which are parallel to the surface of the reflecting body, it would seem that in this case all the incident light ought to be reflected; whereas it is known by experiment that, on the contrary, a great part of it is attracted by the body, and refracted, if the body is transparent, or absorbed, if it is opaque. To reconcile these seemingly contradictory results, we must advert to a property of light, already mentioned, which will hereafter be rigorously demonstrated; it is this; all the luminous particles which compose a ray of light, although moving with equal velocities, are not equally disposed to suffer reflection; for they experience in their physical properties certain regular periodical fits or intermissions, which, like transient magnetism, dispose them at one time to be attracted, and at another to be repelled, by the surfaces of bodies. Now what we have said respecting the effect of repulsion, applies only to those particles which are in the latter state, and which actually suffer reflection. For the remainder either escape entirely the action of the repulsive forces, in virtue of their disposition at the moment of incidence, or they are made to deviate in two contrary directions before and after their passage through the refracting surface; for their actual direction in the medium which they penetrate, is not affected, as we shall also prove.

27. Confining ourselves, therefore, to the consideration of those particles only which actually suffer reflection, let us endeavour to determine geometrically the general circumstances of reflection from the single condition of the existence of a repulsive force decreasing rapidly with the distance; and to simplify the question, let us suppose the reflecting surface plane. Let then *M* be the position of a luminous particle which, moving in free space, approaches the surface *AB*, in the direction *SI*, and begins to be acted upon by its repulsive force; let us represent the proper velocity of this particle by *IM*, and decompose it into two other velocities, one *IN*, perpendicular to *AB*, the reflecting surface, and other *MN* parallel to it. Since this surface is supposed to be perfectly plane, it is evident that the resultant of the repulsive force which it exerts, will be always perpendicular to its direction, and consequently perpendicular to *MN*; for, on account of the small distance at which this force is sensible, we may suppose, as in the theory of capillary attraction, that it proceeds from a plane of an indefinite extent, and then there is no reason why it should be inclined one way more than another; it will, therefore be perpendicular to the plane, and there will be no exception to this, unless it be at the extreme edges where the surface terminates; these form an infinitely small extent which must be considered separately. Now, since for all the rest of the surface the action of the repulsive force is perpendicular to *MN*, it follows that during the whole passage of the luminous particles, the portion *MN* of the velocity parallel to the surface, will sustain neither increase nor diminution, but remain precisely the same. But it will be otherwise with the part *IN*, which is perpendicular to the surface. This part will be directly opposed to the repulsive force, and will be resisted by it freely at first, when it is beginning to come within its action; but afterward with increasing energy, till the whole motion of the luminous particle in this direction is destroyed. When this has taken place, the particle can go no further, and the repulsive force acting upon it directly, will turn it in the opposite direction and gradually restore to it all the velocity of which it had before deprived it; till finally, having passed without the sphere of this action, the luminous particle will fly off in a straight line with its recovered velocity. Fig. 24.

28. Thus from the instant in which the luminous particle begins to be resisted by the repulsive force till the instant when it becomes parallel to the reflecting surface, it is constantly acted upon by two forces, of which one *MN* is constant and parallel to the surface, while the other, perpendicular to this same surface, is equal at any moment to the excess of *IN* over the intensity of the repulsive force at the point where the luminous particle is found at that moment. If this force suffers no fits or intermissions in its action, the velocity *IN* will be constantly retarded. Then according to the principles of mechanics, the luminous particle in its descent will describe the first branch of a curve, which will have for a tangent the primitive direction of the particle, and will terminate at the point *s* where the perpendicular velocity is wholly destroyed. But if the repulsive force has intermissions in its action that render it more feeble at less distances, which seems not improbable, the luminous particle will describe, not a regular curve, but a waving line, represented in figure 25, till its motion finally becomes parallel to the surface in *s*, where it is acted upon by the constant force *MN*. From this point, the particle, being always repelled, will begin to recede with a velocity regularly or periodically accelerated; and being acted upon by the constant force *MN*, parallel to the surface, it will describe the second branch of a curve convex or waving towards the surface, like the first; but always symmetrical with it, since the forces which urge the particle are the same at equal distances from the surface on each side of the point *s*. From the symmetry of the two branches of the curve, the last tangent $R'T'$ will make with the reflecting surface the same angle as the first; and as the distance at which the repulsive force begins and ceases to be sensible is extremely small, the curvilineal portion of the path, which is included within the same limits, must also be exceedingly small; therefore, reflection will seem to take place directly at the point *s*, where the directions of the incident and reflected rays intersect each other. Fig. 25.

29. In what precedes, we have considered the reflecting body as in void space, and have thus referred reflection solely to the repulsive force of the particles of the body. But if we suppose it surrounded with air or water, or any other material medium, the particles of this medium must necessarily exert upon the light an analogous action. To determine what will then hap-

pen, let us consider a luminous particle, arrived at the first medium, and at a certain distance from their common surface. We can always imagine the action (whether repulsive or attractive) of the particles of the second medium to be decomposed into two parts, *M* and *M′* — *M*, one of which *M* is equal to the action of the particles of the first medium at the same distance, and the other *M′* — *M* is the excess of the action of the second medium over that of the first. Now if the second medium possessed only the force *M*, the effect would be the same as if there were no change of medium, and the luminous particle being equally attracted or repelled in every direction, would move on without any change of velocity. This is the case with homogeneous media, as water and glass, within which no reflection takes place. The effect of the first part *M*, therefore, is always destroyed, and there remains to be considered only *M′* — *M*, the excess of the action of the second medium, which falls under the case first examined.

30. As to the greater or less value of this difference, it is found not to depend on the density alone; and we shall soon be made acquainted with media, which act with greater energy than others of the same or greater density. All this is agreeable to the general phenomena; for if the action of bodies upon light, and of light upon bodies, is analogous to chemical affinity, it is natural to suppose that it depends on the chemical nature, or even the form of the particles.

After what we have now said, it will be readily perceived that we may form artificial heterogeneous media, in the interior of which, nevertheless, no reflection shall take place. This is done, for example, at least very nearly, when two pieces of glass are made to cohere by means of a lamina of thick oil of turpentine, interposed between them; for if the junction be accurate, the surface of separation of the two glasses is not perceptible, and no reflection is produced. The oil of turpentine acts, therefore, in this case like the glass itself; and it is on this account immaterial whether the contiguous surfaces of the glass be polished or unpolished. In the latter case, the liquid fills all the cavities of the glasses, and supplies the place of the particles which are wanting, thus giving to the glass a more perfect polish than it could receive from art. We have another example of this property, by throwing some small irregular pieces of borax into olive oil; for these pieces, on account of their inequalities and the roughness of their surface, do not transmit the light regularly when placed in the air; but when immersed in olive oil, the oil fills up their inequalities and they become transparent; and so little reflection takes place at the common surface of the two substances that the limits of their separation are scarcely distinguishable.

It will be readily understood, also, that a body transparent in itself, may be made less transparent and even opaque, by separating its particles from each other, and introducing between them a medium, which acts very differently upon the light. This takes place, for example, when transparent liquids are made to foam by the introduction of air; for there is no doubt that the action of air and of these liquids upon the light is very different, since when their surface is covered with air a very brilliant reflection is produced. Now the air being introduced between the particles of the liquid, as many successive reflections take place as there are bubbles, or breaks in the continuity of the liquid; and all the incident light being thus turned back or dispersed among the particles of the body, this body becomes opaque; but its transparency will be restored by restoring the continuity of its parts, and this is the case with froth when by losing its air it returns to the state of water. The same phenomenon occurs in a porous stone called *hydrophane*, which is perfectly opaque when dry, but becomes translucent on being saturated with water, because its action upon light approaches more nearly to that of water than to that of air. Hence we see that transparency and opacity are not qualities which belong to the matter of bodies, but which depend solely on the arrangement of their particles. This will be still more strikingly confirmed, when we examine by experiment the manner in which the repulsive force begins and increases with the thickness of the body, to the limit at which it ceases to be affected by the addition of new laminæ.

DIOPTRICS.

General Laws of Simple Refraction.

31. Having examined the phenomena attending that portion of the incident light which is reflected at the first surface of bodies, we shall now consider the portion which penetrates them.

This, when the incidence is oblique, instead of continuing on in a straight line, deviates from its first direction; and the phenomenon is called *refraction.*

In all bodies, not crystallized, the refracted ray remains simple, and continues on in the plane of incidence. As to the extent of the deviation, it depends on the difference between the nature and density of the medium which the light leaves and those of the medium it enters.

If the two media are of the same nature and of equal density, the ray suffers no refraction and continues on in a straight line. If they are of the same nature but of different densities, the luminous ray on entering the more dense, is attracted towards the perpendicular to their common surface. Finally, if both the nature and density of the media are different, these two elements conspire in the result, and the ray approaches the perpendicular in that medium which exerts the strongest action upon the light. These facts we shall establish by experiment.

When a piece of money *M* is placed at the bottom of a vessel *AB*, of which the sides are opaque, it is invisible except to the Fig. 26. eye situated within the cone of direct rays *RR′*, proceeding from it, and bounded at *A* and *B* by the edges of the vessel. But if the vessel be filled with some liquid, the piece of money becomes visible in a much more open cone, such, for example, as *OSO′*, although the cone of rays proceeding from M is the same as before. These rays, therefore, must be bent outward from the vessel on entering the air; and consequently they are removed farther from AN, the perpendicular to the common surface of the liquid and the air, remaining always in the vertical plane which contains the incident ray AM and the perpendicular AN.

We may take another example no less familiar. If a straight g. 27. stick TT' is plunged obliquely into tranquil water of which the surface is AB, the stick appears to be broken at the point I where it touches the surface, and the prolonged part, although comprehended in the same vertical plane with the part which is without, seems to approach nearer to a horizontal position. To explain this phenomenon, let us suppose the eye situated at the point T, that is, at the upper extremity of the stick. If the rays from the liquid came to it in a straight line, the other extremity T' would be seen in TI produced, and not where it really appears, at T''. Now we have already remarked that we see objects in the direction of the rays which they send to the eye; since then we see the point T'' above its true place, it follows that the ray $T'I'$, which renders it visible to us, passes above $T'I$, and follows the broken direction $T'I'T$, Consequently, if we draw, through the point of incidence I', the perpendicular $NI'N'$ to the surface BB, we shall see that the luminous ray $T'I$ on passing out of the water into the air, diverges from the perpendicular as in the last example; but it still remains in the vertical plane which contains the angle of incidence.

The deviation would take place in a contrary direction, if the ray passed out of air into water, in which case it would approach the perpendicular. To prove this, take a tub of a rectangular form, the sides of which are of glass; and let $ABCD$ represent a g. 28. horizontal section. Then having filled it with water, by means of a heliostat make a ray of horizontal light SI fall obliquely upon the side AB. Closing the window-shutter in order to darken the room, it will be easy to find the direction of the refracted ray IR. For we have only to move along the side opposite to the point of incidence CD, a small circle of paper or rough glass till it intercepts the emerging ray. The point R being thus found, and the line RI being drawn to the point of incidence, this line will be the direction of the refracted ray; and comparing it to the incident ray SI, we shall see that refraction has made it approach the perpendicular *NIN'* to the surface of incidence *AB*.

The phenomenon being thus established, it is important to determine the ratio which exists, for every angle of incidence, between the inclination of the incident ray to the perpendicular, and that of the refracted ray ; so that, knowing the one, we may calculate directly the other. For this purpose we fix accurately the points *S* and *R* in the incident and refracted rays, and measure their distances *SN*, *RN'*, from the common perpendicular *NSN'* ; and we measure also the parts *IN*, *IN'*, of this perpendicular, which are intercepted by the perpendiculars *SN*, *RN*. Having constructed the two right-angled triangles *SIN*, *RIN'*, we shall know the angle *SIN*, formed by the incident ray with the perpendicular, which, as in reflection, is called the *angle of incidence*. We shall have likewise the angle *RIN'*, formed in the interior of the liquid, by the prolongation of the same perpendicular, with the refracted ray *IR*. This is called the *angle of refraction*. We recognise the two following laws discovered by Descartes ; (1.) *The incident and refracted rays are always comprehended in the same plane, perpendicular to the common surface of the two media ;* (2.) *The sine of the angle of incidence has to the sine of the angle of refraction a constant ratio for all angles of incidence, when the media are the same.* This ratio is called *the ratio of refraction.*

This beautiful law is the fundamental principle of all dioptrics. Indeed, when the direction of the incident ray is given, as well as the position of the refracting surface, we may always deduce the direction of the refracted ray, either immediately, if the surface is plane, or if it is curved, by considering the incidence as taking place upon the tangent plane. Afterwards, if the form of the refracting medium is given, we may follow the refracted ray to the interior, and determine the point where it will meet the opposite surface, as well as the angle it will form with the tangent plane ; hence we may calculate the angle of emergence, and the direction of the emergent ray.

32. The importance of this principle requires, therefore, that we should endeavour to establish it with the utmost possible exactness ; we shall presently point out the method to be pursued. We shall first, however, take notice of a remarkable phenomenon which always accompanies refraction.

This phenomenon consists in a dilatation of the refracted ray in the plane of refraction, and its dispersion through an angular space, the vertex of which is at the point of incidence. This angle is then filled with rays of various colours; for if we place within it a piece of white pasteboard or of ground glass so as to intercept all the refracted light, we shall see painted upon it an oblong spectrum, in which may be distinguished all the colours of the rainbow, and arranged in the same order, the violet and red being outermost, the yellow and green in the middle. The violet rays suffer the greatest refraction, the red rays the least, the green rays less than the violet and more than the red. For the sake of brevity, I shall designate these rays by the colours they give to bodies. But it is evident that they are not in themselves either violet or green or red, and that these names express only the sensations they produce in us.

This phenomenon is called the *dispersion of light;* it is more sensible in the same medium in proportion as the angle of refraction is greater; and in different media, at the same angle of incidence, in proportion as the refractive power is greater. No experiments can be made on refraction without producing it, and it is for this reason that it is noticed here. But, we shall not at present, examine the phenomenon more in detail; we shall confine ourselves in what follows, to considering the refraction of the yellow or green rays, which are nearly at a middle point between the others.

33. It ought also to be mentioned that there exist substances in which light is not refracted in a single beam, but in two separate and distinct beams, each having its proper dispersion. This takes place in all bodies regularly crystallized whose primitive form cannot be geometrically reduced to a cube. One only of these follows the law of Descartes. The cause of the other is subject to a much more complicated law, which Huygen's discovered from a large class of crystals, and which has been found to apply to crystals in general. This law we shall investigate in its proper place; but at present we shall confine ourselves to the consideration of the first kind of refraction which takes place in all bodies, and is called *ordinary refraction.* The other kind, which is called *extraordinary refraction*, takes place only in certain bodies.

Exact Determination of the Ratio of Refraction in Solid Substances.

34. The manner in which we just now measured the ratio of refraction and recognised its constancy, can only be considered as an approximation, intended to indicate the general law of the phenomena. This ratio must now be exactly determined. For this purpose, the most simple method is, to construct a right, triangular prism of the transparent substance which we wish to observe, then to measure the deviations of the luminous ray in passing through it under different angles of incidence, and to see if they may all be calculated according to a constant ratio of refraction.

Let *ABC* be a section made in the prism by a plane perpendicular to its edge. Fig. 30. In this plane let us imagine a luminous ray *SI*, falling upon the prism at the point *S*, and refracted in the direction *II'*. By the first law of refraction, the two straight lines *SI*, *II*, are situated in the same plane perpendicular to the refracting surface; they lie therefore in the plane *ABC*. The refracted ray, after passing through the substance of the prism, will meet the second surface in *I'*, and as it passes into the air will be refracted anew in the direction *I'O*, comprehended also in the plane of the section *ABC*. Then an observer, situated any where in this direction, as at *O*, would receive at the same time the refracted ray *I'O* and the direct ray *OS*, coming immediately from the luminous object. If light suffered no deviation in passing through the prism, these two rays would be confounded in one. Their departure from each other, *SOI*, is therefore caused by the refraction of the first; and in any given position of the luminous object, the prism, and the observer, the deviation *SOI'* will depend directly on the law of refraction. To prove this law, as stated by Descartes, we have only to employ it in calculation, and then to compare the results with observation. We may in fact thus determine the angle of emergence *CI'O*, when we know the angle of incidence *BIS*, and the ratio of refraction; and reciprocally, we may calculate the ratio of refraction when we know these two angles. The calculation of this ratio, therefore, for different angles of incidence, will prove if it be indeed constant, as stated by Descartes; and this is in fact found to be the case with the utmost precision.

35. This truth being once established, a single measurement of the deviation *I′OS*, produced by a prism of a given angle, under a known angle of incidence, enables us to calculate the ratio of refraction of the substance of which the prism is composed. This measurement may be made in several ways. For example, if the substance be solid, we have only to form a prism of it, whose angles are measured by the reflection of light, then fixing its base on a support capable of being levelled with screws, we place it in relation to the distant signal *S*, as represented in figure 30, and then placing the instrument divided in *O*, so as to see successively with the same glass the object *S*, first directly, and then through the prism, we measure the angle of deviation *SOS′*. We also determine by observation the angles of incidence and emergence *SIB*, *OI′C*; with these data and a knowledge of the refracting angle of the prism, we can calculate the ratio of refraction.

We may arrive at the same result without the distant signal, by using the circular goniometer employed to measure the angles of crystals. For this purpose we must apply the prism to the glass *GG*, figure 31, using all the adjustments and verifications which we have pointed out to make the edge of the two faces coincide with the central axis of the circle; and observing, moreover, that the central glass used as a support must be thin and its two faces perfectly parallel. Then having placed the index *S* in such a situation that the light which enters through it shall be refracted by the prism, we move the other index *O*, till the eye placed behind the opening *O′*, perceives the image of *S′* by refraction. When this takes place, the division of the circle will give the measure of the angles which the incident and refracted rays *S′C′O′C′*, form with the two surfaces of the prism of which the position and inclination are known. With these data, we determine by calculation the ratio of refraction for the substance of which the prism is composed.

If the light of the sky, admitted through the hole in the index *S*, be too feeble to give, after refraction, a distinct image of this hole at *O*, we may invert the direction of the rays by placing the flame of a taper, lamp, or any bright light, beneath *O*, and then placing the eye at *S′*, behind the other index which we move till the rays of refracted light pass through it.

We have recommended that the two faces of the central glass used as a support, should be perfectly parallel; otherwise, the prismatic form of the glass will cause a deviation in the refracted light, which we should falsely ascribe to the substance which we were examining. But if they are parallel, the ray will resume the same direction on passing into the air out of the glass which it had at its incidence, or it will penetrate a prism placed upon the glass in the same direction as if nothing were interposed. These facts we infer from the constancy of the ratio of refraction, and they are deduced from the calculus and confirmed by experiment.

It is also necessary that the glass should be thin, and that refraction should take place near the edge of the prism, that the incident and refracted rays may be considered as departing exactly from the centre of the circular division. If extreme exactness be required, it is easy to correct any little defect of centring. I have confined myself here to pointing out this mode of measuring the angles of refraction, because we have already made use of it in reflection.

In these experiments, the dispersion of light which always takes place, produces an infinite number of shades, among which may be distinguished seven stronger and more definite than the others, viz. red, orange, yellow, green, blue, indigo, and violet. Since these colours are separated in the emergent ray, it is evident that the different parts of this ray have different degrees of refrangibility, which may be estimated by the extent of their deviations. Thus we find the refrangibility least in the red rays, and that it goes on increasing in the order of the colours as we have named them, till it becomes greatest in the violet. We find, moreover, by varying the angles of incidence, that the ratio of the sine of incidence to the sine of the refraction, is constant for each of the different colours, though not the same for all; but it is difficult to determine this by the simple observation of the absolute deviations, because we are never sure of directing the eye in our different experiments to the same shade. For this reason I merely state in this place the constancy of the ratio of refraction as a thing very probable, and shall hereafter endeavour to establish it rigorously by other means.

Nevertheless the simple knowledge of the unequal refrangibility of the different colours, will furnish us now with a very important remark respecting the manner in which they ought to appear when we look at a luminous point through a refracting prism. Let *SI* be an infinitely thin white ray, proceeding from a point *S* at an infinite distance, and refracted by the prism *ABC*. After its emergence, it will be divided into a beam *VI′R*, of which the most refracted side *VI′* is violet, and the least refracted *RI′* red, the other shades being distributed between these two. But if the eye of the observer be situated somewhere as at *O*, in the red ray produced, it is plain that it will receive no other of the coloured rays contained in the branch *VI′R*. Now if through *O* we draw a line *O i′*, parallel to *VI′*, it is plain that the observer will receive in this direction a violet ray from the white ray *S i*, which falls upon the prism parallel to *SI′*, proceeding also from the point *S*; and with respect to the first refracted beam *SI*, the observer receives only the red ray *I′R*; so of the last, he will receive only the violet *O i′*. But other incident rays between *SI* and *S i* will send to the eye the intermediate shades, and it will thus perceive all the colours of the spectrum, as if the whole refracted beam *RI′V* had been received upon a piece of white pasteboard. The most refrangible rays will always be those which appear to deviate most from their primitive direction, and consequently those most distant from the base *BC* of the prism; this property will regulate the distribution of the colours with respect to the observer, and reciprocally, the order of the colours will indicate their greater or less refrangibility. Fig. 32

This remark, although very simple, it is highly essential to retain; for it will be of use in a great variety of cases, where we are obliged to infer from the order of the colours, this greater or less refraction.

Determination of the Ratio of Refraction in Liquids.

36. THE method which we have now explained might be equally applied to liquids if they could be formed into prisms. Now this is easily done by confining them within prismatic vessels of which the sides are formed of plane parallel plates of glass; for the definite directions of the rays not being changed

by their passage through these plates, the refraction which takes place, and which we observe, is produced entirely by the liquid. Indeed, such vessels while empty do not sensibly affect the images of objects, at least when the glasses are well made, and the luminous point at a great distance compared with their thickness.

But to adjust these glasses to each other, and to form of them a vessel capable of containing liquids, they must be attached or luted together. If screws are used to confine them, it is difficult to prevent the liquid from escaping. If they are luted, the lute is often attacked by the liquid, and the refraction thus affected. The inconvenience is still greater if we wish to observe volatile liquids, as ammonia, the essential oils, and most of the acids. Happily all these difficulties may be avoided by a very simple method invented by M. Cauchoix and myself.

37. We begin with taking a square plate of glass, about ⅖ of an inch in thickness, and two inches in breadth. It is of little importance whether the glass be pure or impure, opaque or transparent. The plate is perforated at its centre by a cylindrical canal of about ⅕ of an inch in diameter; it is finally cut into a prism, as represented in figure 33, and the two sides are carefully reduced to plane surfaces and polished. Two plates of glass, also made plane and polished, being then laid upon these faces with a slight pressure, they adhere of themselves, by the effect of the attraction at a small distance, which is denominated capillary. We thus form a true hollow prism without lutes, in which we can enclose all kinds of liquids without their undergoing any alteration.

To introduce them with ease, and without being obliged every time to remove the glasses, we form in the thickness of the prism a lateral canal *a b*, which is closed with a glass stopple, made smooth with emery; and finally that the glass plates may adhere more perfectly, and not slide on the solid faces of the prism by the motions which we are obliged to give it, they are confined by triangular frames of copper which are slightly pressed against their surfaces by means of screws.

38. Supposing that the glass plates employed are perfectly parallel, it is evident that this apparatus offers us a true prism for liquids, through which the refraction of these substances may be observed as if they were solid, and with accurate results. The Fig. 31. prism is placed upon the goniometer, or upon any instrument

which serves to measure refraction, and we observe the deviation of the luminous rays as in a solid prism, regarding only those which pass through the cavity in which the liquid is enclosed.

Determination of the Ratio of Refraction in Aeriform Substances.

39. The refraction of the gases is observed in the same way as that of liquids, by introducing them into prismatic vessels, the faces of which are closed by parallel plates of glass; but there are few particular modifications depending on the constitution of these substances.

The gases have much less density than solids or liquids, and their refraction is much less at the same angle. To render it sensible, therefore, we are obliged to increase considerably the refracting angle of the prism in which they are confined. Borda had one constructed with an angle of 145° 7′ 28″, with a large cylindrical, hollow tube of glass, the two ends of which were cut into a prismatic form, and closed by glasses with parallel faces, carefully luted. The tube was pierced at bottom, and provided with a cock, capable of being attached either to an air-pump or to a receiver, by which means a vacuum might be produced within the prism, and the gases under examination introduced. We have before said, that for the same substance the refraction is changed by a change of density; but the density of gases varies rapidly with a change of temperature or pressure. To be able to compare the results of different experiments, we must take into account these two elements.

40. To measure the pressure, we attach to the prism a vertical tube *TV*, communicating with its interior, and enclosing a syphon barometer the open branch of which is long enough to permit the mercury to rise to a level, when a vacuum has been produced in the prism. The height at which the mercury of this barometer is supported by the gas within, determines the pressure. To ascertain the temperature, we might insert into the prism a small thermometer; but it would be necessary to place it in the middle of its capacity, which would intercept the light; on this account it is better to suspend two very sensible thermometers without the prism, and very near it, or even in

Fig. 34.

contact with its faces. The temperature of these faces, as indicated by the thermometers, may be taken without sensible error for that of the gas and air which touch them within and without; for we know how very easily gases acquire the temperature of surrounding objects. We take every precaution, moreover, that the temperature of the place where our experiments are performed, may vary but little during the experiment, and especially that it may vary very slowly.

This prism is then mounted upon a foot perpendicular to its length, by which it is fixed in a horizontal position. The place of observation and the object which we look at should be chosen in such a manner that this object may lie in the horizontal plane passing through the centre of the prism. We then observe the deviation with a repeating circle, the limb of which is also placed in the same plane, at first by approximation, and afterwards exactly, by the condition that the upper telescope, being turned from the direct object to the refracted image, they shall both be found to be on the same horizontal wire, stretched in the interior of the tube. To verify this horizontal position of the wire, it is well that the signal be placed in one of the faces of some large building which presents in its construction long level lines, by one of which we may be governed. Then the best of all sights is a vertical lightening rod which throws a dark line upon the vault of the heavens.

41. Here as well as in the case of solids and liquids, the method of observation consists always in directing the upper telescope of the circle alternately toward the direct object and the refracted image, in order to measure the angle of deviation. But as the deviation for gaseous substances is always extremely small, even with the large prism here supposed, it is necessary, in order to obtain its value exactly, to multiply our observations, and to take the mean of the results, that opposite errors may balance each other. This is done by a repeating process, founded principally on turning the prism from right to left, and from left to right alternately, so as to admit of our observing the deviation successively with the same telescope in these two positions, as is represented in figure 35.

We naturally attempt first to measure the refraction of atmospheric air. In this case we exhaust the air from the prism by means of the air-pump. This operation does not produce an absolute vacuum; but when the density of the interior air is very much reduced, so as to support the barometrical column at the height of only a small part of an inch, this height is observed, and account taken of it in our calculations. We have, therefore, a prism void, or nearly void, of air, immersed in the surrounding atmosphere; the luminous rays must consequently, in penetrating it, suffer a deviation, determined by the excess of the refracting power of the exterior air, and this in fact takes place. If the upper telescope of the circle is first directed immediately to the object through the air, when we afterwards come to interpose the prism, the deviation is considerable; this is the effect of the refraction of the air. If the telescope be brought again to the object, by moving the limb, and the prism be then turned half round, the deviation is doubled, and the object twice as much displaced. For example, in our experiments, the prism was placed in one of the chambers of the Luxembourg facing the observatory, whose lightening rods were the points of sight. The turning of the prism carried the wire of the telescope from one side of the building to the other; or, to speak more exactly, the telescope remaining immoveable, the edifice seemed to move to the right and to the left of the wire the whole of this distance. Yet we could perceive no sensible dispersion, though undoubtedly there was one produced; but it was too small to be perceptible.

42. If we would observe the refraction of the air at different densities, the process is the same; we only exhaust the air to the proposed limit, which is indicated by the interior barometer.

When we wish to observe other gases, we must first exhaust the air from the prism as far as possible; observe the density of what remains, and then introduce the gas. This introduction is effected by means of a pneumato-chemical bath of water, or of mercury, if the gas is liable to be dissolved in water. It is necessary that the prism, and the vessel containing the gas, should be connected by a double stop-cock, as in the weighing of gases, in order to avoid the water-bubbles which might make their way into the neck of the instrument.

If we wished to obtain a dry vacuum in the prism, or dry gases, we should place in the glass tube which surmounts it a quantity of caustic potash to absorb the humidity. When this and similar substances act in a void, the absorption is almost instantaneous; but in the air or in gases, a certain time is necessary for the vapours to be precipitated and to combine with the alkali. If, on the contrary, we wished to observe the refraction of aqueous vapours, it would be necessary to employ every means to moisten the air in the place where we make our observations, by sprinkling water, suspending wet cloths, and especially by raising the temperature; but we must avoid introducing these vapours into the prism, for being deposited upon its faces they would affect the passage of the light.

In all that we have said, we have supposed that the glasses which form the faces of the prism have their two surfaces exactly parallel. When great care is used in their construction this is perhaps nearly true, but it is highly improbable that the condition is ever rigorously fulfilled. But as the refraction of glass is very powerful, while that of the air is feeble, it is easy to see that an error of this kind must very much affect our results. To determine its effect, we open the stop-cock of the prism or even detach the glass tube which surmounts it, in order to give free access to the external air. We then observe the deviation in these circumstances, as we should with the prism void or filled with gas. If the surfaces of the glasses are exactly parallel, the object will not be removed from its place by turning the prism, since the interior and exterior air of the prism will be exactly homogeneous and of equal density; but if we observe any deviation, it will necessarily be produced by a defect of parallelism; and this quantity must be added, with its proper sign, to each of our other observations; for it is with this as with all very small quantities, of which the partial effects are only to be added to each other to obtain the total effect.

Having now explained every thing which concerns the arrangement of the apparatus and the manner of making the observations, it remains only to determine the ratios of refraction of the air and of gases. This is a simple subject of calculation.†

---

† For the necessary formulas see Biot's *Traité de Physique.*

Of Spherical Lenses.

43. The methods which we have employed to calculate the deviations of the luminous rays in their passage through prisms terminated by plane faces, may be applied to the general case in which the refracting medium is terminated by any curved surfaces whatever. For here, as in reflection, we may compare the luminous rays to mathematical straight lines, whose refraction for each point of the surface takes place in exactly the same way as it would do in the tangent plane. It is sufficient, therefore, to calculate the positions of this plane for each point of incidence, in order to determine the deviation of the luminous ray; and this calculation is always possible when the form of the surface is given.

In the common applications of optics it is not necessary to make our calculations so general; for in our experiments we make use of spherical glasses only, since there are no other forms which can be exactly and easily executed; it is sufficient, therefore, to analyse and calculate the refractions which these produce. To do this with all possible simplicity, and to comprehend the results indicated by analysis, we must first take a general view of the sort of glasses in question and make ourselves acquainted with their principal properties.

If we imagine a straight line, or *axis*, drawn through the centres of the two spherical surfaces which terminate such a glass, and that a cutting plane then passes through this axis, we shall have the *profile* of this glass, which, according to the directions of the curvatures that may be given to the two faces, will necessarily have one of the forms represented in figures 36, 37, 38, 39, 40, 41. These different forms are distinguished by the following names which are generally adopted.

Fig. 36. (1.) The double-convex glass. This glass is called a *lens*, (the Latin word for a lentil, which it resembles in shape,) and the name has been extended to all the other spherical glasses.

Fig. 37. (2.) The plano-convex lens. We speak of the concavity or convexity, always, in relation to objects situated without the glass.

(3.) The concavo-convex. The difference of these two forms is, that the first is thinner at the edge than at the centre; and the second, on the contrary, is thicker at the edge than at the centre. We shall soon see the peculiarities which result from this difference of construction. Fig. 38, 39

(4.) The plano-concave. Fig. 40.

(5.) The double-concave. Fig. 41.

All these forms of lenses agree in this, that the planes tangent to the two spherical surfaces which terminate them, are at first parallel to each other at the points A_1, A_2, where the lens is pierced by its axis; from this point to the edge of the glass, the angle of the two tangent planes goes on increasing more and more, and symmetrically on each side of the axis. A luminous ray, which passes through such a glass, is refracted precisely as it would be in passing through a prism formed by the two planes tangent to the points of incidence and emergence. A spherical lens of any form whatever, may be considered therefore as an assemblage of such prisms, or as a prism of a variable aperture, the refracting angle of which, being nothing at the axis A_1A_2 of the lens, afterwards goes on increasing to its edges.

Accordingly all the forms of spherical glasses which we have been describing, may be divided into two classes, according as the base or the point of the refracting prism is turned toward the axis A_1A_2, of the lens. The first class will comprehend the figures 36, 37, 38; the second the figures 39, 40, 41.

It is easy to understand the influence of this different arrangement of the prisms upon the course of the luminous rays. For if we suppose a beam of incident rays parallel to each other and to the axis A_1A_2 of the lenses, it is evident that all those of the first class will refract these rays toward the axis A_1, A_2, while those of the second class, on the contrary, will turn them from it. Thus the first will cause the incident light to converge, and the latter will make it diverge; these two kinds of lenses are hence called *converging* and *diverging glasses.*

44. Let us examine more particularly the manner in which these phenomena are produced, and let us begin with the first kind of lens, of which we have a general representation in figure 42. Among the rays which compose the incident beam parallel to the axis A_1A_2, there is one SA which coincides with this axis itself. This passes through the lens at the points where

the two surfaces which terminate it are parallel. Moreover, its incidence and its emergence take place perpendicularly to these two surfaces. It therefore suffers no deviation, but passes on in its original direction SA_1A_2F. But it is not the same with the incident rays situated at a little distance from the axis. These suffer a very slight refraction on account of the small refracting angle of the prism through which they pass. They will, therefore, cut the first ray somewhere as in F. As the incident rays depart from the axis, their deviation increases, and they will cut each other successively in $F_1, F_2, \ldots\ldots$, and the assemblage of all these intersections, supposed, when taken two and two, to be infinitely near to each other, forms in general two branches of a curve which begin at the point F, where the rays nearest the axis cut each other, and terminate at the point F_2 in the last ray which passes through the edge of the lens. These curves are named *caustics*. But when the surfaces of the lens include but a few degrees of the spheres upon which they were formed, we find by experiment that the greater part of the rays meet at F, rather than in any other place, so that the curve FF_1F_2 is almost entirely concentrated in that, to which we give the name of the *principal focus*. The distance of this focus is sensibly the same for every lens, whatever be the distance of the faces which are presented to the incident rays.

ig. 43. 45. Reasoning in the same way with respect to diverging lenses, of which the general form is given in figure 43, it will be seen that they must form two branches of a curve FF_1F_2, also symmetrical above and below the axis; but the *principal focus* F of the rays near the axis falls on the same side of the lens with the incident rays; and there is no real concentration of light at this point, nor at any other point of the curve of intersection. This curve then indicates only the imaginary place of meeting of the emergent rays produced.

In all the figures which we have yet examined, the lenses are represented as perfectly symmetrical about the axis A_1A_2, so that this axis contains also the centre of figure of their exterior surface. In this case, the glass is said to be *exactly centred*; and this is a very important condition in all optical experiments, as we immediately perceive. When it is not fulfilled, the thickness of the lens at its edge is necessarily unequal, as appears in ig. 44. figure 44, in which A_1A_2 is really the common axis of the two spherical surfaces while B_1, B_2 is the apparent axis drawn through the centres of the two circles which form the exterior contour of the glass.

Hence it follows that converging lenses are necessarily centred when they are sharp at the edges; for their thickness at these edges being nothing, it will evidently increase uniformly on every side toward the centre of the figure, where the two surfaces will be parallel. When we have learned how to determine by experiment the position of the foci, it will be seen that we may make use of this determination with much accuracy to verify the centring in every kind of lens.

46. After what we have said above concerning the formation of caustics, it will be readily perceived, that in lenses as well as in mirrors, the concentration of the rays will be so much the more perfect as they pass nearer to the axis of the lens. Hence in optical instruments, we are often obliged to cover the edges of the lenses and a portion of their surfaces with opaque circular rings, which are called *diaphragms*. The luminous rays fall then only on the circular and central portion of the lens which has not been covered. The diameter of this remaining portion is called *the aperture of the glass.*

In optical experiments which require much accuracy, this aperture is made very small compared with the radii of curvature of the lens, and no rays are admitted but such as are very little inclined to the axis which joins the centres of its surfaces; these are the only means of obtaining distinct and well defined images. It hence results, that both in their incidence and their emergence, the luminous rays meet the surface of the lens almost perpendicularly; which reduces the deviations they experience, and greatly facilitates the calculations by which we determine them.

47. To fix the circumstances of the passage of the rays and the formation of the image geometrically, let us first consider a single radiant point S, placed before the first surface of a spherical lens. Through this point and the axis of the lens, draw a plane cutting the glass in the direction of one of its profiles A_1A_2MM. It is always to be understood that the rays proceeding from the point S are, during their whole course, very little inclined to the axis A_1A_2X, and that their points of incidence and emergence I_1, I_2 are very near this axis in com- Fig. 45

parison with the radii of the two spheres to which the surfaces of the glass correspond.

48. If several spherical lenses are placed on the same axis with the first, and the beam of light from the point S passes successively through them all, it is evident that such of the rays as are comprehended in the plane represented in the figure will continue in that plane, since it is perpendicular to the surfaces through which they pass. But the rays which vary from this plane either above or below, will pass successively into different planes of incidence and refraction, which, it would seem, must render their course very difficult to be calculated. Fortunately, this calculation is unnecessary when the angles of incidence and emergence are small, as we must always suppose them in optical instruments; for these rays are collected into very nearly the same foci with the others, so that it is sufficient to follow out the first rays in order to determine the place where the image of each radiant point is formed. Hence we have only to consider the course of the rays comprehended in the plane drawn through the radiant point and the common axis of all the lenses.

Here, as in the case of spherical mirrors, all our results may be deduced from the principal focal distance, and the method is the same. It is this distance, therefore, which we must first determine; and this is easily done when we know the radii of the two surfaces of the lens, and the ratio of refraction which belongs to its substance. The focal distance is equal to the product of these two radii, divided by their difference, and by the ratio of refraction diminished by unity. This supposes the curvatures turned in the same direction; if they are in a contrary direction, we must take the sum of the radii, instead of their difference. Fig. 38, 39.

Fig. 36, 41.

This being laid down, we may easily find the focus of any radiant point situated either in or without the axis. For, let S be this point and MAA_1M the profile of the lens, which I represent by a straight line to indicate that it is supposed to be very thin. Through the point S draw next the incident ray SI parallel to the axis AA_1X; this ray after the two refractions, will pass into the principal focus F, so that IF will be the direction of the emergent ray thence resulting. Draw now another incident ray SA, directed to the centre of figure of the lens; this will pass through the glass without deviation, since Fig. 46, 47.

the thickness being supposed infinitely small, and the two surfaces at *AA*, parallel, the lens has at this point the effect of an infinitely thin plane glass. It only remains, therefore, to produce *SA* in a straight line, till it cuts the first emergent ray in *f*; the point *f* will be the common focus of these two rays, and it will be the focus also of all those which proceed from the same radiant point *S*. Figure 46 represents the effect of this construction for the converging lens, and figure 47 for the diverging lens. Expressing our operations algebraically, we obtain a general formula for determining the length of the focal distance *Af*, and the position of the focus, for all possible curvatures of the surfaces, and all situations of the radiant point. Hence it is easy to determine the images of objects which have finite dimensions, for we have only to apply the same construction to all the cones of rays proceeding from the different points which compose it. We shall thus find the foci of these several cones, and they will together form the image of the object.*

---

* As the formula indicated in the text is one that is very often used, I shall give an explanation of it here, making use of figure 47. We designate by Δ the distance *AS* of the radiant point from the centre of the lens, supposed to be infinitely thin. We denote also by Δ' the distance of this centre from the focus *f*, supposed on the same side with *S*. We call r the ray of the anterior surface of the lens situated on the same side with the radiant point *S*, and r' the ray of the second surface, both surfaces being considered as having their concavities turned towards *S*; and lastly, we designate by n the ratio of refraction for the kind of glass of which the lens is made; the distance Δ' will always be connected with Δ by the following relation,

$$\frac{1}{\Delta'} = \frac{1}{\Delta} + (n-1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

In this formula the rays r, r', are considered as positive when the surfaces to which they belong are concave towards the radiant point. If, on the other hand, one surface is convex towards this point, we must give its radius the negative sign. Also the distances Δ, Δ', are considered positive when they are situated on the side of the radiant point, as in figure 47. Therefore, if one of them, Δ' for instance becomes negative by the disposition of the values of Δ, r, r', this wi signify that the focus *f* is formed on the opposite side of the len consequently beyond it, as in figure 46, and not on this side, a

Determination of the Images produced by Diverging Lenses. Use of these Lenses in correcting shortsightedness.

49. We shall first apply the method to diverging lenses. Let MAM be such a lens, of which A is the centre of figure; and place the object SS′ before its surface, at any distance whatever, provided it be such that the angles of incidence shall not exceed the limits supposed in our approximations. If from the extremity S of the object, we draw the line SA to the centre of figure of the lens, the cone of incident rays proceeding from the point S, will have for its axis SA, and its focus will be found somewhere in this straight line, and on the same side of the lens (since it is diverging), for example, in *f*. The focus will be found in the same way in S′A as at *f*′; and these two foci comprehending between them all the others, *ff*′ will be the image of the object. It will always be erect and smaller than the object, since it is comprehended between the sides of the angle SAS′, and nearer its vertex A. Moreover, the absolute value of its distance from the lens will always be less than the principal focal distance AF, and so much less according as the object itself is nearer the glass. Fig. 48.

According to this construction, when the luminous rays, proceeding from the same point S or S′ of the object, have passed through the lens, their course is exactly the same as if they had set out from the corresponding point *f* or *f*′ of the image. There-

---

in figure 47. This always happens, for instance, in converging lenses, when Δ is infinite, which renders $\frac{1}{\Delta}$ nothing. For then, according to figures 36, 37, 38, the ray r of the anterior surface becomes negative; and whether the other lens be convex or concave, $\frac{1}{r} - \frac{1}{r'}$, will remain a negative quantity. We shall then have

$$\Delta' = -\frac{r r'}{(n-1)(r-r')};$$

this is precisely the rule enunciated in the text for the calculation of the principal focal distance AF.

fore, if a spectator had his eye situated at OO on the other side of the lens so as to receive all these rays or only a part of them, he would see neither the points S, S', nor the points between them, but their image ff'; and his eye would be affected in the same way as if the object had really become smaller, and had been transferred to the place where the foci are formed. He will thus see this imaginary object erect, diminished and brought nearer. But although these may be in fact the only elements of the sensation produced in the eye, yet we do not estimate correctly the distance and magnitude of the image, because our judgment is affected at the same time by other considerations, altogether independent of the direction of the luminous rays.

To convince ourselves of this singular fact, we may take any diverging glass, for example, that nearest the eye in an opera glass, which is usually double concave; and look through it at objects which we will suppose to be very distant compared with the focal distance of the glass. When the eye is placed at a proper distance from the posterior surface, we shall see a very distinct image of these objects. It will appear erect like them and smaller, but instead of supposing it near the glass and in the focus ff', where it is really formed, it will seem to us more distant than the object itself. This is because the sensation of the visual angle, and that of the greater or less divergency of the luminous rays which reach us, are not the only elements from which we estimate distances. We add to these, without being sensible of it, the impressions which we may have of the absolute dimensions of objects. A man seen successively at the distances 20, 40, and 60 yards, appears always of the same absolute magnitude. Nevertheless the rays of light which render him visible at these different distances, cross each other on entering the eye under very different angles, since they are to each other nearly as the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$; so that were we to judge only from the openings of these angles, the apparent magnitudes would seem to us to decrease in the same ratio.

50. This habit of connecting the idea of absolute magnitude with the sensation of the visual angle in judging of the distance of objects, is derived from the experience of our whole life, and it becomes as rapid as sensation itself; or rather, the sensation which is transmitted to the mind when we look at an external object, is the compound result of these different elements. But

the involuntary application which we make of them, deceives us when we look through a diverging lens; for the objects which we had just viewed with the naked eye, of the absolute magnitude and distance of which we were consequently able to form a pretty accurate judgment, being suddenly presented to the eye with much smaller dimensions, we do not hence conclude simply that their images are smaller, but that they are further removed from us; and no reasoning can prevent this conclusion being formed even when we know theoretically that it is false.

With the exception of this case, in which our senses are but poor judges, observation perfectly confirms the results indicated by theory; this we may verify not only when the object is at a great distance, as we have just now supposed, but also when it is gradually brought near. If it be brought, however, very near the lens, it must be very small, and the refracting surfaces must be held nearly perpendicular to the rays which proceed from it; otherwise we should exceed the limits of incidence and emergence supposed in our approximations.

In performing these experiments, we find that in order to see the image distinctly, we must place the eye at a certain distance from the lens, and this distance varies with different eyes. If the eye be brought nearer, the image becomes larger and confused. If, on the contrary, it be removed, the image becomes smaller and more difficult to be seen distinctly. This results from the fact, that the eye is itself an optical instrument which cannot concentrate the rays with sufficient exactness for perfect vision, except when they fall upon its surface within certain limits of incidence.

Suppose, for example, that the luminous point S, forms its Fig. 49. image in F, and that this image appears distinct when the eye is placed at OO; then vision is produced by a cone of rays FOO which has for its base the surface OO of the pupil, and for its vertex the point F. If the eye be brought nearer, suppose to $O'O'$, then the pupil intercepts a more diverging cone of rays, and consequently the rays FO' which form the exterior surface of this cone, fall upon the eye with a greater angle of incidence. If this angle become so great that the eye cannot concentrate all the rays upon its retina, vision is necessarily confused; and this in fact takes place when we bring the eye too near the lens, and consequently too near the focus F, the common centre of the emerging rays. If, on the contrary, after having found the point where vision is most perfect, we remove the eye further from the lens, the image, which remains always in the same place, is at a greater distance from the eye. It ought, therefore, to appear smaller and less distinct in its outline, like other objects when removed from us; and this actually happens.

51. The least distance at which objects are seen differs with different eyes. Those persons who are *shortsighted* are obliged to approach the focus *F*; those who are *longsighted*, on the contrary, find it necessary to go to a greater distance from it. The reason of this will be readily understood when we remember that this focus, from which the rays diverge, is to the eye as it were an object situated in *F*; and thus each one, in order to see it distinctly, is obliged to place his eye at the distance at which he would see a common object most distinctly. This distance is commonly from eight to ten inches, when we wish to distinguish the minute parts of small objects; but some persons cannot distinguish objects placed at so small a distance; and, on the contrary, there are others with respect to whom this distance is only two or three inches. The latter are called *myopes*, or *shortsighted* persons, and the former *presbytes* or *longsighted.*

52. It is not to be understood that distinct vision is, in every case, strictly confined to a particular distance. On the contrary, the eye is endued with a power by which it is enabled to adapt itself, within certain limits, to the different distances of objects. But beyond these the image is confused, and vision becomes imperfect. Thus persons possessing the longest sight, cease to distinguish the minute parts of objects when placed at a considerable distance. But these parts disappear to shortsighted persons, even when the objects are very much nearer.

The defect of shortsightedness may be corrected by placing a diverging lens between the object and the eye, as represented in figure 49. For by means of such a lens, there is substituted instead of the real object the image formed at its focus, and we have only to give to the lens a focal distance equal to that of distinct vision of the organ for which it is designed; then by bringing it near the eye, the shortsighted person will see distant objects as distinctly as if they were situated near him, although he refers them to their true places. But he cannot use the same glasses, at least, when brought close to the eye, for the purpose of seeing very near objects; because the foci of the rays which proceed from them being formed still nearer the glass, the image of the objects would be too near the eye to be seen distinctly without fatiguing it. It is necessary, therefore, to employ lenses of a greater focal distance; or, which is better, to dispense with them entirely, since it is these near objects which shortsighted persons perceive most distinctly with the naked eye, and to reserve the use of diverging glasses for distant objects. The principal focal distance ought, moreover, to exceed a little that at which small objects are seen most distinctly; for the eye would be soon fatigued by the too near approach of the image. Such is the purpose and operation of those *spectacles* by which shortsighted persons remedy the imperfection of their sight.

It is plain that the glasses in question would be useless and worse than useless to the longsighted who cannot perceive near objects distinctly, on account of their being too close to the eye; for, the foci of diverging lenses being always nearer than the object itself, the evil would only be increased. We must, in this case, therefore, seek some means of removing the image beyond the object which produces it; and this purpose is effected by means of converging glasses, as we shall soon see when we have studied their properties.

Method of determining the Images produced by Converging Lenses. Use of these Lenses in correcting the defective Vision of longsighted Persons.

Fig. 50. 53. Let MAM be a lens which we now suppose to be converging; and let SS' be the object placed beyond the focal distance of parallel rays. Through its upper extremity S draw the ray SA to the centre of the lens. This ray will be the axis of the pencil of rays proceeding from the point S; and the lens, being supposed to be very thin, the focus of this pencil will be situated somewhere beyond the lens with respect to the point S, as at f, for example, in SA produced, as appears from our general construction. All the emerging rays which come from the point S will meet in this point from which they will pass on and diverge as if from a real object situated in the same place. If we repeat the same construction for the other extremity S', we shall find its focus f' in the axis $S'A$ produced; and in the same way we obtain all the foci of the intermediate points; whence we have ff' an inverted image of the object. The image is inverted because the foci are formed beyond the point where the axes of the pencils cross each other.

This image will become sensible if it is received upon a screen of white pasteboard or ground glass placed in ff'; it may also be seen immediately, if the eye be situated beyond this point, at the proper distance for seeing distinctly a real object occupying the same place.

54. If the luminous object SS', be situated at a very great distance, the image will fall on the opposite side of the lens, near the principal focus F. This furnishes us with the means of determining experimentally the focal distance of converging glasses. As the object approaches the lens, the image recedes from it increasing at the same time in magnitude. When the object is removed only by a quantity double the principal focal distance, Fig. 51 the image will be of the same magnitude; if it be brought nearer, the image will be removed, the dimensions being increased; finally, when it arrives at the principal focus the image will be Fig. 52 removed to an infinite distance. This result might be easily foreseen, since the object and image may always be made to change places; if the object, placed at an infinite distance, has its image in the principal focus; reciprocally, when the object is placed in the focus, the image will be at an infinite distance. Between these two limits, the image is always inverted.

The object, being brought nearer continually, will fall at length between the principal focus and the surface of the lens. Then Fig. 53 the image, according to our general construction, passes to the same side of the lens. It is now larger than the object, more distant and erect; as the object approaches the lens, the image approaches also, the size being diminished; till finally the image and object unite and coincide throughout upon the surface.

55. These are the results of theory, and they are completely confirmed by experiment. Take, for example, any converging lens, as the object lens of an opera-glass, and look through it at an object which we will at first suppose at a great distance relative to the focal distance of the lens. Then, plac- Fig. 50. ing the eye properly we shall see an inverted image of this

object, which, as has already been remarked, we can render sensible by receiving it upon a screen of pasteboard or ground glass. But although this image really falls on the same side of the lens with the observer, yet it is referred by him to the opposite side, and conceived to be at a greater or less distance than the object, according to the circumstances by which his judgment is influenced in other cases.

56. When we are thus placed at the point where the image is most distinctly seen, if we measure the distance of the eye from the lens, we shall find that it is equal to the focal distance of parallel rays, plus the ordinary distance of distinct vision for the eye of the observer. This is another proof that the eye is placed so as to view the image as if it were a real object; and it receives from the image similar impressions. This opinion is every way confirmed; for if the eye be removed further from the glass, the image appears diminished and its minute parts are less easily distinguished, like those of a real object which is removed from us. If, on the contrary, we approach the image, it becomes irregular and confused, like that of an object which is brought too near the eye. In this last case, the image appears to be increased in size, like an object seen very near; and we are led to believe that it is brought nearer. It finally becomes altogether confused and indistinct when the eye arrives at the focus itself. But, which is very remarkable, as we approach still nearer the glass, the image is formed again, erect and very much confused. Its direction is not again changed as the distance of the eye from the glass diminishes, but it becomes less indistinct, and at length we see the object tolerably well, with its natural outlines and dimensions, when the eye is at the surface of the glass; especially if we contract the aperture of the pupil by looking through a small hole made in a card.

In these last experiments the rays are convergent when they approach the eye; and since the image is still perceived, it is a proof that vision may also take place in this way, though with incomparably less distinctness than when it is produced by divergent rays.

57. But it may here be asked why the image appears erect and becomes less confused as the eye approaches nearer to the glass. To resolve these different questions clearly, let us first take the simple case where the radiating object is reduced to a

luminous point very far removed. This takes place, for example, when we look through a converging lens at Venus, or any very brilliant star.

In this case, if the eye is first placed in *OO*, beyond the focus Fig. 54 of parallel rays, and at a proper distance for distinct vision, we shall see a very clear well defined image of the star. This sensation is produced by a cone of divergent rays which has its vertex at *F*, and *OO*, the aperture of the pupil, for its base. As the eye approaches the lens, the base of this cone remaining always of the same magnitude, it intercepts a greater number of rays, and of rays which form among themselves a greater angle. On account of this divergency the eye is unable to collect them all into the same focus on the retina; and consequently, they form upon this membrane a small circular image, such as a luminous circle would produce placed without the eye. The image of the star is thus gradually enlarged, and forms a disc which increases in size as the eye approaches the focus. Finally, when the eye arrives at *O′O′*, that is, at the focus, this disc is equal in magnitude to the lens itself, because the aperture of the eye then admits all the rays which the lens has refracted. But as the eye approaches still nearer the glass a certain portion of the rays are lost; and those which first escape, being those which depart farthest from the axis of the lens, they are also the most convergent. Hence, when the eye is very near the glass, the only rays which can enter the pupil have very little convergence, and consequently the object begins to appear less confused. This being applied successively to all the points of an extended object, we perceive why it appears more distinct as the eye approaches the glass. The position is erect, since the luminous pencils do not cross each other before they enter the eye.

58. Having thus verified the results of theory for very distant objects, we shall now consider those which are placed at a less distance. In this case we also verify the other phenomena indicated by theory, and we find them the same, whichever of the faces of the lens is presented to the incident rays.

It will hence be readily understood what advantage is derived from the use of converging lenses by long-sighted persons who see near objects confusedly. For, if the object *SS′* be too near Fig. 5 the eye *OO* to allow the rays, proceeding from it, to come to a focus upon the retina, we have only to interpose the convex lens

MAM, so near the object as to bring it within the principal focal distance AF of parallel rays, and having such a convexity as to throw back the images of the points S, S', to the foci f, f', precisely at the distance fO, $f'O$, where the eye sees most distinctly. This is the way in which convex glasses enable long-sighted persons to see near objects distinctly. They are particularly important to aged persons, who are commonly long-sighted, and who are able by means of this useful invention to read, write, and in general to execute any kind of work which requires to be placed near the eye, as if the focus of vision was not affected by age. But they are obliged to dispense with these glasses in looking at distant objects, because they make the rays converge too fast, and thus occasion indistinctness of vision, as we have already shown.

Of Magnifying Glasses and Simple Microscopes.

59. The expedient above mentioned is useful also to watchmakers, engravers, and generally to those persons whose business requires much care directed to small objects. In this case, however, it is not to remedy defects of sight, but to magnify the objects,
56. and render the minute parts more perceptible. To understand this use of lenses, let OO be the eye, and SS' the object which we wish to see distinctly and in large dimensions. If the latter condition were the only one sought, it might be fulfilled by simply bringing the object nearer the eye, for the visual angle subtended increases as the distance diminishes. But the image would appear confused since the object is brought within the limits at which distinct vision naturally takes place. To remedy this inconvenience, we have only to place close to the eye a convex lens, and to bring the object within the focal distance of the glass precisely so far that its image shall be thrown back to the distance Of, Of', of distinct vision. This is always possible; for the distance of the image from the glass may vary from nothing to infinity, according to the distance of the object. When we have found by several trials this proper distance, the image ff' will be seen distinctly; and, moreover, the visual angle fAf',

which it subtends at the eye placed in contact with the glass, is equal to the angle *SAS′* under which the object would appear to the naked eye, if it could be seen distinctly at so small a distance as *AP*. We shall thus obtain the double advantage of seeing the object distinctly, and of seeing it under a larger angle. But from the very circumstance of never having seen it in this way, the judgment which we form of its real magnitude is not modified by any previous experience of the ratio of the distance to the visual angle; and as we see it under a much greater angle than it subtends to the naked eye, although at the same distance at which we endeavour to place it in order to see it distinctly, it must appear to be magnified in all its dimensions. And this is constantly found to be the case. Convex lenses fitted to produce the effect in question, are called *magnifiers*.

60. The magnifying power is evidently determined by the ratio of the absolute magnitudes of the image and object; and this ratio is the same, by the construction of the figures, as that of their distance from the glass, since they are both comprehended between the sides of the same angle of *fAf′*. Thus the enlargement takes place because the image is formed at a greater distance from the glass than the object, and on the same side of it. These two conditions cannot be fulfilled, except with converging lenses.

61. We have supposed the eye to be very near the surface of the glass. This position renders our considerations more simple; and affords a wider field, that is, it allows us to take in with the same glass, a larger extent of objects. But after what we have said, it is evident that this condition is not indispensable to the obtaining of magnified images. Indeed, whenever the image, although larger and more distant than the object, is too near the glass for distinct vision, by removing the eye we can always find the exact point where it may be clearly and distinctly seen.

Large magnifying glasses are sometimes employed in this way by aged persons placed at some distance from the eyes, which enable them to read. They are also used in examining the minute parts of maps. It is obvious that these glasses must be very large that both eyes may look through them at once. It is necessary, moreover, that the focal distance should be so great that the axes of the luminous pencils proceeding from the same object toward the two eyes, may not make too great an angle with the common axis of the two surfaces. The magnifying power produced by these glasses, necessarily varies with the distance at which they are held from the object to be examined, as appears from the preceding considerations; and their effect is the more perfect according as they are held near the object, since on the supposition that we look through the centre of the glass, the luminous pencils which enter the pupil of each eye, pass through the glass under small angles of incidence, agreeably to what has been taken for granted in all our approximations. But the condition of seeing with two eyes obliges us to incline the pencils to the axis of the lens more than if we used but one; and this circumstance, together with the facility with which we alter the magnifying power by varying the distance of the object from the lens and of the lens from the eye, makes this instrument liable to injure the sight. It is, therefore, at present rarely used, and we employ instead of it separate glasses for the two eyes.

62. We generally use but one eye with magnifiers and other optical instruments in which we wish to obtain a highly magnified image with the greatest possible distinctness. In this case experiment proves that the best position of the eye is very near the glass. It only remains, therefore, to place the object within the principal focus, at such a distance as to throw the image back to the point of distinct vision; and this is effected after a few trials. The calculus also furnishes rules for determining this position with respect to every lens, and the distance of distinct vision belonging to different eyes, while it teaches us also how to calculate the magnifying power thus obtained. To form a glass which shall magnify a certain number of times m, when applied to the eye, we must give it a focal distance equal to the distance of distinct vision divided by the magnifying power minus unity, that is, by $m - 1$. For example, if we take for the ordinary distance of distinct vision eight inches, which is the usual measure for good eyes, and if a glass is sought which shall magnify fifty times, its focal distance must be $\frac{8}{50-1} = 0{,}163$; if it were required to magnify 100 times, the focal distance must be $\frac{8}{100-1} = 0{,}0808$; which, supposing the two curvatures the

same, would require each surface to be of a less radius than the above. In general, the radii of the surfaces diminish as the magnifying power increases. This is an obstacle to carrying the magnifying power very far with a simple lens. It will be seen also that the focal distance would diminish if the distance of distinct vision were less than eight inches; and it is for this reason that the same glass magnifies less with respect to short-sighted persons, than when used by those who are long-sighted.

63. We thus see how far we can proceed with a single microscopic lens. But by combining these lenses with others of less curvature, we obtain much more powerful effects. By similar combinations of large glasses, or of mirrors and magnifiers, we are able to form arrangements which bring near and magnify the images of distant objects; and thus we obtain what are called *refracting and reflecting telescopes.* We shall make known the theory of these instruments when we have learned how to correct the effect of the decomposition of light in the case of simple lenses, which cause the images to be coloured, and is on this account the occasion of an essential defect.

64. A very simple means of procuring a microscope of considerable power is by making a small hole in a thin metallic plate, and introducing into it a drop of water. This drop arranging itself in the hole, as in a very narrow and very short capillary cylinder, forms at the two surfaces of the plate two spherical convexities, whose diameter is the same as that of the hole. Then the rays which pass through this globule are refracted by it as by a solid glass magnifier of a very short focus. Then placing the eye very near the hole and looking through it at small objects, situated on the other side of the plate, at a very small distance, we find a point where they may be seen distinctly and very much magnified.

65. The lenses obtained by this process are inconvenient, as they are continually changing their curvature, and are soon dissipated by evaporation; this renders them unfit for continued observations. Dr Brewster proposed to remedy the defect by using drops of transparent varnish on a plate of glass, which, by the mutual attraction of the particles, take a spherical shape. Very clear lenses are obtained in this way; but it is difficult to fix them in a metallic plate, and still more difficult to keep them unaltered. By a slight modification Mr Sivright of Edinburgh

has discovered a process that is free from all these inconveniences. It consists in placing small fragments of glass on perforations in a thin plate of platina, and then submitting them to the action of the blow-pipe. The glass thus melted, assumes a spherical shape like the drop of water or varnish; when this is once cold, it preserves its form, and presents a lens well mounted. Instead of employing a plate of platina, we can make use of a wire of the same metal rolled around a fragment of the glass, which serves the purpose of a ring to keep it in place. Platina is preferable to any other metal for this purpose, because it does not oxydate with the heat of the blowpipe, and it adheres very strongly to the sides of the small lens; its point of fusion also is much higher than that of glass, and it may be employed very thin, so that its contact shall less retard the fusion of the vitreous globule.

In these different applications it is supposed that the lenses used are capable of concentrating the parallel rays which fall upon them in a single focus. But this condition can only be fulfilled when the thickness and aperture of the lens are very small compared with the radius of curvature, and it is also, as we have seen above, the only case which our theoretical approximations embrace. It is manifest that in practice we must conform to these conditions in order to obtain distinct images; and experiment soon shows within what limits we are to confine ourselves. We may nevertheless, by a happy choice of curvatures, increase the extent of our glasses, and at the same time, diminish the aberrations of the focus. We may even destroy almost entirely these aberrations by combining several lenses; but the proper arrangement for producing this effect cannot be discovered without the aid of the calculus. It is sufficient in this place to observe that our arrangements tend generally to diminish the angles of incidence and emergence of the rays, at the different surfaces which they are successively to pass through.

Physical Theory of Refraction.

66. The methods above described give the ratio of refraction for any substance in the state in which it is observed. But it has already been remarked that this ratio, in any case, changes with a change of density. The variation is small in solids and liquids, since the dilatations and condensations to which these are exposed are inconsiderable; but it is quite different with respect to aeriform substances, in which every change of pressure and temperature is attended with a marked change of density. While we are ignorant of the relation which exists between the density of a substance and the refracting power it exerts upon light, we cannot compare together the observations made upon the same substance in different states, nor those made upon different substances; nor will it be possible to distinguish what belongs to their densities, from that which belongs to the chemical nature and arrangement of their particles. In order, therefore, to a more thorough acquaintance with these phenomena, it is necessary, by means of the observed motions of a ray of light, to discover the forces which act upon it, and thence to ascertain by calculation how these forces produce the particular results.

Let *AB* be the first surface of a body or of any refracting Fig. 57. medium, and let *SI* be a luminous ray traversing a void in a direction nearly parallel to the surface of the medium, but meeting it at the point *I*. Then this ray will no longer continue its direction in a straight line, but will be refracted in a certain direction *IR*, which will depend upon the ratio of refraction for the medium under consideration. If this were water, for example, the angle *BIR*, formed by the refracted ray and the refracting surface, would measure 41° 18′ 36″, and the incident ray would be turned thus far out of its course. Now, as a moving body can be made to change its course only by a force oblique to its own direction, we must conclude that the particles of light, in approaching the surface of the medium, are acted upon by forces which tend to make them enter it; and moreover, that these forces are directed perpendicularly to the surface; for the case of perpendicular incidence is the only one in which they do not change the direction of the ray.

It is evident that these forces must be sensible only at very small distances above and below the surface. For the luminous ray must begin to bend the moment it begins to be acted upon, and must take a rectilineal direction the moment they cease. Now the space occupied by this inflection is so small as to be incapable of being appreciated by our senses, and the ray appears to be broken suddenly at the point of incidence. The action of the refracting forces is not then sensible to a greater extent.

67. All these results concur to prove that the refraction of the luminous rays is produced by the affinity of the particles of the body for the particles of light; an affinity analogous to capillary attraction, and which, like that, becomes sensible only at very small distances. This conclusion, drawn from the phenomena above stated, appears at first view, to contradict that which we have deduced from experiments upon reflection. For, in that case, the luminous particles appeared to be repelled by the reflecting body, instead of being attracted as we now find them to be. But it must be remarked that the particles which are reflected may not be in the same physical state or in the same circumstances of motion, as those which are refracted. Now the possibility of this difference is sufficient to remove the apparent contradiction in the two opposite consequences, drawn from the phenomena exhibited in these two states of the particles. For when a body *A* acts upon a body *B*, in any manner whatever, this action does not depend simply upon the state and nature of *A*, but also upon those of *B*. We shall hereafter see this difference in the state of the luminous particles confirmed by abundant proofs, and we shall learn also in what it consists.

Let us now determine from these phenomena the conditions to which the attractive forces are subjected. For this purpose, let us suppose a luminous particle *M*, placed at any distance without a homogeneous refracting medium, and so modified as to escape the repulsive action of the reflecting forces. Then this particle will be sensible only to the attraction of the medium, which draws it perpendicularly to the surface *AB*, as before demonstrated. Moreover, it will be drawn with the same intensity, at the same distance, whatever part of the surface it approaches, whether it be at *M*, or at *m*, or at μ. For, as this kind of action is sensible ig. 58.

only at very small distances, provided the points M, m, μ, are not situated infinitely near the extremities A and B, of the medium, the luminous particles, supposed to be in these points, will be attracted with as great a force as if the medium were indefinately extended; and the intensities of these attractions must be equal throughout, since we suppose the medium to be homogeneous. Also, in order that we may have to consider, in the action of the medium, only the progressive variations depending upon the distance, let us suppose that it is not crystallized; in which case no account need be taken of the modifications of the attractive force, which might result from the figure of the particles, or their arrangement. Then supposing the distance $a\,A$ to be that at which the attraction of the medium begins to affect the luminous particles, the line $a\,b$, parallel to AB, will represent the exterior limit, at which the luminous ray begins to be bent. It is manifestly not necessary to regard this limit as rigorously and mathematically exact; for, mathematically speaking, the attractive action must extend indefinitely; but since, beyond a very small extent, it becomes so exceedingly feeble that its effect is insensible, we may express this circumstance graphically, by indicating near the surface a certain very small distance, as the limit at which the ray begins sensibly to bend.

Let us suppose now that the luminous particle M, having passed this limit approaches nearer the medium. It will then be more forcibly attracted, and the attraction will increase until the luminous particle reaches the surface of the body. But when the particle has traversed this surface, the attraction of the medium will begin to diminish, and will diminish progressively according as the particle penetrates into the interior of the medium. Suppose it to have arrived at a certain depth M', for example. In order to ascertain the relative intensity of the force which then attracts it, draw through the point M', the line $c'\,d'$ parallel to the surface AB; and below this line draw another parallel $A'B'$, at the same distance from M' with AB. Then the two equal portions of the medium, which are limited on one side by the line $c'\,d'$, and on the other by AB, $A'B'$, will attract the particle M' in contrary directions, and will attract it equally. These two parts, therefore, will counterbalance each other; so that the particle will be attracted only by that portion of the medium situated beyond $A'B'$. If the thickness of this remaining part exceeds the limit at which the attractive forces are sensible, the intensity of the attraction will be the same as when the particle was without the medium, and as far distant from it as it has now penetrated. For, by reason of the small distance at which the attraction is sensible, the portion of the medium which is cut off by the plane $A'B'$, is always infinitely thin; and the rest of the medium, since it exceeds the sphere of activity of the forces, may be considered as indefinite.

It results from this reasoning, that the limits of the interior and exterior attractions are equally distant from the surface. When the luminous particle has attained this depth, if we cut off from the medium the portions which counterbalance each other above and below, the rest of the medium is too far distant to produce any sensible attraction.

Resuming the results at which we have arrived, we perceive that the incident ray SI preserves its rectilineal direction till it arrives at the first limit $a\,b$, where the attractive forces begin to be sensible. From this point the action of the forces begins to bend the ray into a curve, to which its primitive direction is a tangent at I, and whose concavity is turned towards the interior of the medium, as shown in the figure. This curve continues to the interior limit $a'\,b'$, where the influence of the attractive forces again becomes insensible. Then the ray takes the rectilineal direction, derived from the refraction, and which is the prolongation of the last tangent to the curve it has described. The extent of this curve is too small to be appreciated by our senses, and it is necessary to suppose it enlarged in order to compare the direction of the incident with that of the refracted ray.

68. To do this completely we must be acquainted with the law, according to which the attractive force increases, as the luminous particles approach the refracting surface. Of this law we know nothing, except that it must produce a very rapid increase of velocity, and an equal increase at equal distances, through the whole extent of the surface. Happily these general data are sufficient to conduct us to very important results.

In order to represent the increase of attractive force in the most general manner, let us divide the space in which it is sensible into an infinite number of very thin zones by the lines $c\,d$, Fig. 59.

ef, *gh*, &c., parallel to the surface of the medium and to the first limit *ab*. Then let us suppose that in each of these zones the intensity of the attractive force is sensibly constant, so that it shall only increase in passing from one zone to another. Continue this construction into the interior of the medium as far as to the second limit, where the attractive force ceases to be sensible. This being done, if we do not establish absolutely any relation between the successive values of the force for these different zones, there can be no law so general that it may not thus be represented. The conformity will be the more perfect, the more we multiply the zones, and it will be entirely so, by supposing their number infinite. We can then employ this supposition for the purpose of representing the progress of the attractive forces; and if we thence deduce results which are independent of the number of zones, we may be certain that they belong also to the attractive forces themselves, whatever be the law by which they are governed.

The whole is then reduced to considering what takes place, when a particle infinitely small, projected into void space with a certain direction and velocity, traverses a zone comprehended between two parallel planes, where it is urged by a uniformly accelerating force. For if we resolve this problem for the first zone, we shall be able to calculate the direction which the luminous particle will have acquired there, together with its increase of velocity. We shall then proceed to the calculation for the second zone with these data, and thence to the third, and so on through the whole thickness in which the attractive forces are sensible.

This problem is precisely that of the motion of projectiles in void space, supposing them acted upon by gravity alone. By resolving it we find that for the same substance the ratio of the sine of incidence to the sine of refraction, is constant under all possible inclinations, as experiment has shown us. We find also, that this ratio is the same as that of the velocities of light, after it has penetrated the body to a sensible depth and before entering it. This initial velocity is always more feeble than the other, if the ray passes from a vacuum into a material substance, so that in this case it is accelerated by being refracted.

Finally, the analysis, by discovering all the particulars of the phenomenon, enables us, by experiment, to ascertain, if not the absolute value of the attractive force, at least a quantity proportional to it in each body. This quantity is the square of the ratio of refraction diminished by unity, and divided by the density of the refracting substance. Newton gave it the name of *refracting power*. Its estimation supposes that the luminous particles, in their passage through bodies, are acted upon only by forces peculiar to ponderable material particles, while such imponderable and intangible principles as those of electricity and caloric, do not contribute either directly or indirectly to produce or modify this action. The only means of judging whether this supposition is correct, is to compare the results derived from it with those deduced from observation.

69. Let us now consider what takes place when the luminous Fig. 60. particle approaches the surface of emergence A_2B_2, which, for the sake of simplicity, we will suppose parallel to the first surface. The attractive forces, exerted by the body near these two surfaces, will have the same extent of action. Let us then represent, by the parallels $a_2\,b_2$, $a_3\,b_3$, the interior and exterior limits where these forces cease to be sensible. This being done, while the luminous particle is approaching the interior limit $a_2\,b_2$, it will continue to move in a straight line with the velocity derived from refraction; but when it has passed this limit it will begin to be attracted again towards the interior of the refracting body, by forces varying with the successive zones through which it passes. In order to calculate the effects which these forces will produce upon it, it is only necessary to decompose its velocity into two others, the one parallel to the surface of emergence A_2B_2, and the other perpendicular to it. Now it is evident that the action of the attractive forces will not change the first velocity which is perpendicular to their direction; but it will tend constantly to diminish the normal velocity to which it is parallel and opposite, since this velocity tends to bring the luminous particle without the body, while the attractive forces have a contrary tendency.

Consequently, from the instant the action of these forces begins, the path of the luminous particle will be in a curve convex towards the surface of emergence; but on account of the similarity of position which we have supposed between this surface and the surface of entrance, the actions experienced by the particle will be exactly equal, so that in approaching the second, surface it will not lose what it had acquired in departing from the first. It will therefore reach A_2B_2, with the same velocity which it had at AB; and then, passing this limit, it will continue to be affected by the action of the medium ABA_2B_2, which will retard it, and will tend to draw it inward, but with an energy continually decreasing in proportion as its distance becomes greater; until at length this action entirely ceases to be sensible, when the particle has reached $a_3\,b_3$, the exterior limit of the attractive forces, after having gradually lost all the increments of velocity which it had acquired at its entrance into the refracting medium. Then the particle, being no longer influenced by the attraction of this medium, will continue to move with its primitive velocity, in the direction of a straight line, tangent to the last element of the curve it has described; and this tangent, on account of the parallelism of the two surfaces, will be parallel to the primitive direction of the particle before it entered the refracting medium.

70. Let us now consider the case where the surface of emergence A_2B_2, instead of being parallel to the first surface, is inclined to it at a certain angle. Then the limits of the action of the medium will still be near this surface, and we can likewise represent them by the straight lines $a_2\,b_2$, $a_3\,b_3$, drawn at the same distance as before. Moreover, the succession of actions exerted by the medium, at different distances from the surface of emergence, will also be the same. But the normal velocities which these actions will have to oppose will be different; for, upon repeating here the decomposition which furnishes these velocities, it will be seen that they become more feeble in proportion as the direction of the ray approaches the surface of emergence. If their value is still such that the attractive forces are not sufficient to destroy it, the luminous particle will traverse all the zones where these forces are sensible, will pass their exterior limit, and having regained all its primitive velocity, will move off in a straight line in such a manner that the sine of its emergence will be to the sine of its interior incidence in a constant ratio, and this will be the same that it has at the surface of entrance. But it is also possible that the luminous particle may reach the surface of emergence with such an inclination, that its normal velocity, which alone tends to make it emerge, will be entirely destroyed by the continued and increasing attraction of the medium, before it has passed this surface; or there may remain sufficient Fig. 61.

velocity to make it pass, and then this remainder may be destroyed by the retarding action of the medium before it has attained the exterior limit a_3 b_3, where this action ceases to be sensible. In both cases, the luminous particle, in proportion as its normal velocity is diminished, will yield to the action of the other component whose intensity does not suffer any diminution, so that its path will be bent more to the surface of emergence to which this component is parallel. The moment this component comes to act alone, the normal velocity being destroyed, it will cause the luminous particle to describe a small element of a straight line parallel to the surface of emergence; after which the attractive action of the medium, experiencing no obstruction, will draw the luminous particle inward by a course symmetrical with that by which it approached the surface of emergence, and will communicate to it in a contrary direction the same degrees of normal velocity which it had before destroyed, until the particle, having reached the interior limit a_2 b_2, where the attractive forces cease to be sensible, will continue its course in a straight line, with the same velocity it had before entering the strata of variable attraction, and in a symmetrical direction; so that the angles of reflection and incidence will be equal.

71. This analysis makes known the origin of the change from refraction to reflection, which is actually observed in certain circumstances at the second surface of bodies; and teaches us, moreover, that this change is not limited to a single incidence, but may be distinguished into three cases. 1. When the reflection takes place in the substance of the refracting medium between the interior limit of the attractive forces and the surface of emergence; 2. When it takes place at the surface of emergence; 3. When it takes place between this surface and the exterior limit of the attractive forces, the luminous particle emerging first from the refracting medium, and afterwards entering it. This last case is that in which the angle of emergence, calculated according to the ratio of the sines, becomes equal to 90°; it is also that which determines the interior incidence the nearest to the normal, under which the total reflection takes place. We shall see, hereafter, that these different limits become very manifest from observation, although the spaces in which they are comprehended are altogether insensible; insomuch that the ray appears

to be broken suddenly and at the same point in these different cases.

Hitherto we have considered the bodies subjected to experiment, as contiguous to a void; we shall now see what takes place when the light passes from one refracting medium into another. It is easy to combine these two phenomena; for it is sufficient to remark that in this case the luminous particle is attracted, not only by the medium which it leaves, but also by that which it is to enter.

Let *M* be the first medium, *M′* the second, and *A′B′* their common surface. While the particle is at *M*, so far from the surface *A′B′*, that the attraction exerted by the first medium is equal on all sides, and that of the second still insensible, it will move in a straight line with the constant velocity which the action of the first medium has impressed upon it. But when it has arrived within the limits of distance where the intensity of the attractive forces becomes variable, it will begin to experience simultaneously the action of the two media, drawing it in contrary directions; that of the medium it has left tending to retard it, and that of the medium to which it is approaching having a tendency to accelerate it. Accordingly it will be influenced only by the difference of these actions, which are both directed perpendicularly to the common surface *A′B′*. Hence we see that if the medium which it is about to enter, acts at an equal distance with greater force than the other, it will always enter, and after passing, in a curvilineal direction, beyond the interior limit $a_3\,b_3$ of the attractive forces, it will at length go on in a straight line with a constant velocity. But if, on the contrary, the first medium acts more strongly upon the ray than the second, we may consider the luminous particle as moving in a medium, whose action will be simply the difference of action of the two media, and consequently retarding, *A′B′* becoming the surface of emergence. Then we shall find all the cases of emergence and of interior reflection which we have already considered. Thus the incidence may be such that the particle shall be refracted and afterwards move in a straight line into the second medium; but it is likewise possible that the refraction may be changed into reflection; and here, as when the particle emerged into a vacuum, the reflection may take place, either in the interior of the first medium, between the interior limit $a_1\,b_1$ of the Fig. 62.

attractive forces and the common surface $A'B'$; or at the surface $A'B'$ itself; or lastly, in the interior of the second medium, even to the second limit a_3 b_3 of the attractive forces. But generally, when the ray passes from one medium to the other, whichever be the medium that acts most strongly upon the light, the ratio of the sine of incidence to the sine of refraction will be constant, and the reciprocal of the ratio of the velocities in the two media.

72. Let us now endeavour to realize these results by experiment, and see whether they are conformable to theory. In the first place, as to the constancy of the ratio of refraction and its value, we can easily verify them in the following manner. Having measured the deviation effected by a prism under a determinate incidence of the luminous rays, extend over one of its surfaces a stratum of some transparent substance, rendered throughout of the same thickness by being pressed with polished glass, or by being ground and polished. It will be found that the addition of this stratum to the prism does not alter the observed deviation, the incidence remaining the same; wherefore the ratio of refraction which the light experiences in penetrating the stratum, is exactly counterbalanced by that which takes place at the second surface where it touches the glass, so that it penetrates the glass in the same manner as if it had entered it directly. This is at once a consequence and a verification of the properties deduced theoretically.

Fig. 64.

In order now to observe the progressive effect of the incidence upon the interior reflection construct a prismatic vessel $AABB$, whose anterior and posterior faces, formed of glass, may be gradually inclined the one to the other, and remain exactly fitted to two parallel planes of copper, in such a manner as to form a prismatic vessel of a variable angle. Then, having filled this vessel with water or any other liquid, dispose vertically the anterior face BB, and direct perpendicularly upon it a fixed horizontal ray SI, obtained from a heliostat. The ray thus penetrating the liquid with a perpendicular incidence, will continue its course in a straight line till it reaches the second surface of the prism, where it will experience, in general, a partial reflection, which will carry a portion of it into the interior of the liquid, and thence out to O'' through the horizontal surface. The rest, experiencing the action of the refracting forces, will come out refracted in the direction $I'O$, if it is capable of emerg-

ing; and if not, this also will be drawn into the interior, making
the angle of reflection equal to the angle of incidence, and will
go to join in O'', the portion partially reflected. Then, in order
to follow out the phenomenon, render the second face AA verti-
cal, and having formed a liquid plate with parallel faces, the
ray will be seen to pass on without any deviation whatever, as
if there were no prism interposed. But as the angle is opened
a little the refraction will begin to take place, the emerging
ray being turned upward. This deviation will increase in pro-
portion as the angle is enlarged, which will bring the interior
incidence nearer to a coincidence with the second surface.
We shall at length arrive at a point in which the obliquity will Fig. 65.
be such, that the emergence will be hardly possible, the ray at Fig. 66.
its departure grazing the surface of the second glass. Beyond
this term the ray will no longer emerge, but will be entirely
reflected inward. This will take place when the sine of the inte-
rior incidence, reckoned from the normal, is equal to unity divid-
ed by the ratio of refraction. Then the total reflection will con-
tinue to take place inward, under every enlargement of the angle
of the prism, this enlargement bringing the interior incidence
always nearer to a coincidence with the second surface. These
details are strictly conformable to the results furnished by the
theory of the attractive forces. Fig. 65.

73. One circumstance must here be mentioned which will be useful hereafter; at the moment when the emergence is on the point of becoming impossible, it does not cease suddenly and at once for the whole ray; but the part of this ray which produces the sensation of red is the last to disappear. Such indeed must be the result, if, as we have already remarked, this portion is composed of rays less refrangible than the rest, and whose ratio of refraction consequently differs less from unity.

It may be asked how it happens that a certain number of particles, under all incidences undergo an interior reflection, even when their velocities, decomposed perpendicularly to the refracting surface, are sufficient to make them emerge. It arises from this, that beside the velocity of translation, which alone we have here considered, these particles are affected also by particular physical circumstances to be made known hereafter, which favor the action exerted upon them by the reflecting forces of the second surface; an action from which the other particles are free, being less fitted to yield to it.

74. When a prism of a variable angle, like that we have described is not at hand, the effect of interior reflection may be observed in a very simple manner. Place a triangular prism Fig. 67. ABC, between the light and the eye in such a manner that the incident rays SI, entering by the first face AB, may be reflected at the base BC, and then pass out through the posterior surface AC. If we would attend to all the particulars of this phenomenon, the eye must first be placed sufficiently high to allow the objects situated below the base BC to become visible by refraction through the angle BCA. In this position the rays SII', which come from objects situated above AB, are refracted also in I', and emerge into the air above the prism, with the exception always of the feeble portion which experiences in I' a partial reflection. But if the eye be lowered a little there will be found a position in which the objects placed below BC cease to be visible by refraction; and, on the contrary, the exterior objects, situated above AB, are represented on the base BC, as in a mirror. It is because the refracted rays II', becoming, in this case, too oblique to BC to be capable of emerging into the atmosphere, are all reflected inwards; and as no ray, coming from the objects situated under BC, can, upon being refracted by the prism, depart so far as these from the normal $I'N'$, none can any longer reach the eye. Thus, when this total reflection has once commenced, it will continue under all the greater obliquities.

75. In order to fix the exact limits where these objects disappear, consider $N'I'I''$ the greatest possible angle of refraction for the rays which enter the prism by the base BC; and draw the emergent ray $I''O$, resulting from the second refraction through the face AC. Then, if we draw through the vertex of the angle C, the line CO' parallel to $I''O$, it is manifest that no ray coming through the face BC, can enter the angle $O'CC'$, comprehended between this straight line and the prolongation of the base of the prism. Accordingly, while the eye is situated within this space all the objects below BC will be invisible by refraction.

76. It is to be remarked that the phenomena of total reflection can never be observed when we look through the same face of the prism at which the rays enter, as is shown in figures 68 and 69. For, if the prism be turned as in the first of these figures, the refracted ray II' can never be totally reflected at the second

surface *CB*; and if, on the other hand, the prism be turned as in figure 69, the reflected ray *I′I″* will cease to be capable of Fig. 69. escaping through the first surface *AB* before its total reflection at the second *AB* begins to takes place.

Let us recur now to the mode of observation employed above; Fig. 67. but instead of leaving the whole base *BC* in contact with the air, put a drop of water on one of its points *E*. Then, as the prism Fig. 70. is turned slowly about its axis, the eye being placed at *O*, so as to receive the reflected rays *SII′* coming from the clouds, it will be seen that the total reflection begins to take place on every part of the base *BC*, which is contiguous to the air, before taking place at the point *E* where the drop of water is placed; so that this remains some time visible through the thickness of the prism by means of the rays which emanate directly from it. As the eye is lowered towards the base *BC*, to render the rays more oblique, the total reflection gradually begins at *E*, and the drop disappears.

77. All these phenomena are the necessary consequences of he theory. When the refracted rays *IE* fall upon the base *BC* with an incidence nearly perpendicular, they do not experience a total inward reflection, even at the point of the base contiguous o the air. Reciprocally every ray coming with this incidence rom objects situated below the base of the prism, enter it and come to the eye. But when, upon lowering the eye, the refracted rays *IE* become more and more oblique to the base *BC*, we at length reach a point where the action of the air in contact with this base is not strong enough to transmit them; then, they will be totally reflected, after departing from the prism, provided they do not exceed the exterior limit of the attractive forces of glass and air. But the incidence at which this phenomenon takes place, is not the same for the point *E* where the drop adheres, because the attractive action of the water upon light is much stronger than that of air; which renders the normal forces, tending to make the luminous particles emerge, much more powerful. It is thus necessary to lower the eye more towards the base *BC*, in order that the interior reflection may take place at the point *E* where the drop is situated; and it is only when such an inclination is effected that the drop entirely disappears. At this instant there is a certain number of rays *SI*, coming from external objects, which pass through the drop at an infinitely small distance from the

base of the prism, and which being then drawn into the interior of the prism by refraction, reach the eye. When the drop is of a transparent liquid, these rays in their short passage through it are not sensibly weakened. Their brightness in this case prevents us from distinguishing the feeble light which is still sent to the eye by the infinitely thin liquid stratum embraced by the attractive forces. Accordingly the drop disappears.

The limit would manifestly be different if the substance applied to the base *BC* were opaque, even though its refracting force were otherwise the same as that of water. For then the rays *SI*, which must penetrate it in order to be reflected would be arrested by its opacity. In this case, we should begin to see the total reflection only at the moment when the refracted rays *IE* are reflected at the limit of the two media; which requires an interior incidence approaching more nearly to this surface. Although the difference of thickness by which this case of reflection is distinguished from the preceding, is infinitely small, or at least, incapable of being appreciated by our senses, yet its influence upon the angles of incidence and reflection is very perceptible. If, for example, the substance applied to the base of the prism be bees-wax, which can be rendered alternately transparent and opaque, by being subjected to the flame of a candle and then suffered to cool, it will be found that the total reflection, in this last case, will begin at an incidence at least eight degrees nearer the reflecting surface than in the other. Laplace was the first who pointed out this difference of limit required for interior reflection in transparent and opaque bodies. He has afforded the means of calculating the incidences in both cases, according to the theory of attractive forces; and Malus has realized these results in the experiment which we have described; adding at the same time the exact measure of the angles, which prove that the limits of incidence, given by observation, agree exactly with the theoretical ones. This agreement, and the constancy of the ratio of refraction, are two strong circumstances in favor of the hypothesis of the emission of light, especially when we consider by what secret links these two results are connected together.

73. We are indebted, moreover, to Malus for another experiment, which, without being susceptible of measurement, affords a striking example of the action of the attractive forces in the

phenomenon of total reflection. He placed upon the base of a prism of crown glass a drop of ink, which being at first transparent, permitted total reflection under certain angles of inclination. But gradually, as this drop increased in density by drying, greater degrees of obliquity were necessary in order to make it disappear; and at length becoming entirely opaque, it rendered the phenomenon impossible by reason of the great refracting force of the metallic particles which enter into its composition. If the glass of the prism had been of a kind possessing greater refracting power, perhaps the reflection would have been possible; and then the incidence at which it took place would have determined the ratio of refraction of the ink, though it had become impermeable to light. In general, it appears that we can determine, by this method, the ratio of refraction even of opaque bodies, when we can apply them to the second surface of a transparent prism, possessing a greater refracting power than the bodies themselves. But it is necessary carefully to calculate the total reflection as it begins to take place at the common surface of the body and the prism. This ingenious process was suggested and applied by Dr Wollaston; but it was Malus who first remarked the necessity of a different calculation for the results of experiment, according as the body applied is opaque or transparent; and this he did from the formulas for the two reflections given by Laplace. We hence derive a simple method of finding very nearly the ratio of refraction of a given liquid, when the quantity possessed is quite small. We have merely to apply a drop of it to the second surface of a prism, and observe if it disappears sooner or later than the drops of other liquids whose ratio of refraction is known.

79. Having thus explained the methods of determining by experiment the refracting powers of all transparent bodies, and also those of opaque bodies, when they can be brought into contact with a transparent body of a greater refracting power than themselves; it remains to bring together the results obtained in this way for different classes of substances, in order to see whether we can discover any relation between their refracting forces and their chemical composition. This is the purpose of the following table formed by Newton.

Refracting Substances.	Ratio of the sine of incidence to the sine of refraction for yellow light. n.	Values of the quantity. $n^2 - 1$.	Density of the refracting substance. ρ.	Its refracting power. $\frac{n^2 - 1}{\rho}$.*
False topaz (sulphate of barytes)	23 to 14	1,699	4,27	3979
Air	3201 to 3200	0,000625	0,0012	5208
Glass of Antimony	17 to 9	2,568	5,28	4864
Selenite (sulphate of lime)	61 to 41	1,213	2,252	5386
Common Glass	31 to 20	1,4025	2,58	5436
Rock crystal	25 to 16	1,445	2,65	5450
Iceland spar	5 to 3	1,778	2,72	6536
Common salt (muriate of soda)	17 to 11	1,388	2,143	6477
Alum (sulphate of potash)	35 to 24	1,1267	1,714	6570
Borax (borate of soda)	22 to 15	1,1511	1,714	6716
Nitre (nitrate of potash)	32 to 21	1,345	1,9	7079
Dautzick vitriol (sulphate of iron)	303 to 200	1,295	1,715	7551
Oil of vitriol (hydro-sulphuric acid)	10 to 7	1,041	1,7	6124
Rain water	529 to 396	0,7845	1,00	7845
Gum Arabic	31 to 21	1,179	1,375	8574
Alcohol	100 to 73	0,8765	0,866	10121
Camphor	3 to 2	1,25	0,996	12551
Olive Oil	22 to 15	1,1511	0,913	12607
Linseed Oil	40 to 27	1,1948	0,932	12819
Spirits of Turpentine	25 to 17	1,1626	0,874	13222
Amber	14 to 9	1,42	1,04	13654
Diamond	100 to 41	4,949	3,4	14556

From inspection of this table it appears that substances of very different densities may have equal refracting forces; and also that a substance of less density than another may yet possess a greater refracting power. Accordingly, as we before observed, the action of bodies upon light does not depend upon their density alone, but also upon the chemical nature of their particles. It appears, moreover, that the substances whose refracting power is greatest, are in general, the resins and oils; and as that of distilled water is little inferior to them, it is natural to conclude that there must be in water some inflammable principle analogous to that of which the resins and oils are composed. The same conclusion must be extended likewise to the diamond whose refracting power is still greater. These striking suggestions did not escape the sagacity of Newton; and he did not hesitate to make them known. For this great man, who practised the greatest severity in his experiments, and the most cautious reserve in his conjectures, never feared to follow out the consequences of a truth however far it might lead him.

---

* The numbers belonging to this last column have been multiplied by 10000 to avoid decimals. In the first column, the modern names are subjoined to those used by Newton. The great weight of the stone which he calls *false topaz*, leaves but little doubt that it is the sulphate of barytes.

If we would discover what this principle is which is common to the oils and resins, and which gives them so powerful an action upon light, we have no better means of doing it than by determining the refracting powers of gaseous substances; for, since almost all bodies with which we are at present concerned, are composed of gases combined together, we shall thus have the advantage of surveying them in their most general elements. This, as before mentioned, Arago and myself have done. In what manner the observations were made and what were their immediate results, we have already stated; it remains now to make known the refracting powers thence deduced.

Refracting Power of the Gases for the Temperature of 32°, and the pressure of 29,92, deduced from a collection of Observations.

Gases.	Density, that of atmospheric air being unity.	Value of $n^2 - 1$.	Refracting power, that of air being unity.
Atmospheric Air . .	1,00000	0,000589171	1,00000
Oxygen	1,10359	0,000560204	0,86161
Azote	0,96913	0,000590436	1,03408
Hydrogen	0,07321	0.000285315	6,61436
Ammonia	0,59669	0,000762349	2,16851
Carbonic Acid . . .	1,51961	0,000899573	1,00476
Carburetted Hydrogen	0,57072	0,000703669	2,09270
Hydrogen more carburetted than the preceding	0,58825	0,000630300	1,81860
Muriatic Acid Gas .	1,24740	0,000879066	1,19525

All the densities collected in this table are the results of our own experiments. The values of the refracting powers would be somewhat different, if we were to substitute the densities given by MM. Dulong and Berzelius.

From inspection of this table, it appears that the refracting power of hydrogen very much exceeds that of any other gas, and even that of any other substance hitherto observed. This element exists in great abundance in resins, oils, and gums, where it is united with carbon and oxygen; and is accordingly that which gives to these substances the great refracting power which Newton so justly remarked. This influence of hydrogen is eminently exhibited in ammonia, which is composed of hydrogen and azote. The refracting power of this gas is double that of air, and therefore, according to the first table, exceeds that of water.

But let us go further; and, since each substance seems to carry with it its peculiar character in all the combinations into which it enters, and to retain, to a certain extent, the force with which it acts upon light; let us endeavour to estimate under this view the influence of the constituent principles which belong to any given mixture or composition.

80. If we should attempt to discover these relations for any other substance but light, we should meet at once with insuperable difficulties, arising from the combination and degree of condensation of the constituent principles; for, although the chemical action is exerted only at very small distances, yet these distances admit of being compared together; so that the greater or less distance between the particles cannot fail to alter the intensity of the action. These variations, further modified by the figure of the particles, must render the ratios between the compounds and their elements, extremely complicated; and though we cannot estimate the effects thus produced, it is manifestly on account of them that the two have not the same properties. But, according to the most probable theory of light, the extreme minuteness of its particles, compared with the distances by which the particles of bodies are separated, must render the effect of small changes in these distances less sensible. Accordingly the refracting powers of bodies must differ little from those of their constituent elements, especially if these powers are to be ascribed simply to the action of the material, ponderable particles, and if their energy is not modified by the greater or less abundance of imponderable principles, united with them, which the act of combination almost always changes.

In order now to determine what must be the influence of each element, it must be remembered that the refracting power of a body is a quantity proportional to the sum of the attractive forces which it exerts upon light at different distances, and with a density equal to unity; this sum being reckoned from the instant the luminous particle begins to be sensibly attracted by the body, to the instant it arrives at the surface. If then we suppose that the proper action of each element is not altered in the combination, it will follow that at each distance, taken within these limits, the total attraction experienced by the luminous particle, will be equal to the sum of the attractions exerted upon it by the different elements composing the attracting body; and the part of the effect due to each element, will be proportional to the product of its proper refracting power by its mass, that is, by the ponderable quantity of this element which enters into the combination.

81. Let us, in the first place, try this law in those simple cases in which there is little or no condensation. Atmospheric air will afford a convenient example. It is known that this air when dry, contains 0,21 by bulk of oxygen gas; the rest is a mixture of azote, carbonic acid, and perhaps other gases in imperceptible proportions. For the sake of simplicity, we will take into consideration only the azote and carbonic acid; and will suppose 0,784 of the first, and 0,006 of the second, these quantities being always reckoned by bulk. I adopt these proportions, because they agree very nearly with the results obtained by analyzing atmospheric air, and because they satisfy the values of the densities determined by experiment. Now in order to find the ponderable quantities of each element which enters into the volume 1, each fraction of this volume must be multiplied by the density of the gas of which it is composed. We shall thus have

Oxygen . . . 0,210 × 1,10359 = 0,231754
Azote . . . . 0,784 × 0,96913 = 0,759798
Carbonic acid . 0,006 × 1,51901 = 0,009114

1,000666

This sum is nearly equal to unity; and that is what it ought to be, for it must express the specific gravity of the mixture, that is, of atmospheric air, which we have taken for unity. The error is like those that occur in observations of which no account is to be given. By neglecting it, the results which we have calculated will be precisely the ponderable quantities of the three elements which compose a mass of atmospheric air equal to unity. It remains then to multiply each of these by the refracting power which corresponds to its nature, and we find

Oxygen	0,199682
Azote	0,785693
Carbonic Acid	0,009157
Refracting power of the composition	0,994532

The sum of these numbers expresses the refracting power of atmospheric air, deduced from its constituent elements. To be perfectly exact, it ought to be equal to unity; the error is equal to 0,005468, or about five thousandths of the total value. It would not amount to three tenths of a second in the elevation of the pole at Paris; and this difference may arise from errors almost inevitable in making experiments; for the preceding result, depending on the specific gravity of the gases, their purity, and the refractions they produce, is connected with a great number of operations which are liable to error.

Accurate and numerous analyses of atmospheric air, made in different climates, and under circumstances the most various, have shown that its ponderable principles are throughout the same, and that they exist always in the same proportions. It follows from this, that the refracting power of atmospheric air is also the same throughout the earth, since it is determined by the partial refracting powers of its constituent elements. Accordingly, the tables of refractions calculated for one latitude, may be employed in all climates, regard being had to the variations of density produced by change of pressure and temperature. As to the difference which may arise from the moisture of the atmosphere, we shall show hereafter that it is sensibly nothing, and that it may be wholly neglected.

82. Let us now consider the cases in which the constituent elements are united by actual combination; and to proceed gradually, let us begin with affinities that are not very powerful. We have a very suitable subject in ammoniacal gas, composed of hydrogen and azote, whose combination does not prevent its preserving the gaseous state. In this example, as in the preceding, according to the analysis of M. Berthollet, the result is such, that if the compound had not been analyzed, and we had hardly known the nature of its elements, its refracting power would have indicated exactly its composition.

Proceeding now to more intimate combinations, we calculate in the same manner the refracting power of water from its constituent elements, hydrogen and oxygen. According to very accurate experiments made by MM. Guy-Lussac and Humboldt, water is composed of two parts by bulk of hydrogen to one of oxygen; which, reduced to weight, gives for 1000 parts, 117 of the first to 883 of the second. Each of these quantities being multiplied by the value of the refracting power belonging to it, the sum of the products gives 1,53545 for the refracting power of water, that of air being unity. The refracting power actually observed by Newton, is found to exceed this by about ⅛ of its total value. Thus oxygen and hydrogen gas, being condensed into water, exert upon light an action sensibly stronger than when in a state of simple mixture. The same experiment made upon other compounds gives similar results; that is, the change from a gaseous to a compact state, always produces a sensible increase of affinity. But this is particularly remarkable in the diamond, which, according to experiment, has a considerable refracting power, although from the most accurate analysis, it appears to consist entirely of carbon, a substance which in a gaseous state, exerts only a very feeble action upon light; at least if we may judge from carbonic acid, of which it is one of the constituent elements.

Still the refracting power, deduced from chemical composition, will be found to differ very little, if at all, from the truth, when the state of the body undergoes only slight changes. The rule may indeed be employed with perfect safety in the case of mixed liquids, even those which exert a very strong action upon each other, as water and alcohol. We have seen that the mixtures of the gases presented a similar agreement.

From what has been said, it seems probable that the refracting power of water in a state of vapour, differs very little from water in a liquid state; and hence we are able to calculate the influence which its presence in the air has upon atmospherical refractions. We thus find that the excess of refracting force of this vapour over air, is almost exactly compensated by its inferior density; so that on the whole, the refraction produced by masses of dry and humid air are sensibly the same, the pressure and temperature being supposed equal in the two cases. Indeed, this equality has been proved by experiment, and has been found so exact that it was impossible to perceive any difference in the two cases. We hence deduce a practical principle of great importance to astronomers, which is, that their observations are independent of the hygrometric state of the air; for, if moisture had any appreciable effect, it would render the calculations in astronomy much more complicated. Nevertheless, by a mode of observation still more delicate than the one here made use of, M. Arago has ascertained that aqueous vapour has a refracting power a little less than that of water in a liquid state; and that in this respect there is a difference also in all other vapours, when compared with the liquids from which they are derived. Happily this difference is so small in the case of water, that if strict accuracy require that we should take account of it, still experiment shows that it may be neglected.

83. In like manner we learn that change of temperature produces no perceptible change in the refracting powers of the gases; for the deviations observed in summer and in a room where the moisture and temperature are artificially varied, agree exactly with the results of calculation from refracting powers deduced from observations taken in winter. It is probable, however, that we should find a difference in this respect, if we were to operate at such degrees of cold as to admit the reciprocal action of the gaseous particles to become sensible by their approach. It is also possible that in a liquid condensed by cold, the attractions resulting from the configuration of its particles might modify sensibly their action upon light. Perhaps we may attribute to this cause the curious fact observed by M. Arago, that the refraction of water in a liquid state increases in proportion as its temperature is reduced, even when its density is diminished by the dilatation it experiences below the temperature of 39°.

84. Hitherto we have considered the motion of light in homogeneous media; but we may likewise conceive of media of different strata and composition, with a view to discovering what takes place when a ray of light traverses them. This problem is solved by means of the same principles which guided us in calculating the course of luminous particles as they approach the surface of bodies, or as they penetrate their substance to a small depth. We divide the medium, traversed by the light, into such a number of strata as to allow the density and composition of each to be regarded as uniform. Then, in each

of these strata, the path of the luminous particles may be considered as a parabola, whose direction is determined by the actual intensity of the refracting power; and, continuing the calculation from stratum to stratum, we can ascertain successively the path of each luminous particle from the direction in which it is projected.

Suppose *ABCD* such a medium, composed of horizontal strata, Fig. 71. whose ratio of refraction is at first constant for a certain height, and then goes on decreasing by insensible degrees, till at length it again becomes constant, but less than before. We shall have an instance of such a medium, if we conceive in the same vessel a thick stratum of sulphuric acid covered by a stratum of pure water, a disposition easily effected if we pour the water in first, and then introduce the acid by a glass tunnel, the end of which, drawn out into a narrow tube, is directed to the bottom of the vessel. The acid being heavier than the water, extends under its surface and raises it in proportion as its quantity increases. Now these substances having a considerable affinity for each other, tend to combine together, and the combination in fact takes place in the lowest strata of the water which lie immediately upon the acid. But by reason of their plane figure, the attraction which they exert upon each other, as well as the difference of their specific gravities, prevents them from changing places; and the strata of the two substances which are in contact, being always the same, the combination can only be propagated from one to the other very slowly. Hence in spite of the strong affinity between water and sulphuric acid, if we do not make use of a very large vessel, we often find that after a whole day the upper strata of the water remain unmixed with the acid. Here then is a medium composed of strata that are parallel, heterogeneous, of different refracting powers and powers decreasing with the height. For the ratio of refraction of concentrated sulphuric acid exceeds that of pure water, and it carries this property into the strata with which it enters into combination. Let us now suppose that the experiment is made in a rectangular vessel of thin glass, in order that its vertical sides may present transparent parallel surfaces which will not change the regularity or the course of the luminous rays. Conceive now a radiating point situated at *A*, on one of the sides of the vessel, at such a height that the strata of acid which correspond to it, shall so exact that it was impossible to perceive any difference in the two cases. We hence deduce a practical principle of great importance to astronomers, which is, that their observations are independent of the hygrometric state of the air; for, if moisture had any appreciable effect, it would render the calculations in astronomy much more complicated. Nevertheless, by a mode of observation still more delicate than the one here made use of, M. Arago has ascertained that aqueous vapour has a refracting power a little less than that of water in a liquid state; and that in this respect there is a difference also in all other vapours, when compared with the liquids from which they are derived. Happily this difference is so small in the case of water, that if strict accuracy require that we should take account of it, still experiment shows that it may be neglected.

83. In like manner we learn that change of temperature produces no perceptible change in the refracting powers of the gases; for the deviations observed in summer and in a room where the moisture and temperature are artificially varied, agree exactly with the results of calculation from refracting powers deduced from observations taken in winter. It is probable, however, that we should find a difference in this respect, if we were to operate at such degrees of cold as to admit the reciprocal action of the gaseous particles to become sensible by their approach. It is also possible that in a liquid condensed by cold, the attractions resulting from the configuration of its particles might modify sensibly their action upon light. Perhaps we may attribute to this cause the curious fact observed by M. Arago, that the refraction of water in a liquid state increases in proportion as its temperature is reduced, even when its density is diminished by the dilatation it experiences below the temperature of 39°.

84. Hitherto we have considered the motion of light in homogeneous media; but we may likewise conceive of media of different strata and composition, with a view to discovering what takes place when a ray of light traverses them. This problem is solved by means of the same principles which guided us in calculating the course of luminous particles as they approach the surface of bodies, or as they penetrate their substance to a small depth. We divide the medium, traversed by the light, into such a number of strata as to allow the density and composition of each to be regarded as uniform. Then, in each

elastic force sufficient to sustain the weight of the superior strata. Now we have seen that the ratio of refraction of the air depends upon its density simply; consequently the strata situated immediately over the bar, will refract the light less than those immediately above them; and these again will refract it less than those which follow, until by a progressive but rapid gradation, we arrive at strata whose distance from the bar removes them from the influence of its temperature; from this point the ratio of refraction will be sensibly constant. Hence it is clear, that if through such a medium we look horizontally at distant objects, situated in the direction of the bar and at a small height above it, we shall see two images, the one superior and upright, through the stratum of uniform density; the other inferior and inverted, through the strata of variable density. This curious observation, together with many others relating to the same subject, is due to Dr Wollaston.

85. The cause of these phenomena being once known, we can easily imitate them by methods of our own, which, besides the advantage of presenting them with certainty, have likewise that of exhibiting them in their full developement. A convenient apparatus for this purpose consists of an iron vessel of a rectangular shape, suspended about five feet from the ground by means of iron wires. In the direction of the sides of this vessel, at the distance of fifty or sixty feet, are placed small signals of paper of a triangular form, the vertices corresponding very nearly with the visual ray which grazes these sides. This done, we fill the vessel with coal partially ignited. Then, if we place the eye in the direction of the sides of the vessel, at the end opposite the signals, we shall see the image become double, as the temperature is raised, presenting distinctly two white points, one direct and the other inverted, which will approach each other in proportion as the visual rays come nearer grazing the sides, and at length join each other; and this reflection, produced by the excess of temperature of the air in contact with the heated iron, takes place, as well on the vertical sides of the vessel as on the horizontal bottom.

86. The same effect is sometimes very conspicuous in the strata of air contiguous to a dry and sandy soil, strongly heated by the sun. The density of the air, in this case, increases from the surface to a certain height, commonly very small, after which

it becomes for a short space sensibly uniform, and at length decreases very slowly, according to the common constitution of the atmosphere. If we suppose an observer placed in a stratum of mean density, and looking at a distant object situated also in this stratum, he will be able to see it in two ways; directly through the uniform stratum which intervenes, and indirectly by the rays reflected from the inferior stratum. These rays at first directed from the object towards the terrestrial surface, with a certain inclination, enter the strata of less density, are refracted there, taking a direction more nearly approaching to a horizontal line, whence they rise, and traversing the superior and denser strata which attract them, pass on to the eye of the observer. There will then be two images of the object, the one upright, by direct vision, the other inverted, by reflection. If the object be insulated on the dark ground of the sky, an inverted image of the sky will surround also the reflected image of the object exactly as when objects are represented by reflection on the surface of water.

87. We are hence able to explain a very curious phenomenon, known to French mariners under the name of *mirage*,† and which the French army had many opportunities of observing in their expedition to Egypt. The land of Lower Egypt is a vast horizontal plain, the uniformity of which is interrupted only by some small eminences which serve for the sites of villages, and which are thus rescued from the inundations of the Nile. Morning and evening, the appearance of the country agrees with the actual disposition and distance of the objects; but when the surface of the soil is heated by the presence of the sun, the ground seems to terminate at a certain distance by a general inundation, and the villages beyond resemble islands situated in the midst of a large lake. Beneath each village is seen its inverted image exactly as it appears in water. As we advance, the limits of this inundation appear to recede; the imaginary lake which seems to surround the village retires; and at length entirely vanishes, while the appearance is reproduced with respect to another village more distant. Thus, according to Monge, from whom this account is borrowed, every thing conspires to complete an illusion which is sometimes painful, especially in the

---

† The English word *looming* is sometimes used to denote this appearance, generally however it is employed in a more restricted sense.

desert, since it tantalizes one with the show of water at the very time when he has the greatest need of it.

A similar appearance is exhibited at sea in very calm weather. A ship seen at a distance and in the horizon, sometimes presents two images, one direct, the other inverted. The second answers exactly to the first, and is equally distinct; indeed the effect is precisely that of a reflection from a mirror. Hence the name, *mirage*, which mariners have given to this phenomenon. As it is produced by the difference of heat between the water and air, it is commonly seen after sudden changes of temperature; since the density of the water at the surface of the sea does not admit of its taking these variations so rapidly as the air. But on the other hand, the temperature of the water, and the evaporation which is constantly going on at its surface, prevent it from taking so high a temperature as the sandy surface of an arid soil. For these reasons, the phenomenon of double images occurs rarely at sea, and continues only for a short time; whereas in Egypt and on some sandy plains in which the same circumstances present themselves at the same height of the sun, it occurs every clear day.

88. M. Mathieu and myself observed at Dunkirk, on the seashore, many phenomena of this kind, and I have given the mathematical theory of them.* I have proved that the consecutive paths or trajectories, traced from the eye of the observer, interest in their second branches, in such a manner as to form a caustic, below which no point can be seen. The curve *LT* represents this caustic, and *DMS* is the limiting trajectory, drawn from the eye of the observer, a tangent to the ground. I call it limiting trajectory, because it limits the height where the inversion takes place. In this figure all the points situated above the trajectory can send only one image to the eye of the observer. Those situated in the space *SLT* send two, the superior being erect, and the inferior inverted. Lastly, those situated below the caustic in the space *MLT*, not being capable of sending any, are invisible; so that a moving object, a man, for example, withdrawing successively to different distances, will present the successive appearances represented in figure 73. Fig. 72.

Theory and experiment agree in proving that no very considerable difference of temperature is necessary in order that these appearances may be produced. Two or three degrees of Fahrenheit's thermometer are sufficient, when the place of observation is a level and extended plain, which allows the luminous rays to be prolonged without obstacle, and thus to manifest the curvature of the trajectories described by them. Of this kind was the station selected at Dunkirk, it being a sandy beach situated in the downs, near fort Risban; and the observations were also favored by the existence of a great number of distant objects, such as steeples, trees, and cottages, which, rising like so many signals above the dry soil, manifested the course of the rays by the appearances they presented. Moreover, the phenomenon of the doubling and inverting of the images, presented itself almost every day, arising from differences of temperature not exceeding three degrees of Fahrenheit.

---

* Memoirs of the Institute for 1809.

89. If we suppose a like difference of temperature to exist between two strata of air, not placed vertically one above the other, but laterally contiguous, the phenomenon will still present itself; only it will take place in a horizontal direction perpendicular to the common surface of the two strata. The late M. Jurine and M. Sorret observed an appearance of this kind on the lake of Geneva, at a place rendered very narrow by the approach of the opposite shores. Now upon examining the configuration of these shores, of which the one situated on the south side is bordered by high mountains, and the other, situated on the north side, is exposed to the rays of the sun, it will be easily perceived that such a difference of aspect, in calm weather, or even when a moderate wind is blowing parallel to the direction of the shores, may occasion for a moment, in the mass of air incumbent on the lake, a lateral inequality of temperature, and consequently of density; and this may produce inflexions in the luminous rays which traverse the mass of air in a longitudinal direction sufficiently near the limit where the lateral variation of density takes place.

90. It sometimes happens also, that distant objects seem suspended in the air; the image in this case is single, erect, and apparently not accompanied with any inverted figure.*

---

* This phenomenon is called by the French *suspension*, and by the English *looming*. It has been shown in the memoir above cited, that when this phenomenon occurs, the second inverted image exists, but is infinitely reduced, so that we see only the erect image which is detached, on the inverted image of the sky.

When vision thus takes place by trajectories convex towards the earth or sea, the reflection is negative; the apparent horizon is much lower than it ought to be, for the height where the observation is made. Mariners ought, therefore, to be on their guard with respect to this phenomenon which tends to occasion considerable errors in latitude. It is found, indeed, by experiment, that these errors often amount to four or five minutes. The apparent horizon will therefore be too low when the sea is warmer than the air. If, on the contrary, its temperature is lower, its density is increased, but by a much more rapid law than ordinary; and the apparent horizon is sometimes raised very considerably. These errors may be avoided by observing the height of the stars, not above the horizon of the sea, but above an artificial horizon, placed without the inferior strata, where the extraordinary variation of density always takes place. But this expedient is not always easy even on land; and on board of vessels it is altogether impracticable, on account of the motion of the sea. In this case the error may be corrected, by taking, if it be possible, the distance of the two opposite horizons of the sea. The excess of the sum above two right angles, will give double the apparent depression of the horizon which it is necessary to employ in the calculation. Thus we shall know this depression by taking half the result. Unfortunately this observation of the two horizons, proposed by Dr Wollaston, appears very difficult to be taken with accuracy; and besides, we can seldom expect to find the differences of temperature between the air and water to agree sufficiently to render the two depressions equal. If it is not in our power to correct the error to which we are liable under these circumstances, it is at least well to be apprized of its existence and tendency, in order that we may not be misled by it.

Double Refraction.

91. It has already been stated, that a beam of light in traversing crystallized bodies, is generally divided into two portions of which the one, called the *ordinary ray*, follows the ordinary law of refraction assigned by Descartes; while the other, called the *extraordinary ray*, is subject to very different laws.

According to a very ingenious remark, first made by Dufay and confirmed by all succeeding observations, this phenomenon takes place in all crystals, except those whose primitive form is the cube or some of its geometrical derivatives. The separation of the refracted rays, proceeding from an incident beam, is more or less considerable, according to the nature of the crystal, and the direction in which the light traverses it; but this last variation is subject to the same law for all substances. In general, there are two directions, and only two, in which the separation of the two rays is infinitely small or nothing, and their velocity the same. These two directions are called the *axes of the crystal.* In proportion as the refracted rays deviate from the axes, their velocities of transmission become different; and this diversity is manifested, in general, by the separation or divergence which takes place in the rays, when the obliquity of incidence or emergence is the same. The inequality of the velocities increases according as the refracted rays form greater angles with the two axes; and it is the greatest possible when these angles are both right angles, that is, when the refracted rays become perpendicular to the plane of the two axes. Among the different kinds of crystals there are some in which the angle formed by the two axes is absolutely nothing; these may be regarded therefore as having but one axis. It is thus indeed that we did at first consider them; but it is better to imagine two axes, coinciding with each other, in order to preserve the analogy, and for the purpose of comprehending all crystals under one law. This case, however, is evidently more simple than the general one, because the inequality of the velocities depends entirely upon a single angle; on this account it will be examined first; and to make the process still more easy, we shall take, for an example, the rhomboidal carbonate of lime, commonly called Iceland spar; which, besides being remarkable for a double refraction extremely energetic, has likewise the advantage of being frequently met with in collections of minerals.

92. The crystals of this substance present cleavages in three directions, remarkable for their evenness and the facility with which they are effected. They are hence often separated of themselves into an infinity of small rhomboidal solids, having each six faces, parallel two and two, and formed of similar par-

allelograms. Each of these rhomboids has six acute solid angles Fig. 74. and two obtuse. These last are formed of three plane angles equal to each other and equally inclined. In the acute angles formed by two faces, the inclination is 74° 55′, and it is consequently 105° 5′ in the other angles. Malus measured these inclinations by the reflection of light. Dr Wollaston obtained the same result; and I also have had occasion to confirm the accuracy of this measurement. The primitive form adopted by Haüy is, in like manner, a rhomboid, similar to the above, as respects the angles, but in which we suppose all the faces equal to one another; so that each becomes a perfect rhombus.

93. If we place one of these rhomboidal solids over a printed page or a piece of writing, and look at the characters through its thickness, the whole appears double; so that each radiating point under the crystal, sends two images to the eye, and consequently causes each ray to become two rays. Whence, reciprocally, each incident ray is divided into two portions in traversing the rhomboid. Indeed this may be verified by causing a single ray of light, obtained from a heliostat, to pass through the crystal; for two distinct rays are found to emerge from the refracting medium. Now in order to measure the divergence of these rays and determine their course, a very simple method has been devised by Malus. On a large ivory plate upon which we place the rhomboid, we trace, in very fine black lines, a right-angled triangle *ABC*, whose smaller side *BC*, is, for instance, one tenth of *AC*. If we look at this triangle through the rhomboid, Fig. 75. we shall see it double in whatever manner we place the eye; and for each position there will be a point *F* in which the line *A′C′*, the extraordinary image of *AC*, will cut the hypothenuse *AB*, which we suppose to belong to the ordinary image. If we take then, in the same triangle, a length *AF′* equal to *A′F*, the point *F′* will be that whose extraordinary image coincides with the ordinary image of the point *F*, considered as belonging to the hypothenuse *AB*. The ordinary ray coming from *F*, and the extraordinary ray from *F′*, are then confounded together after their egress, and send only one emergent ray to the eye. Reciprocally, a natural ray which proceeds, according to this last direction, from the place where the eye is, and which is directed towards the crystal, is refracted there into two portions, one of which ends in the point *F*. and the other in *F′*, of the

triangle, drawn under the base of the rhomboid. Indeed the motions of light are such as to present this reciprocal effect in rays which go and return by the same course, and which thus undergo the same successive modifications only in an inverse order. But here it is possible and proper to verify the phenomenon directly by experiments with the heliostat. Accordingly, if the lines *AB*, *AC*, are each divided, for instance, into a thousand equal parts, and every tenth division numbered, a mere inspection of these numbers will be sufficient to indicate what are the points of *AB* and *AC*, whose ordinary and extraordinary images coincide; consequently, if the position of these lines and that of the triangle are known with respect to the edges of the base of the rhomboid, we shall always know, in each case, to what part of this base the point *F* corresponds, and to what the point *F′*. Thus, in order to construct the refracted rays, it only remains to determine on the superior surface, the position of their common point of emergence *I*. This is done by stretching over the superior surface of the rhomboid, two fine silk threads, crossing each other at right angles, and observing the divergence of the images, when the common emergent ray passes the point of the surface where the threads cross. But we must also know the direction of this ray, that we may be able to assign the incidence under which the observed divergence takes place. This, likewise, is easily obtained by drawing in the plane of the triangle at which we look, four straight lines, divided and disposed rectangularly, but without the space covered by the rhomboid. At the moment when the divergence of the images is observed, we note the points of the divisions on which the threads are projected. The planes drawn through these points and through the threads, being prolonged to the eye, will evidently indicate, by their intersection, the direction of the emergent ray. Fig. 74.

94. Suppose now, that such an apparatus as that above described, is applied to a crystal of Iceland spar, terminated by natural rhomboidal faces. Among all the positions which can be given to the rhomboid on its plane, there is one which merits particular attention, because in it the extraordinary refraction takes place, as well as the ordinary refraction, in the plane of incidence itself. To ascertain this position, conceive a plane drawn through one of the sides, as *AC*, of the divided triangle *ABC*, perpendicular to the superior and inferior surfaces of the rhomboid; then

place the eye in this plane, and turn the crystal gradually upon its base, until the two images of the right line *AC*, take a common direction; and present a partial superposition, as in fig. 76. Then, as we know that the ordinary image always remains in the common plane of incidence and emergence, the extraordinary image which coincides with it will be found there, in like manner. This coincidence is observed when the straight line *AC*, the object of view, bisects one of the obtuse plane angles of the rhomboid, or is parallel to such bisecting line. Then the divergence of the two images, perpendicular to the plane of incidence, becomes nothing; and consequently, whatever be the forces which produce the extraordinary refraction, it is certain that their resultant is comprehended wholly in this plane. On this account it has received a particular denomination, namely, *the principal section of the rhomboid.* If we suppose the crystal on which these experiments are made to have the precise primitive form which belongs to carbonate of lime, the bases of the rhomboid will be perfect rhombuses; and then the principal section will contain the small diagonals drawn upon each base through the vertices of the obtuse angles. The plane of this section will then cut the rhomboid so as to form a parallelogram *ABA′B′*, in which the sides *AB*, *A′B′*, are the small diagonals Fig. 77. of the opposite rhombuses, and *AB′*, *A′B*, the edges which connect them in the rhomboid. The line *AA′*, drawn through the two solid obtuse angles *A*, *A′*, is called the *axis of the crystal*; it is equally inclined to all the faces, and forms with them an angle *A′AB* of 45° 23′ 25″, the angle *B′AA′* being 63° 44′ 45″. To this we are to refer all the phenomena of extraordinary refraction, if we would obtain for them symmetrical and simple laws. Now this line exists only in crystals of Iceland spar, whose form presents outwardly the relations of symmetry which belong to the primitive particle. It is likewise necessary to conceive its existence in each small element of all these crystals, whatever be the exterior configuration which art or nature has given to the sensible mass resulting from their aggregation, and it forms there the same angle among its faces. If, in one of these small elementary rhomboids, we imagine a plane normal to one of the faces and dividing its obtuse angle into two equal parts, this plane will be parallel to all the similar planes which can be drawn through each of the other elements of the whole mass, whatever be the figure which terminates it; and this is the direction thus considered in the elements themselves, which constitutes the *principal section* of the crystal, and which has the property already given to this term. In general, a crystal of a sensible size, is nothing more than the union of an infinite number of small crystals, all of the same form, and arranged in a similar manner, so that their homologous faces are all directed the same way. A mass thus aggregated, must therefore exhibit properties identically the same in all the minutest parts which can be subjected to experiment. And reciprocally, this absolute identity of properties is the specific mark which essentially distinguishes and characterizes the crystalline state.

95. This being well understood, let us take any rhomboid whatever of Iceland spar, and examine first how double refraction takes place in the plane of the principal section. All the general phenomena are represented in figure 78, where *SI* represents the incident ray, *IO* the extraordinary ray, and *NI* the direction of the normal at the point of incidence. When the incidence is perpendicular, the ordinary ray is confounded with the prolongation of the normal, and traverses the crystal without deviation; but the extraordinary ray is broken at the point of incidence, and is turned more or less at *E* towards the small solid angle *B'*. This effect is general for all incidences, as represented in the figure, and it always determines the situation of the extraordinary ray with respect to the other.

The conclusion at which we have arrived, is, that there exists in the crystal a particular force, which takes from the ordinary ray a portion of its particles, and repels them towards *B'*. But what is this force? We shall soon see that it seems to proceed from the axis of the crystal; that is, if through each point of incidence we draw a line *IA'*, parallel to this axis, and representing, consequently, its position in the first strata where the ray is divided, all the phenomena take place as if there proceeded from this line a *repulsive* force, which acted only on a certain number of luminous particles, and tended to turn them from its direction. This force always urges the rays towards *B'*, because the refraction which the crystal causes them to undergo always brings them on this side of the axis, whatever be the side of the normal towards which they are directed in their incidence.

* * *

96. Let us pursue this idea, which is not opposed to the few observations we have made; and to verify it by direct proof, let us cut the crystal by two planes perpendicular to its axis, in such a manner as to form thereby two new faces *a b c*, *a′ b′ c′*, parallel to each other. Now if we direct a ray *SI*, perpendicularly to these faces, it will enter the strata parallel to the axis of the crystal. Accordingly, if we suppose the repulsive force to proceed from this axis, it will be nothing, and the extraordinary ray, incapable of diverging from the axis, will coincide with the ordinary ray. This is what actually takes place; for we observe, in this case, only a single image situated in the prolongation of the normal to the two faces of incidence and emergence. Fig. 79.

We find also, by experiment, that the image remains single when the second face is inclined to the axis, the first remaining always perpendicular to this axis and to the incident ray. This would take place, for example, if we should take away only the first solid angle *A* of the primitive rhomboid. An incident ray *SI*, which entered the anterior surface *a b c*, under a perpendicular incidence, would continue its course parallel to the axis as above; and, on emerging through the second face it would be refracted in the air in a single direction *I′R′*, depending on the law of ordinary refraction; that is, on the constant ratio of the sines. Hence we conclude, reciprocally, that an incident ray *R′I′*, which passes with the same angle of incidence from the air into the prism, would be refracted parallel to the axis, in a single ray, and would emerge at *I*, in the same manner. This also is confirmed by experiment. If we cut a prism in this manner, and place the eye opposite to the face perpendicular to the axis, in such a manner as to receive only the rays which come in this direction, all the images of external objects are single; they only experience on their borders that diffusion which has been adverted to, and which belongs to the general phenomena of the decomposition of light by prisms. But if the repulsive force which produces the extraordinary refraction, really proceeds from the axis, as these phenomena indicate, it can become nothing only when the refracted ray is parallel to it. The cutting which we have just determined, must, therefore, be the only one by which a prism, formed of a crystal having only one axis, can give single images. This too is confirmed by experiment; and we may make use of the property in question to ascertain the position of the axis in any small portion of such a crystal.

97. Let us resume the plate cut with parallel faces perpendicular to the axis. We have seen that a ray coming from without is not divided when it enters with a perpendicular incidence; but with oblique incidences it must be divided, since it forms in this case, a certain angle with the axis from which the repulsive force proceeds. Moreover, in Iceland spar, the extraordinary ray must be carried farther from the axis than the ordinary ray. This is what actually takes place; and, besides, with an equal incidence the extraordinary refraction is the same all round the axis; which shows that the repulsive force acts equally at equal distances on all sides of the axis.

98. In every crystal with one axis, there exists one direction only which presents such phenomena; that is, the rays which traverse the crystal in this direction are not divided, even when they emerge from a prismatic face. Moreover, when the refracted rays deviate from this direction, they depart from each other as if they were separated by a force proceeding from the axis. But the action of this force is not always repulsive. I have found that there are crystals in which the extraordinary ray is attracted towards the axis, instead of being repelled. This requires that we should divide crystals of one axis into two classes, one of *double refraction attractive*, the other of *double refraction repulsive.* These denominations are merely enunciations of the facts observed.

99. If in a crystal of one or the other of these classes, we cut in *any direction whatever*, a plate with parallel faces, we shall find that this plate has also a section in which the two refractions do not deviate from the plane of incidence; and conformably to what we have observed in Iceland spar, this section is always that which is made through the axis of the crystal perpendicularly to the two faces of the plate. In generalizing this idea, we designate by the phrase *principle section of any face*, in crystals of one axis, the plane which is drawn through the axis of the crystal perpendicularly to this face.

In all these crystals the connexion of the phenomena with a resultant force, either attractive or repulsive, is manifest. The influence of this force is sensible in all directions, in which we would observe the relative course of the two refracted rays. To give a simple example, and at the same time one which affords many useful applications, let figure 80 represent a plate cut in the form of a rectangular parallelopiped, one of whose edges, *AA′*, is directed according to the axis of the crystal; and let us measure the two refractions in the two rectangular directions *AB*, *AA′*, by first making the plane of incidence perpendicular and then parallel to the axis *AA′*. This can be done by the apparatus already explained, and also by various other analogous methods which may be easily imagined. Accordingly let us see what are the results.

100. In the first position in which the incidence is in the plane *AB a b*, perpendicular to the axis *AA′*, the two refracted rays, ordinary and extraordinary, do not deviate from this plane; and both are refracted according to the law of Descartes, in such a manner that the sine of refraction and the sine of incidence, reckoned from the normal *IN*, are to each other in a constant ratio. But this ratio is not the same for the two refractions. For instance, in Iceland spar and in all other crystals of a double repulsive refraction, the ordinary ray, with the same incidence, is nearer the normal than the extraordinary ray. This is shown in figure 81, where the refraction in the face *AB a b*, is represented separately. The contrary takes place in crystals of a double attractive refraction, where the extraordinary ray is Fig. 82. more refracted and brought nearer the normal than the other.

101. The constancy of the ratio of the sines for each kind of refraction, when crystals are cut in this way, may be ascertained from the divergence *EO*, which the two refracted portions present, as they emerge from the plate at the second surface. But it will be better to establish this law at once by the consequence which follows immediately from it; and this consequence is, that if the extraordinary refraction, when a plate is thus cut, is actually subjected to a constant ratio of the sines, we can determine this constant, as well as that of ordinary refraction, by cutting a prism *ABDA′B′D′*, whose edges are parallel to the Fig. 83. axis, and observing the deviation which it produces in the two images of the same external object. Moreover, since the direction of the cutting is indifferent for ordinary refraction, it is evident that this single prism will serve to ascertain the constants for the two refractions. For one as well as the other, the constancy of the ratios will be demonstrated, if all the incidences

give the same value, and if this value is still the same for all the prisms which can thus be formed about the axis. This remarkable result extends to all crystals of one axis hitherto observed. In Iceland spar, for instance, Malus found that the ratio of the sine of incidence to the sine of refraction, for ordinary refraction, is 1,654295, and for extraordinary, 1,4833015. Hence we see that this last refraction produces a less deviation than the other, conformably to what has already been stated with regard to all crystals of a double refraction repulsive, in the direction of the cutting and with the supposed incidence. And this result well agrees with the idea of a repulsive force proceeding from the axis; for, in the case observed, the ray refracted extraordinarily remains perpendicular to the axis, for the whole extent of its passage through the crystal. Thus the repulsive force which proceeds from the axis has no tendency to make it deviate from the plane of incidence; it only opposes itself to the motion of. the luminous particles, and retards their velocity according as they begin to penetrate the crystal. Its action in this respect, is, therefore, directly opposed to that of the ordinary attractive forces which tend to increase the velocity of the particles. Thus the effect which hence results upon these particles, must be the same as if the refracting force of the crystal were diminished; that is, the deviation of the ray must be less, as observation actually proves it to be. In crystals of a double refraction attractive, on the contrary, the attractive force, proceeding from the axis, unites with the refracting force of the medium to augment the velocity of the luminous particles, and the deviation is increased; so that the extraordinary ray approaches the normal more than the other. In rock crystal, for instance, the ratio of the sine of incidence to the sine of refraction is found to be 1,547897, for ordinary refraction, and 1,557106 for extraordinary, in conformity with the preceding statement.

Fig. 80. 102. Let us return now to the rectangular plate, and suppose the plane of incidence to be in the face *AA′ a a′* which contains the axis. The force proceeding from the axis *AA′* is exerted still in this plane, and therefore does not tend to make the extraordinary ray deviate from it. Now we can easily ascertain whether this ray will diverge more or less from the normal than the other; for if we represent separately the section of the plate made by the new plane of incidence, figures 84 and 85, *AA′*

being the axis, *SI* the incident ray, and *IO* the ordinary ray, it is evident that the force proceeding from the axis, will tend to increase the angle *AIO*, if the crystal be repulsive, and to diminish it if the crystal be attractive; so that the extraordinary ray *IE*, Fig. 84.
will, in the first case, approach the normal more than the other, Fig. 85.
and in the second, less, which is conformable to observation.

103. All the other examples which can be imagined offer only repeated applications of the same principles. The consideration of the attractive or repulsive force proceeding from the axis, will always indicate faithfully the direction of the deviations. But passing over these repetitions, we proceed to the more general case, where the crystal has two axes separated by a sensible angle. Then the rays which follow one or the other of the axes will not be divided, through whatever faces they emerge. For every other interior direction the rays experience a separation which indicates an inequality in their velocities; and the extent as well as the direction of their divergence are exactly the same for similar deviations with respect to each of the two axes. Hence we conclude that the force which divides them acts equally about each of these two lines.

Here a new circumstance peculiar to crystals of two axes, presents itself, which renders the phenomena more complicated, and the ascertaining of their general law more difficult. This circumstance, discovered by M. Fresnel, consists in the fact, that neither of the two refracted rays here conforms to the simple law of refraction established by Descartes, according to which the sine of refraction bears a constant ratio to the sine of incidence. Whence it follows that neither of them can be considered as having in the crystal a constant velocity in whatever direction it is transmitted, this constancy being peculiar to the law of Descartes.

The existence of this variation in the two velocities may be established, as M. Fresnel has shown, by a very simple and purely negative proof. Indeed, if there be exerted in the interior of a body, a refraction of a constant velocity, this mode of refraction will impress upon the incident rays deviations exactly equal, at equal incidences, whatever in other respects be the direction of the rays and that of the cutting according to which they are introduced. Hence, if we cut from the substance we would subject to experiment, several small pieces in different directions, and fit them all together in such a manner as to form one prism whose faces are perfectly plane, the deviations experienced by the luminous rays in traversing this prism, ought to be the same in all its parts at an equal incidence; so that, for example, the image of a straight line, parallel to the prism, will not be broken, but will still appear a straight line, when viewed by refraction. Now if we form such prisms with pieces of the same topaz, or any other crystal of two axes, cut in different directions, we shall find that this equality of deviation does not take place for either of the two refractions, and therefore we must conclude that neither is produced by a constant velocity.

But though this experiment is sufficient to establish the variableness of the velocity, it will not furnish its absolute measure with sufficient precision, on account of the uncertainties to be met with when we attempt to compare together the refractions of the images, transmitted by the contiguous parts of the prism; uncertainties which arise from the difficulty of rendering the surfaces of incidence and emergence perfectly plane, as well as from the irregularities which the refracted images present near the borders of contiguous pieces, where it is necessary to bring them when we would compare them together. It is in vain to think of observing the absolute values of the different deviations, because the errors we should be liable to commit in estimating them, might bear a comparison with the differences of refraction, always very small, which we propose to measure. But there exists an element, easy to be observed, which being intimately connected with the law of the velocities, may lead indirectly to the discovery of this law, since it presents various conditions which the law in question must satisfy. This element is the divergence of the two refracted rays, proceeding from the division of the same exterior incident ray, which penetrates the crystal, either by natural faces or by artificial ones, the direction of whose cutting can be known. Indeed this divergence being always very feeble in all substances hitherto observed, and its gradual variations being always far from rapid about each direction of incidence, it follows that the difference of the two velocities on which it depends, must also be very small compared with their absolute value; so that we may consider the divergence of the two rays as sensibly proportional to this difference; and, moreover, in the analytical formulas which express it, we may neglect the variations of the two velocities, and employ their absolute values as constant, and as they are deduced by the law of Descartes from any measures of absolute deviation. Since the divergence of the rays thus affords a very proper means of discovering the law of the difference of velocities, we must seek some method of observing this divergence with the greatest precision.

This is effected even for crystals in which double refraction is the most feeble, by means of the following apparatus, with which a great number of experiments have been made on all kinds of crystals, both of one and two axes.

It consists principally of two ivory rules *AX*, *AZ*, divided into Fig. 171. equal parts and disposed at right angles to each other. The first *AX*, is placed on a horizontal table; then the other *AZ*, becomes vertical. A column *Hh*, whose superior and inferior bases are parallel and formed of glass plates, moves upon the division *AX*, and may thus be placed at different known distances from the division *AZ*.

This disposition is sufficient when the two refractions to be observed, take place in the prolongation of the plane of incidence, as is the case for ordinary refraction. Now this happens in all crystals under certain particular circumstances of incidence and cutting. As this is the most simple case and will serve to make known the method, it will be first explained.

104.. If the substance we wish to examine has a very powerful refraction, we can, as Malus did, confine ourselves to the forming of parallel plates of it, on which the operation may be performed as already mentioned. But this case being very rare, we in general suppose the crystal to be cut into a prism; and also, to render the refraction more sensible, we commonly give to this prism a large angle *BCD*, for instance, a right angle, which has the particular advantage of simplifying the calculation. But light cannot traverse immediately the two faces of such a prism when it is placed in a medium of so small a refracting power as the air, because the rays entering by the first face *BC*, are reflected inward when they arrive at the second; and, on this account, we fix to this second face *CD*, a prism or parallelopiped of glass *CDFG*, whose refracting angle *D*, is nearly equal to the angle *C* of the crystal prism, so that the anterior face of the crystal *CB*, and the posterior face of the glass *DG* shall be nearly parallel. The junction of the two prisms is effected by heating them and melting between their surfaces some small grains of very pure mastic in tears, which are pressed out into a very thin transparent lamina. This lamina after cooling is sufficient to make the two surfaces adhere strongly and to determine the passage of light from one to the other, so that vision becomes possible through the double prism. Then we place this prism on the superior base of the column *H h*, applying it by its face *BC*, to the column, which requires that the face *CF* of the glass should extend a little beyond *BC*, or should be in its prolongation; and in order that the crystal may remain fixed before the division *AZ*, in the situation judged most convenient for the experiment, we first put on the edge of the glass a few small drops of inspissated spirits of turpentine, which are sufficient to make it adhere. This being done, we place the eye at *V*, behind the posterior face of the glass, and look through the double prism at the vertical division *AZ*. In virtue of the two refractions which the crystal causes each ray, emerging from it, to undergo, it appears double; so that, properly speaking, we see two scales of division, placed one above the other, in a common direction, at least while the lateral deviation is nothing, as we now suppose it. But the inequality of the two refractions, added to that of the distance, causes the homologous marks of of the two refractions to separate unequally from each other. If in certain parts the deviation is half an interval between the marks, a little further it is a whole interval; and at this place the two divisions will coincide, the coinciding marks, however, not being homologous. Further on, the coincidence ceases, and the marks of the two divisions again separate; but at some distance from this, their deviation being increased to an entire interval, the two divisions again coincide. If, for example, the division marked 451 coincided, in the first case, with the division marked 450, so as to give a deviation equal to one of the spaces; at the second coincidence the deviation would be two spaces; that is, if one the divisions was 502, the other would be 500. It would be three spaces if the coinciding divisions differed from each other by three units, and so on. Now, to follow out the consequences, let us take one of these cases, the second, for example. Since the marks 502 and 500 coincide, when seen through the double prism; this proves that the ray emanating from the mark 502, and subjected to the system of refraction which produces the image *e*, reaches the eye at *V* in the same direction with the ray emanating from the division 500, and subjected to the system of refraction which produces the image *o*; and as the divergence of these two rays could not be produced or modified by the glass prism, since the prism exerts upon them one and the same kind of refraction, not being crystallized; it follows that if the two rays 502 and 500, coincided on arriving at the eye, they coincided also while traversing the glass; and consequently they coincided at their emergence from the crystal. It is this condition which furnishes a very precise method of verifying the law of divergence of the two rays in the crystal, depending on the difference of the velocities. Indeed, we can determine at once the direction of incidence of each of these two rays. If we take *OI*, for instance, we know that it comes from the point *O*, whose position is known on the vertical division, and that it comes thence to the point of incidence *I*, whose position is likewise determined on the column by its height and distance from the vertical division. We have analogous data for the incident ray *E i*, subjected to another system of refraction, whether we consider its point of incidence the same as that of *OI*, or calculate the small difference between these points, having regard to the thickness of the crystal prism, as I shall explain hereafter. Now, if we suppose the laws of the two refractions *o*, *e*, to be known, or only indicated by some hypothetical analogy which we propose to verify, we have only to follow at once through the crystal, the ray *OI* subjected to the refraction *o*, which may be done according to the law which we adopt for this refraction, and thus trace it to its emergence from the surface at *I'*. Then we consider the ray *e* which enters the crystal by this surface, as derived from the same exterior ray *I'I''*; and tracing this ray through the prism to the first surface, by the law of refraction to which we suppose it subjected, it ought, on emerging from this surface to coincide with the incident ray *E i*. It may be proper to remark that this condition and the observation itself, are altogether independent of the greater or less refracting force of the glass prism. This prism serves only to receive the rays refracted in the crystal, and to render their emergence possible.

105. In this example we have supposed the prism to be cut in such a manner, that the refraction shall take place in the

plane of incidence as in ordinary refraction. This is the most simple case, and is much the most convenient for determining the elements of double refraction in each crystal, when the law of this refraction is supposed to be known. But to verify the law itself or any one of its particular consequences, it may be necessary to observe the refraction in planes oblique to the cutting of the prism. For this purpose we adapt to the vertical division, the additional appendage represented in figure 172. *A′C′* is a metallic plate divided into millimetres,* and terminated at *C′* by a circular plane *P′P″P‴*, divided into half degrees. *RR* is another plate divided also into millimetres and moveable about *C′*, the centre of the circular division. On this plate and in the direction of its length, is traced a fixed straight line *RR*, which passes through the centre *C′*. When we wish to observe the lateral deviations, we apply the branch *A′C′* horizontally to *AZ*, the vertical division of the principal apparatus, and fix it there by a clasp which shall keep it in a direction perpendicular to this division, as represented in figure 172. Then looking obliquely at the rule, *RR*, through the crystal prism, and with a point of incidence which we determine, on this prism, by means of a mark or some other small signal, we turn the rule slowly about its centre till its axis *RR* is found to have exactly the direction of double refraction, at the place we are considering; which happens when the axis seen through the prism, no longer appears laterally double, its image *o*, *at this point*, falling upon its image *e*. When we have attained this position, we leave the rule there and note those of its divisions whose divergence is such as to coincide with each other. Suppose this to take place in the most perfect manner possible at *E′*, *O′*. Then the numbers of the marks *E′*, *O′*, will give their distance from the centre *C′* of the circle; the circular division will measure the inclination of their axis *RR* to the horizontal axis *C′A′*; and the division traced upon this axis will give the distance of the centre *C′* from the vertical division *AZ*. Accordingly, the position of the two points *E′*, *O′*, will be known with respect to the vertical division, and their divergence will be known by the coincidence observed upon *RR*. We shall then have all the elements of the double refraction belonging to the position of the prism, and the direction of the rays, which we have chosen for the experiment; and we can compare these results with the theory.

---

* A millimetre is 0,03937 of an English inch.

In performing this operation, we must not omit to fix with care the position of the point of incidence on the anterior surface of the crystal; for this position is one of the indispensable elements of the calculation, since it determines the direction of incidence of the plane. We may, as already mentioned, employ a mark made upon the prism, or a silk thread extended on its surface, or we may paste on it a small strip of paper to limit the rays whose simultaneous incidence is observed. Similar means may be used to fix the heights of the points of incidence, when we observe the coincidences on the vertical division itself without lateral deviation; but in this case the strip of paper which serves as a limit, must be directed horizontally.

We may also observe coincidences on the horizontal division *AX*, which serves as a base to the column *Hh*. In this case, the points of incidence on the first surface of the crystal prism are limited by the same met[illegible]ds.

106. Finally, it is necessary, as far as possible, to render the edges of this prism sharp or thin, that the corrections relating to its thickness may be very small or even insensible. Indeed they would be altogether nothing, if the observation were made through the edge itself, since then the two refracted rays would have only an infinitely small space to pass through in the crystal, in order to arrive at the second surface, where they unite and emerge simultaneously. For the same reason, we must not, in our experiments, place the column *Hh* very near the vertical division on which the coincidences are observed, nor must we employ too short columns, when we make observations on the horizontal division, because the corrections for thickness, hardly sensible at a moderate distance, become more sensible when the distance is very small.

To these precautions must be added that of making the faces of the prisms as plane as possible, because this is very favorable to clearness of vision. It is also necessary to measure the angles of the faces by the reflection of light, making use of the goniometer of Wollaston, or that of Malus, or some similar instrument.

107. There are two principal advantages of the method now

described; in the first place, the precision of the measurement extends to the divergence itself of the two rays, and not simply to the absolute incidence, in which a small error is seldom of any importance; in the second place, the alternations of coincidence and separation of the marks perform, in a manner, the office of verniers, by enabling us to fix with extreme accuracy the point where each coincidence is most perfect. The precision is rendered still greater by the facility with which we can move the column *Hh* to a greater or less distance from the vertical division which enables us to vary the extent of the coincidences, and thus to fix the precise distance at which each takes place most exactly.

108. According to what has been already observed, the course of the ray which experiences one of the two refractions, is the element from which we set out in calculating the course of the ray which experiences the other refraction. The first determination is easy in crystals with one axis, where one of the refractions always follows the law of Descartes; and in this case, we set out from the ordinary refraction, as being most simple. It is then necessary to determine the constant ratio which is its characteristic. This can be done simply by the above apparatus. For this purpose, it is sufficient to cut in the crystal prism a face so little inclined to the others, as to admit of our observing the Fig. 173. refraction directly through the angle thus formed. Then we put this angle on the column *Hh* by one of its faces, and place the eye behind the other, at *V*, for example, and observe by refraction the ordinary image of *O*, one of the points of division, and note at the same time to what point *R* the prolongation of the emergent ray *I′V*, which comes to the eye, corresponds. The positions *O*, *R*, being thus known, with the height of the point of incidence and the refracting angle of the prism, we can calculate the ratio of refraction.* It is only necessary to be well assured that the refraction which we observe is of the ordinary kind. As to this we may satisfy ourselves either by verifying the constancy of its ratio under different incidences, and with every variety of direction as to the cutting; or by observing the position which the ray thus refracted takes with respect to the other, in virtue of the law of extraordinary refraction,

---

* For the formulas see Biot's *Traité de Physique*, tom. iii, p. 209.

supposed to be known; or lastly, by means of a modification acquired by the luminous rays in the act of double refraction, which Malus, to whom we are indebted for the discovery, has called the *polarisation of light.* This modification, being different for the two rays, will enable us to distinguish them; and for this purpose it is sufficient, as we shall show hereafter, to observe the refracted division through a thin plate of tourmaline. The preceding method of observing refraction admits of a dispersion; but if the prism be not very open, and if we do not place it very far from the division which we observe, this will not prevent the refracted marks from being still distinguishable with sufficient clearness. Finally, we avoid these inconveniences by placing before the eye a coloured glass, which allows only one kind of light to pass, as red, for instance. It is necessary, however, in this case, to reduce the results by calculation to what they would have been if we had observed the refraction of the mean rays.

109. In crystals of two axes, where the two velocities are variable, there are also some modes of cutting, in which one of the velocities is found to have a constant value for all directions comprehended within it; so that they offer the same facility as crystals of one axis, for determining the constants of double refraction. Suppose, for example, in the plane of the two axes, the two straight lines at right angles to each other, of which the one is a middle or intermediate line between the axes, the other, called the supplementary line, dividing the supplement of their mutual inclination into two equal parts. If we form in the crystal two plane sections at right angles to each other, of which the one is perpendicular to the middle line, and the other to the supplementary line; one of the velocities will be constant in the first section, and the other in the second; so that we shall have in one or the other of these directions a constant ratio of the sines. These remarkable results were first made known by M. Fresnel, as the consequences which he had deduced from his speculations respecting the nature of light.

110. But there is another and a direct method by which we can verify this constancy of the velocities in certain sections of a crystal or other substance. We cut an artificial face perpendicular to each of these sections; we then cause a ray to be reflected inward with different incidences, and see if the angle

of reflection is found to be equal to the angle of incidence, under all inclinations; for this equality is a consequence of the constancy of the velocity and is a characteristic by which it is known. To determine, therefore, whether it takes place, we have only to adapt to a parallelopiped of the substance under examination, another parallelopiped formed of glass or any other substance producing simple refraction which has been wrought at the same time with the first, in such a manner as to have all its faces common, and compare at one view the directions of the images of a small distant object, as the flame of a taper, for instance, reflected inward at the second surface of both bodies. For if, as we have supposed, the distance of the observed object be very great, compared with the dimensions of the double parallelopiped, the two reflected images must present equal deviations when the velocity of the reflected ray is constant in each of the two substances; and a small difference between these velocities, like that which is capable of being produced by substances hitherto known, will have absolutely no sensible effect in disturbing this equality, as may be easily ascertained by calculation. It is sufficient then to observe whether this equality exists, and continues through the progressive different inclinations which may be given to the reflected and incident rays by moving the double parallelopiped before the eye.

111. To avail ourselves of the operations performed by the preceding methods, as well as by other similar ones, it is necessary that the position of the axes of the substance on which we operate, should be determined with exactness in each of the fragments observed. This is easily done when the substance has been already examined, and the direction of its axes indicated with reference to some one of its natural faces; for then it is sufficient to separate a fragment at the natural cleavage, which will make known either this face itself, or some other related to it according to the laws of crystallization; so that we shall know the position of the axes in the fragment thus obtained, and consequently in the rest of the mass from which it is taken. When the substance which we observe is new, or when the position of its axes is not yet known, the fixing of these lines by double refraction alone, might require many ineffectual trials, especially on account of the aberration caused by the lateral deviations; but the polarisation of doubly refracted rays, offers for this purpose

a simple and sure indication by the aid of which the axes may be directly discovered with ease and accuracy. We may suppose, therefore, that their direction has been determined by this or some other process, and that it is actually known in the crystals which we wish to subject to observation.

This being done, suppose that by means of the apparatus above described, or some other similar method, we have determined for a certain crystal, the divergence of the rays for different directions about its axes; it remains to ascertain the progression of these divergencies, and form their general expression in relation to the axes of the crystal. Huygens, as before stated, has done this for Iceland spar, by means of a remarkable law which he connected with the theory of the undulations of light; but this same law has since been deduced by Laplace, from the theory of its materiality.

If we regard light as a material substance emitted from the luminous body, the refraction of the rays which traverse transparent bodies, is produced by the attractive forces, which the particles of these bodies exert upon the luminous particles, forces whose effect is sensible only at very small distances, and which in this respect are precisely similar to those which belong to chemical affinity. Accordingly, when a ray of light penetrates a refracting surface obliquely, the curved portion of the path which it describes, has only an infinitely small extent, inappreciable by our senses; so that the ray appears to be suddenly broken and changed, as to its direction, at the point of incidence. Now since the curve which it describes is imperceptible, we cannot deduce from its form, the nature of the forces which act upon the luminous particles at each point, as we discover the law of universal gravitation, from the form of the orbits described by the planets and comets; and we are accordingly reduced to the alternative of conjecturing the nature of these forces, from indirect inductions, and then of verifying it by the agreement of the results with experiment. Newton has done this for ordinary refraction, or where the velocity is constant, by considering each luminous particle which traverses a refracting surface, as acted upon before, during, and after, its passage, by attractive forces, sensible only at very small distances, and emanating from all the particles of the refracting medium. This definition specifies nothing respecting the law according to which these forces diminish in the space through which they are sensibly variable; it allows us only to calculate the resultant for each distance, and to suppose them constant when the distance becomes sensible. Now these data are sufficient to enable us to calculate, not the variable velocity of the luminous particles in their curvilinear motion, nor the nature of this motion, but only the ratio which exists between the velocities and the definite directions which they take, either within or without the refracting medium, when the distance of the luminous particles from the refracting surface has become so great, that the course of the ray is sensibly rectilinear; and this comprehends all the limits of distance within which we are able to observe.

112. In the case of extraordinary refraction, produced by crystallized bodies, we have not even this advantage of being able to define the origin of the molecular or elementary force, nor the manner in which it emanates from each individual particle of the crystal; for our representation of the phenomena by attractive and repulsive forces emanating from the axes, is only the indication of a combined result, and not the expression of a molecular action. All that we know, therefore, in this case, or at least all that we can suppose, when we adopt the hypothesis of the materiality of light, is that the forces, whatever they may be, which act upon the luminous rays, in this as in every other case, emanate from the particles of refracting bodies, and are attractive or repulsive; whether they exert a power of the same kind upon all the luminous particles, or whether the nature of this power varies according to the peculiar modifications of these particles. Now in cases where a material particle is acted upon by such forces, its motion is subject to a general mechanical condition, called *the principle of least action.* By employing this together with the particular condition, that the forces are sensible only at very small distances, Laplace has obtained two equations, which determine completely and generally the direction of the refracted ray, for each given direction of incidence, when we know the law of the definitive velocity of the luminous particles, in the interior of the refracting medium, at a sensible distance from its surfaces.

113. In the case of ordinary refraction, the definitive velocity is constant; for the deviation of the ordinary ray is the same in the same body, in whatever direction it be tried, provided

the incidence is the same, and the surrounding medium does not change. Also when we suppose the interior velocity constant, the equations deduced from the principle of least action, show that the refraction takes place in the prolongation of the plane of incidence itself, and according to the constant ratio of the sines, which is in fact the physical law of ordinary refraction in all bodies hitherto observed.

Hence it was natural to suppose that extraordinary refraction was produced by a velocity varying according to the direction of the ray, about the axes of the crystal. Beginning, therefore, with crystals of one axis, we have seen that double refraction is exerted symmetrically about the axis; and that, being nothing in the direction of the axis itself, it attains its maximum when the rays cut it at right angles. For the crystals in question then, we must confine ourselves to the laws of velocity which satisfy these conditions. Laplace has tried the following equation,

$$v'^2 = v^2 + k \sin^2 U,$$

in which v represents the ordinary velocity, v' the extraordinary, U the angle formed with the axis by the extraordinary ray, and k a constant coefficient for each crystal. The substitution of this law of velocity in the equations derived from the principle of least action, gave him at once the law of Huygens. This law was at first completely verified only for Iceland spar; I have verified it also for rock crystal and several other substances. Generally the coefficient k, is positive in crystals of double refraction attractive, like rock crystal, and negative in the others. Its absolute value changes, likewise, with the substance. Sometimes, indeed, it has variations for different specimens of the same kind of mineral, which may be owing to slight differences of composition or structure; but amidst these different modifications, the same law of velocities applies to all crystals of one axis, hitherto observed.

114. Now for crystals of two axes, the extraordinary velocity v' must be supposed to depend upon the two angles U, U', which the axes form with the refracted ray. Analogy, therefore, leads us to try whether the square of this velocity may not also be expressed by a function of the second order, but more general, that is, having relation to these two angles. Now, since

in every crystal of two axes, the two refractions become equal when the refracted ray takes the direction of one or the other of the axes, it follows, that in this case, the extraordinary velocity becomes equal to the ordinary. This condition limits the generality of the function of the second order, and reduces it to the following form;

$$v'^2 = v^2 + k \sin U \sin U';$$

that is, there remains only the product of the two sines.*

By introducing these formulas into the equations derived from the principle of least action, we determine the course of the rays for all cases, and it only remains to see whether this course agrees with the results of experiment. In order to make the comparison in a rigorous manner, I have instituted, by the method of coincidences, and with the aid of the apparatus above described, a great number of experiments upon the white topaz of Saxony, the yellow topaz of Brazil, the anhydrous sulphate of lime and euclase, which are crystals of two axes, and of a form and composition altogether different the one from the other. The measures given by experiment are found to agree with the formulas most perfectly. I have observed a similar agreement in several other crystals, by applying the same law to a different class of phenomena, but yet connected with double refraction. This accumulation of proof leaves no doubt, that the law, indicated also by such strong analogies, is, in fact, the law of nature. It may be remarked, that the law under consideration comprehends, as a particular case, the law of Huygens, for crystals of one axis; since, by considering these crystals as having two axes united in one, we have their mutual inclination equal to zero. Then the two angles U, U', formed with the two axes by the refracted ray, become equal; and the increase of the square of the velocity contains only the square of the sine. In this investigation I have supposed one of the two velocities to be always constant in the same crystal, whatever be the direction of the transmitted ray. This is indeed the opinion generally entertained; and since, by adopting it, and applying to it the principle of least action, all the *phenomena of divergence* which I have observed, were found to be represented with an exactness which left no room for sensible error, even in crystals whose double refraction is the most energetic, I confined myself to some direct attempts to verify this constancy; and finding that they appeared to confirm it, I dismissed all doubt upon the subject. Meantime the experiments of M. Fresnel, which I have myself verified both by his method and by that of interior reflections, already explained, show that the supposition of a constant velocity is not exact. But by two singular circumstances it appears why, by adopting it, the observations are capable of being so faithfully represented. On the one hand, the variableness of the velocities, which increases with the angle formed by the axes, is found to be exceedingly feeble and almost insensible, in the most energetic crystals which have been made use of; and on the other, this variableness is such, that the difference of the square of the velocities preserves the form which I have assigned it; that is, for each couple of rays which have the same direction of interior transmission, it is, according to M. Fresnel, really proportional to the product of the sines of the angles comprehended between this direction and the two axes of the crystal; whence we see that, by reason of the feebleness of double refraction in all known substances, the divergence of the rays, which is sensibly proportional to the difference of the velocities, is found to be expressed as exactly by the false supposition of a constant velocity, as by the true supposition of its variableness.

---

* I presented this law to the Institute with the observation by which it is proved, the 29th of March, 1819.

115. The ideas of M. Fresnel concerning the nature of light, suggested to him a geometrical construction, by which he represented generally the velocities of rays refracted by any crystal whatever. This construction, represented algebraically, gives the following expression for the velocities of two rays which accompany each other in their passage through the crystal, that is, which traverse the crystal in the same direction. Let v, v', be the velocities of these two rays, and U, U', the angles which they make with the two axes of the crystal; and let n and k be two constants, belonging to the two substances. We shall have,

$$v^2 = n^2 + k \sin^2 \tfrac{1}{2} (U' - U),$$

$$v'^2 = n^2 + k \sin^2 \tfrac{1}{2} (U' + U).$$

Whence we obtain generally,

$$v'^2 - v^2 = k\,[(\sin^2 \tfrac{1}{2}(U' + U) - \sin^2 \tfrac{1}{2}(U' - U)],$$

or by developing the multiple sines of the second member, shall have

$$v'^2 - v^2 = k \sin U \sin U';$$

that is, the difference of the squares of the velocities is prop tional to the product of the sines of the angles formed by rays with the two axes, agreeably to what we had before covered by experiment. When the crystal has only one a the two angles U, U', become equal, and then $U' - U$ is c stantly nothing. Hence it follows that the velocity v redu itself to n, and is then constant in all directions. This veloc which depends generally on the difference of the two angles U', answers then to what we call *ordinary;* and the other depending on their sum, is that which we call *extraordin* To abridge the enunciations, we shall hereafter adopt this m of expression.

Although the velocity v is, generally speaking, variable crystals of two axes, yet it becomes constant in these same cr tals, when the refracted ray is comprehended in a plane situa intermediate between the axes and perpendicular to the s plementary line; and this is what the formula represents; then $U = U'$, and the expression for v is reduced to the c stant n. On the other hand, if we draw a plane through supplementary line perpendicular to the mean line, all the r comprehended in this section will form with the two axes, gles U, U', supplementary to each other, and consequently th sum will be always constant and equal to 180°. Thus in section, the sine of the angle $\frac{1}{2}(U' + U)$ will have a const value equal to unity; whence we see that the velocity v' then become constant, and will have its square constantly eq to $n^2 + k$. These are precisely the first results at which Fresnel arrived.

116. If we represent by n' this particular value of the v city v', k will be expressed by $n'^2 - n^2$, and the general pressions for the two velocities v, v', will be

$$v^2 = n^2 + (n'^2 - n^2)\sin^2 \tfrac{1}{2}(U' - U);$$

$$v'^2 = n^2 + (n'^2 - n^2)\sin^2 \tfrac{1}{2}(U' + U),$$

ich the two constants n, n', have now a physical significa-

e do not undertake to say generally that the law given by philosopher, embraces all the phenomena, for we have not xamined the subject sufficiently to hazard such an assertion ; ve are at least convinced that it unites a great number, and it satisfies particularly all those which relate to the diver- e of rays simultaneously refracted. Accordingly we shall t it in what we have to say hereafter with respect to these omena.

7. It follows from the foregoing considerations, that in order able to predict the course of luminous rays, and their de- ons for all possible directions, in a crystallized substance ssing the property of double refraction, it is sufficient to rve these deviations in a single fragment of this substance, rections known with respect to the axes ; and after compar- these deviations with the theoretical formulas, to deduce ce the values of the two constants n and n', of which one esents the ordinary velocity, and the other the extraordinary, the angles U, U', are both right angles ; that is, when the traverse the crystal perpendicularly to the plane which con- the two axes.

ere, as in simple refraction, the velocities v, v', are not rigor- the same for the different simple rays which compose e light. The images also, given by the two velocities, are rally coloured and dispersed as well as the ordinary images, gh not in the same manner. We shall examine the laws of phenomenon in detail, when we come to speak of the decom- ion of light and the analysis of colours.

18. We shall also defer giving a particular description of a arkable fact discovered by Mr J. F. W. Herschel, son of celebrated astronomer; namely, that in the same crystal, axes of double refraction relative to the different simple rays often separated from each other, forming for each species of , couples unequally inclined, though always comprehended he same plane and symmetrically disposed about the same ight line which bisects the angles of all the couples. This ersion has been observed hitherto only in crystals of two . It hence results that with the same incidence, the an- U, U', formed by the refracted rays with the axes, are

different for different species of rays, which circumstance must affect the deviations they experience. Hence, in order to define rigorously all the phenomena, it is necessary to mention the particular species of rays with which they are supposed to be observed. Hereafter, when we use no limitation in this respect, it is to be understood as in the case of ordinary refraction, that regard is had to the mean rays of the spectrum, that is, to the green or yellow; or rather that we are speaking of a phenomenon in which the effect of dispersion is insensible or inconsiderable.

119. In general, for the same species of simple rays, the values of the coefficient k, or $n'^2 - n^2$, are different in crystals of different forms and different natures; whence it follows that double refraction exerted by these bodies is of very unequal energies. We find also that substances of nearly the same composition, act in this respect very differently. Common sulphate of lime, for example, has a very weak double refraction; whereas the anhydrous sulphate of lime has a very strong one. It is true that the primitive forms of these two substances are very different. But the pure carbonate of lime, and the magnesian carbonate of lime, both of which have the primitive form of rhomboids, and rhomboids whose angles are so nearly equal that the difference, though real and measurable, has been doubted, have also double refractions sensibly unequal. For the mean value of k, which is 0,543 for the first, according to my experiments, varies for the second from 0,581 to 0,591; and it is probable that this last variation is owing to a small difference of composition. Some beryls which do not differ externally, except in colour, give also values for k very sensibly unequal; and several specimens have even exhibited phenomena which seemed to indicate the existence of two axes. Hence we may conclude generally that when two crystallized substances differ in their composition or in their primitive form, they differ also in their property of double refraction; and reciprocally, that a difference as to double refraction supposes always a difference in composition or structure, which renders this kind of phenomena very useful in characterizing minerals.

120. The coefficient k, not only experiences variations in its value, in passing from one substance to another, but also in its sign. It is positive for some substances and negative for others.

This circumstance leads us to apply to crystals of two axes the distinction already made of two sorts of double refraction, one attractive, and the other repulsive.

Dr Brewster having succeeded in distinguishing crystals of one axis from those of two, by means of certain phenomena of colour which they present and which will be hêreafter explained, has subjected to this test a great number of crystallized substances; and he has found, in general, that all those which have only one axis, have such primitive forms, that the faces which surround the axis, are disposed about it in a similar manner. These forms are the rhomboid, the regular hexaedral prism, the isoceles octaedral prism with a square base, and the right prism with a square base. All the other forms have two axes, or do not exert a double refraction. M. Sorret, a skilful mineralogist of Geneva, who has made very extensive researches into the subject of the position of the axes, in double refraction, with respect to the faces of the primitive forms, has found that the plane which contains the axes, is always situated in a symmetrical manner in the solid adopted as the primitive form, or in one of its crystallographical derivatives; and the axes are so situated in this plane, as to make equal angles with the faces of the solid. These characteristics of symmetry leave ordinarily very little that is indeterminate as to the possible position of the axes in each form; and they are thus useful in limiting and directing the attempts that may be made to discover them.

121. In attending to the phenomena presented by crystals of one axis, we have remarked that they take place with perfect symmetry on the two sides of the plane which we have called the principal section, and which is drawn through the axis of the crystal and through the normal to the face of incidence, by which the rays enter. In crystals of two axes this symmetry does not generally take place; but we find it when the directions of the two axes are equally inclined to the face of incidence. Conceive then a plane drawn through each of the axes of the crystal, and through the normal to this face; conceive also a third plane, drawn through the same normal, bisecting the angle formed by the two former planes. All the effects of the crystal are symmetrical on the two sides of this intermediate plane; and the two refractions take place without lateral deviation, as in the principal section of crystals of one axis. More-

over, the preceding definition embraces the construction which we have assigned to this section. For, if we suppose the two axes to approach each other until they form one line, these three planes will become one, which will be drawn through the single axis and through the normal to the face we are considering, which is precisely the construction of the principal section for crystals of one axis, according to the definition we have given.

Reflection at the second Surface of Crystals.

122. The theory we have been explaining, is not confined to rays refracted extraordinarily by crystals. It applies also to those which are reflected inward at their second surface. But before entering into a detail of the consequences thus indicated, it is necessary to establish by experiment the principal characteristics of this kind of phenomena.

When a ray of light falls upon the first surface of a crystal, coming through a vacuum or any medium not crystallized, it is partially reflected in a single beam, in such a manner that the angle of reflection, reckoned from the normal, is equal to the angle of incidence. The attractive or repulsive force which emanates from the axes of the crystal, appears to have no influence on this phenomenon; for we may turn the crystal upon its plane in all possible directions without altering the intensity or the direction of the reflected ray. But it is not so with respect to the interior reflection which takes place at the second surface of the crystal. Each ray reflected at this surface, is generally divided into two portions which return into the crystal, the one experiencing the ordinary refraction, the other, the extraordinary, these terms being used in the sense already explained.

123. In order to comprehend the cause of this division, it must be understood that rays, refracted either ordinarily or extraordinarily, when they have penetrated into the interior of the crystal to a sensible depth, acquire such a mode of arrangement of their particles, that they can no longer be divided during their course through this crystal; and experiment proves that they would no longer be divided if they should traverse a second crystal contiguous to the first, and having its principal section

directed in the prolongation of the first. This particular mode of arrangement constitutes what Malus has termed the *polarisation of light.* Now when the particles which compose the same ray, refracted ordinarily or extraordinarily, approach the second surface of a crystal, at a distance sufficiently small to experience the influence of the reflecting forces proceeding from it, it happens, in general, that a certain number of particles are turned by these forces into directions different from those derived from refraction; so that in returning into the crystal by the effect of a total or partial reflection, they become anew susceptible of being divided into two refracted portions, ordinary and extraordinary. I say, in general, for there are certain particular positions, in which the reflecting forces do not alter the arrangement originally given by refraction to the luminous particles; and then the ray is reflected without being divided, or even escapes reflection entirely. It is sufficient for the present, to observe that the forces in question only influence the intensity of the reflected portion, and not the direction derived from reflection. A ray which is reflected single, or which emerges from the crystal without being reflected, would suffer double reflection, if the particles which compose it were otherwise disposed; as may be verified by experiment. Accordingly the direction of the reflection is the first thing to be determined.

124. This determination is easily made with respect to crystals of one axis, in which one of the velocities is constant. It is sufficient to rely on this fact, that the reflected ray must be affected on re-entering the crystal, as a ray would be, which came from without, and whose particles had not originally received any particular disposition. Now in case the crystal has only one axis, the direction of the return is determined by the known reflection of the ordinary ray having a constant velocity. Let I' be the point of Fig. 86
interior incidence, and $O'I'$ the incident ray. If it has suffered ordinary refraction, construct the ordinary reflected ray $I'O''$, which makes the angle of reflection equal to the angle of incidence, on the other side of the normal $I'N'$; then calculate by the theoretical formulas, the direction of the extraordinary ray $I'E''$, which corresponds to it at its departure from the point of reflection I', that is, which has proceeded from the same exterior incident ray; we shall thus have the two reflected rays which result from the division of the incident ray $O'I'$, after reflection.

If, on the contrary, this ray is extraordinary, draw it to the point g. 87. of incidence I'; then calculate by the theoretical formulas the ordinary ray $I'O'$, which corresponds to it on the same side of the normal; and having obtained this, proceed in the calculation as before; thus we shall have the two reflected rays $I'O''$, $I'E''$, into which the given ray is divided. In general, this is the rule; an ordinary and an extraordinary ray which accompany each other in their interior incidence, accompany each other also after reflection.

In the case where a crystal has two axes, the direction of the interior reflection can no longer be assigned, on the ground of symmetry alone, for either of the two rays; since, their velocities being variable, neither of them makes, in general, the angle of reflection equal to the angle of incidence. But in this case, the interior incident ray being given, whether ordinary or extraordinary, we calculate the emergent ray derived from it, then transfer this emergent ray to the other side of the normal, giving it an equal incidence, and calculate the refracted rays which it would produce on re-entering the crystal in this direction. These ought to be the same with the interior reflected rays; for it is every way manifest that the interior reflection takes place at a distance from the surface much smaller than that, to which the forces, dividing the ray, extend; so that the ray when reflected, ought, in its commencement, to be directed as it would be if the medium were not crystallized. It is only when it penetrates further into the crystal, that it is divided anew as if it came from without. Thus it appears that its primitive direction, after reflection, is to be deduced from the inversion of its emergence, as in the case of substances not crystallized. We see that this method comprehends, as a particular case, that just pointed out for crystals of one axis. But notwithstanding this analogy, I do not propose it without diffidence, and it is with the hope that experiments may be made to prove or disprove its correctness.

125. These are the general laws of reflection in the interior of crystals, both when a portion only of the luminous particles suffers interior reflection, and the rest exterior refraction; and when, the interior attraction being more powerful, all the incident particles are drawn inward by the forces which produce the refraction.

Here, as in ordinary refraction, the incidence at which this total reflection begins in each crystal, depends upon its greater or less refracting power, and that of the exterior medium; but we cannot in the same way calculate its limits theoretically, because we are ignorant how the attractive or repulsive force, which proceeds from the axes of the crystal, varies near its surface. It is, therefore, necessary to recur to experiment, and to determine the commencement of total reflection by the impossibility of obtaining an emergent ray. I have given a detailed account of this calculation* for crystals of one axis, and have developed the remarkable consequences which result from it, in relation to the variations which the forces, proceeding from the axes, undergo near the exterior surface of crystals. Crystals of two axes offer similar considerations. One of the best means of ascertaining the nature of these forces, is to place the surfaces of the crystal in contact with transparent media of a greater refracting power than themselves, for instance, with phosphorus in a state of fusion, and to note the limits, as well as all the other particulars of interior reflection in media thus terminated. Perhaps we may be able to measure by this process, the double reflection of opaque bodies, as Dr Wollaston has determined their simple refraction. This investigation requires some theoretical principles which will be detailed in the following section.

Passage of Light through several Contiguous Bodies possessing the Power of Double Refraction.

126. We have supposed the preceding experiments to be made in a vacuum, or in the air, whose proper action upon light is so feeble as to have no sensible effect. We are now to inquire what takes place when the rays which penetrate a doubly refracting crystal, instead of entering from the air, pass out of a medium possessing the power of double or simple refraction.

Let us begin with this latter case, which is less complicated, and for still greater simplicity, let us suppose also that the crystal, into which the ray is to pass, has only one axis, in which

---

* *Traité de Physique.*

Fig. 88. case the ordinary velocity will be constant. Let *AB* represent the common surface of the medium and the crystal, and *SI* the incident ray. We calculate, or construct by the theoretical formulas, the ordinary refracted ray *IO*, according to the common ratios of refraction of the two substances, and precisely as if the second medium were not crystallized. Then, knowing *IO*, we deduce from the formulas the extraordinary ray, *IE*, which accompanies it, and which depends solely upon the position of *IO*, with respect to the axes of the crystal; the two rays, *IO*, *IE*, will be those into which the incident ray *SI*, will be resolved.

127. We pass now to the case in which the first medium is itself crystallized, still supposing it to have but one axis. The given incident ray *SI*, may be subjected, in this medium, either to ordinary or extraordinary refraction. If the first take place, we may proceed as above, employing the ratio of ordinary refraction of the first medium, and performing the calculation, as if were not crystallized. But if the given ray *SI*, is itself subjected to extraordinary refraction, we begin by calculating in the first medium, by the formulas, the direction of the ordinary ray Fig. 89. *S′I*, which accompanies it. When this ray is known, we make use of it, as above, to calculate the two rays *IO*, *IE*, which are derived from it in the second crystal. These will be the directions sought.

It will be seen from this analysis, that, except the case of a too great attractive force in the first medium, these constructions give always, in the second medium, two rays, the one ordinary, the other extraordinary; whereas experiment seems to contradict the constancy of this result, at least when the first medium is crystallized, and endued with the power of double refraction; for then the subdivision of the rays which come from it, as well as the kind of refraction, ordinary or extraordinary, which they undergo in the second crystal, depends upon the position of this crystal with respect to the first. In order to fix these ratios with the more clearness, let us confine ourselves to crystals of one axis. In this case, if the principal sections of the two crystals are parallel, each ray from the first crystal, whether ordinary or extraordinary, remains single in passing into the second, and undergoes there, the same kind of refraction as it did in the first. If the principal sections, instead of being parallel are at right angles to each other, each ray coming from the first crystal, still remains single; but it changes its refraction, from ordinary in the first, to extraordinary in the second, and *vice versâ.* Between these two limits of position, each ray, whether ordinary or extraordinary, coming from the first crystal, is divided into two when it enters the second, and these portions obey the laws indicated by the preceding constructions. But the intensity of each portion depends still upon the angle made by the two principal sections, increasing or diminishing with this angle, according as the motion of the principal sections causes the portion to diverge from or approach to the limit where it ought to disappear. Hence we must conclude, that the formation or non-formation of two portions in the second crystal, depends upon the physical modifications which the particles may have acquired in the first crystal, modifications which render them better fitted to undergo the one or the other refraction in the second, according to the directions of their faces, with respect to its axes; which does not prevent the theory from indicating with exactness, the *directions of translation*, which these particles would take, if their physical state permitted them to be shared between the two refractions. Similar cases have already been presented by refraction at the second surface of crystals; for, in fact, a ray reflected inward at the second surface of a crystal, experiences the same influence, as if it emerged entirely from the crystal to enter another or return into the same.

128. Hitherto we have supposed that the contiguous media were composed of crystals of only one axis. This supposition, by rendering one of the velocities constant, allowed us to calculate its propagation by Descartes' law of the sines being extended to contiguous media of different refracting powers. This expedient cannot be employed when one of the contiguous media is a crystal of two axes; since then its two velocities are generally variable; but we can supply the defect by considering that the rays, at the moment when they traverse the surface of contact, are without the sphere of action of the forces by which the double refraction is produced; so that they are really in the same condition they would be subjected to, if they had passed with their preceding velocity, acquired from an uncrystallized body, into another likewise uncrystallized, but of a different refraction. In this case, their definitive course would be still the same, if, instead of introducing them thus directly into the second

medium, we were to separate this from the first by a small void space parallel to their common surface. Consequently this same supposition applied to the crystals in contact, would give the definitive direction of the rays which would be transmitted from one to the other, in the case where they were contiguous. Thus, when a ray subjected to any law of velocity whatever, is given in the first medium, we can calculate the emergent ray which would result from it, if it were to emerge into a vacuum; which would be done by means of the principle of least action applied to the general expressions of the velocities. Then, by the aid of the same principle, we calculate the direction which this emergent ray would follow, if it penetrated, with the same continued incidence, into the second medium; and this will be the direction which it will actually take when it passes directly from the first medium without any intervening space. Yet, notwithstanding the clearness of the analogies on which the rule is founded, it would be well to institute experiments for the purpose of verifying it.

Construction of Double Image Micrometers.

129. Rochon employed the double refraction of crystals for measuring small angles, in a manner too useful in astronomy and the physical sciences, not to find a place here. Besides, the same apparatus affords a most simple means of ascertaining whether the double refraction exerted by a crystal, is attractive or repulsive.

90, 91. Suppose two prisms *A*, *B*, formed of the same crystal, of one axis, and cut in such a manner, that, in the first *A*, the exterior face *AB* is perpendicular to the axis *AA′*, of the crystal; while, in the second *B*, this axis is the common intersection of the two faces *A′B*, *A′B′*. Suppose further, that the two prisms are equal to each other as to the dimensions of their parts and the size of their angles. Let them be brought into perfect contact with each other, their refracting angles being disposed in such a manner, that their union may form a plate whose exterior faces shall be parallel, as represented in the figures. Then suppose an incident ray LI, directed perpendicularly to the surface of the first prism, and let us see what will take place.

Throughout the interior of the first prism, the ray will move in a straight line without being divided. Its course will not be broken at the first surface, because it is perpendicular to it; and it will not be divided in the first prism, because it is parallel to the axis of the crystal, and the force, whether attractive or repulsive, which emanates from this axis, can have no effect upon it.

When the luminous ray has arrived at I', the common surface of the two contiguous prisms, it will be divided into two rays, on its entrance into the second prism. The ordinary ray will not deviate, because it passes from one medium into another of the same refracting power; but will continue its course in a straight line, and emerge perpendicularly from the second face $A'B'$. It will be seen, therefore, in the prolongation of its primitive direction. But this will not be the case with respect to the extraordinary ray; for this, on entering the second prism, will experience, together with the ordinary refracting action, that of the attractive or repulsive force which emanates from the axis, and will undergo a new refraction resulting from it; this will take place, like the other, according to the simple law of Descartes, from the manner in which the prisms are cut. If the crystal is repulsive, as in figure 90, this second refraction will be less than the first, and the extraordinary ray $I'I''$, in traversing the second prism, will diverge from $N'N''$, the normal to the common surface, more than the ordinary ray $I'O$; and it will consequently, after its emergence into the air, be carried more towards the vertex of this prism. The reverse will take place when the crystal is attractive, as in figure 91; the extraordinary refraction in the second prism being stronger than the ordinary refraction in the first, the extraordinary ray $I'I''$ will diverge from the normal less than the ordinary ray, and will consequently, after its emergence into the air, be carried more towards the base of this prism.

130. We have supposed the two prisms immediately contiguous; but as this perfect contact can never be realized in practice, we cement the two surfaces together, by means of a stratum of inspissated oil of turpentine, or mastic in tears, substances which are transparent, and whose refracting power is nearly equal to that of crown glass. As we make this stratum very thin by pressing the two prisms together as closely as possible, we may suppose its two surfaces parallel. Then the ordinary ray *II'*, in traversing it, experiences exactly equal and opposite refractions, at its entrance and emergence; so that it takes its primitive direction when it enters the second prism; consequently the extraordinary ray, derived from it, takes also in this prism, the same direction as if it had entered it immediately. The intervening stratum has properly no other effect than to determine, by its attractive force, the emergence of the rays from the first prism, and their entrance into the second, under incidences with which these phenomena would be impossible, if it were necessary that the ray, in order to pass from one prism to the other, should first emerge into the air.

Since the ordinary ray, after its emergence from the second prism, preserves its primitive direction, while the other diverges from it, it is evident that an eye placed at *O*, in the prolongation of the first of these rays, cannot receive the second at the same time; but if through the point *O*, we draw a line *O i''*, parallel to the extraordinary emergent ray *EI''*, and if we produce this line through the two prisms, according to the laws of extraordinary refraction, which will give an emergent ray perpendicular to the surface of the first prism, it is evident that this line will represent the direction of an incident ray, parallel to the first which we have considered, and such that the extraordinary ray resulting from it, will pass through the point *O*, where the eye of the observer is situated; while the ordinary ray which accompanies it will not come to *O*; if then the object *L* is sufficiently distant to send thus two incident parallel rays, or only sensibly parallel, the observer at *O*, will see two images, the one ordinary, the other extraordinary, but proceeding from different incident beams. If the prisms are formed of a repulsive crystal, the extraordinary image will be less refracted by the second prism, than the other, and will consequently appear further removed towards its base *BB'*; if, on the contrary, the crystal is attractive, the extraordinary image will be more refracted than the ordinary, and will appear further removed from the base *BB'*. Thus, when we have put together, in the manner above described, two similar prisms, made of any crystal whatever of one axis, we shall know by this indication whether the crystal is attractive or

Fig. 90, 91

Fig. 90.

Fig. 91.

repulsive. Other more simple, but less direct means of attaining the same end will be described hereafter.

131. But how are we to distinguish between the two images, which is ordinary and which extraordinary? The phenomenon of polarisation would furnish us with a very simple method; but if we would make use only of the characteristics of double refraction, we have merely to apply the first surface *AB*, to a vertical glass plate, as that of a common window, for instance, and look round for some object terminated by a rectilinear edge, so situated that the rays which emanate from it, shall come perpendicularly to the glass. The edge of a roof parallel to the window where we operate, and situated at nearly the same height, would sufficiently satisfy this condition. Then, looking at this through the double prism, we shall see it double; but if the two prisms have equal angles, as all our reasoning supposes, one of the two images will always be situated exactly in the prolongation of the direct image, seen through the glass simply. This, then, will be the ordinary image; for only the ordinary emergent rays can be parallel, after their emergence to the incident rays. The other or diverging image will be the extraordinary one; and if we would determine the direction of its deviation, we turn the double prism on the glass, until this image also becomes parallel to the direct image, and observe whether it appears removed towards the base or the vertex of the second prism. In the first case, the crystal is repulsive, in the second, attractive. Another property characterizes also the ordinary image; it is that of being perfectly *achromatic*, that is, devoid of colour. Indeed the rays which form it, experience no more decomposition through the double prism, than if they had traversed a plate with parallel faces. But it is not so with the extraordinary rays. These experience a decomposition of the colours, because their successive refractions are not compensated the one by the other; and thus the extraordinary image is not achromatic.

132. In the preceding observations, the angular divergence of the two images, is measured by the angle $I'O'i''$, under which the direction of the extraordinary rays cuts that of the ordinary rays after their emergence. This angle varies with the nature of the crystal of which the prisms are made, and with the angular opening ABA', which we give them. We can easily calculate

it when we know this opening and the two ratios of refraction; thus we find that it increases according as the two prisms have a greater opening, and the ratios of the two refractions differ more from each other. It is on this account, that of all known substances, the Iceland spar, with the same opening, gives the greatest deviations, being that whose double refraction is the most energetic; and the divergence, obtained with double prisms formed of this substance, may amount to 43° 26′ 50″. It has long been believed that double prisms of rock crystal, could give a divergence of only 30′ or at most 40′. Malus has fixed the limit at 28′; but this arises from the circumstance that we always suppose the prisms separated by a stratum of air. Such a stratum does not influence the direction of the rays when they have once entered the second prism; but the small refracting power of air opposes an obstacle to their emergence from the anterior prism, and the above estimate gives for the limit of the angle, that which permits the ray to emerge from this prism into the air. Theory and experiment agree in proving that this limitation is incorrect. The divergence obtained with double prisms of rock crystal, may amount to more than ten degrees.

133. We may likewise employ prisms cut in directions different from the one above considered; and by following the course of the rays through them, as we have already done, and having regard only to the new directions of the forces which emanate from the axes, we can determine with equal facility, the directions of the emergent beams. Only it must be remembered, that in virtue of modifications impressed upon the luminous rays by doubly refracting substances, the rays thus refracted by the first prism, may or may not be divisible by the second prism, and may experience the ordinary or extraordinary refraction, according to the relative positions of the axes of the crystal, in the two contiguous prisms. But this distinction is unnecessary in the direction of cutting chosen by Rochon, because, the course of the ray being parallel in the first prism to the axis of the crystal, it has no tendency to be refracted one way more than another, in the second prism, and is acted upon there like a natural ray.

134. Suppose, in general, two prisms of any crystal whatever, thus applied to each other; and suppose that we have determined, by experiment or calculation, the constant angle formed by

the two emergent rays, derived from the same incident ray, perpendicular to the first surface. If we produce these two rays to their common point of meeting *c*, we may regard them as the feet of a pair of compasses, whose angular opening is determinate, and the vertex *c* known. If we have a circular disc whose diameter we wish to measure, we have only to place it between these branches and move it along till it touches them; then we can calculate its diameter by knowing its distance from the vertex of the angle. The operation will be the more exact, according as the angle is smaller; for then a very small difference in the diameter of the disc, will produce a very considerable one in the place of contact. Fig. 92.

Rochon applied this process very ingeniously to the measurement of the apparent diameters of the heavenly bodies. For this purpose he introduced the system of two prisms into the interior of an astronomical telescope. Let *A*, represent the object-glass of this telescope, *AF* its axis, *F* its focus, and *SS′* a very distant object, whose first edge *S* we suppose to be exactly in the prolongation of the axis *AF*. The pencil of rays emanating from *S*, and which covers the surface of the object-glass, is concentrated by it at the focus *F*, situated in the axis of this pencil, and forms there a luminous image of the point *S*. The pencil of rays emanating from *S′*, being concentrated in like manner, in the prolongation of its axis *AF′*, forms at *F′*, a small image of the point *S′*, and a similar effect being produced by all the other pencils emanating from the intermediate radiating points, the series of foci *FF′*, forms a small image of the object. Moreover, if the angle *FAF′* or *SAS′*, subtended by the object is very small, and if the object itself is very distant, all the points of the image will be sensibly at the same distance from the object-glass *A*; so that we may consider it in the figure as a small straight line *FF′*, perpendicular to *AF*, the axis of the object-glass. Then the angle *FAF′*, or its equal *SAS′*, will be the apparent diameter of the object, seen from the point *A*. This being well understood, place between the object-glass *A*, and its focus the double image apparatus, in such a manner, that its first surface shall be perpendicular to the axis *AF*. This will not sensibly affect the size of the image *FF′*, at least, if the exterior surfaces of the apparatus are exactly parallel. But it is evident there will hence result two images at the focus. In- Fig. 93. Fig. 94.

deed each incident ray *AF*, *AF′*, will be divided on entering the second prism, and will give an extraordinary emergent ray *cf*, *c′f′*, which will take the direction of emergence assigned by double refraction. Confining ourselves now to the consideration of the axes merely of the two portions, we perceive that the points *c*, *c′*, where the divergence for the axis of each pencil takes place, will be fixed in the prismatic apparatus, at whatever distance we place it from the object-glass; and the angles of deviation *Fcf*, *F′c′f′*, will be likewise fixed. Hence it follows, that if we remove the prismatic apparatus from the focus of the object-glass, the extraordinary image *ff′*, which remains always in the plane *FF′*, will withdraw from the ordinary image; and on the other hand, it will approach the ordinary image, if we bring the prismatic apparatus nearer the focus. Finally, when the movement is such as to bring the points *c*, *c′*, upon the line *FF′*, at the focus itself, the emergent rays, whether ordinary or extraordinary, proceeding from the same pencil, will diverge together at their departure from the same point, and will only produce upon the eye, the effect of a single radiating point; so that the two images will exactly coincide in all their parts.

As we depart from this position, if the constant angle of deviation *Fcf*, exceed *FAF′*, that is, the apparent diameter of the disc, there will be a position of the apparatus, somewhere between *A* and *F*, which will bring the two images *FF′*, *ff′*, exactly in contact, as represented in figure 95. In this case, the ordinary image *FF′*, will be exactly contained between the two sides of the angle *Fcf*, which expresses the constant deviation produced by the extraordinary refraction. If, therefore, its distance *Fc* from the vertex of the angle be measured (which we shall soon see can be done), we shall know its absolute magnitude *FF′*; and as we know also the distance *AF*, which is equal to the focal distance of the object-glass, we can calculate the angle *FAF′*, which it subtends at the point *A*; this will be the apparent diameter of the object. If this diameter amounts only to a very small angle, it is shown by calculation that it is proportional to the distance *Fc*; at least if we always make use of the same object-glass, and the same double prism.

135. The distance *Fc* may be measured by means of a longitudinal division of equal parts, traced on the outside of the tube of the telescope. This tube is divided in the direction of its length,

so that the prismatic apparatus may be moved at pleasure through the whole focal distance *AF*. We begin by determining its position in the focus. For this purpose, we direct the telescope to the object, and move the prisms towards the eye, until the two images formed at the focus exactly coincide. We then read the point of lateral division, to which the index, carried along with the prismatic apparatus, corresponds; this point is the zero, from which the distances *Fc* are to be counted. Suppose, for instance, that it answers to the number 50. When we have observed any object and brought the images into contact, we read anew the point of division to which the index answers. Suppose this number to be 125; then, *for this value of the apparent diameter*, *Fc* will evidently be equal to 125 — 50, or 75 parts of the division. In order to know, once for all, the ratio of this number to the apparent diameter, we employ a circular Fig. 96. or spherical object *SS′* of a known diameter, placed at a distance *AC*, ascertained by direct measurement, or by a trigonometrical operation. We may hence calculate the apparent diameter *SAS′*, subtended by this object, seen at the distance *AS*. This being done, we observe the same object through the prismatic telescope, placing the object-glass at the same point *A*; and when we have brought the two images into contact by the motion of the prisms, we measure on the lateral division the distance *Fc*. Then the ratio of this distance to the apparent diameter *SAS′* is known for all cases, and we can employ it in all our other observations. Or rather we may make use of it in calculating beforehand the apparent diameters which answer to a certain number of values of *Fc*, and write them on the tube against each distance. This is commonly done in instruments intended for use. But instead of expressing the apparent diameters in minutes and seconds, we express the ratio between the distance of the object and its magnitude, which enables us to deduce one of these elements from the other. Thus, from the mean height of the men who compose a body of troops, we can determine their distance; we can likewise determine the distance of a ship at sea, from the supposed height of its mast. Our results, however, are the more liable to error, according as the distance is greater and the object smaller; so that we must not think of employing this expedient for the purpose of ascertaining the distance of the heavenly bodies, although it has been sometimes proposed. Their apparent diameter is the only element to be determined by means of it.

136. Hitherto we have supposed the first edge F of the ordinary image FF', to be precisely in the axis of the object-glass at the moment when we observe the contact. This condition is absolutely necessary in order that the incident ray AI, which, after its division, comprehends the ordinary image, may traverse the prismatic apparatus perpendicularly to its exterior surfaces, which is the only case we have yet considered. But if the observed object were a heavenly body, whose motion caused it to pass successively through the whole field of view of the telescope, what would be the result? Mathematically speaking, the value of the angle $F\,cf$, would no longer be constant in the different stages of its passage. If these variations are insensible, which is the case when the refracting angles of the prisms are very small, we can establish the contact of the two images when the heavenly body has entered the field of view, and it will continue through the whole extent of this field; but if we much enlarge the opening of the angles of the prisms, and the deviation which is consequent upon it, the angle Fcf will begin to vary sensibly for the different incidences which the field of view admits of, and the images after they are once brought into contact, will separate in traversing it. To avoid this inconvenience, Rochon proposed to substitute in the place of double prisms of a great angle an assemblage of several similar prisms, but each of a very small angle, and cemented together in such a manner that all the principal sections should coincide exactly in the same direction. Indeed, in such a system, the separation of the rays increases with the number of double prisms, and the influence of the variation of incidences on the divergence of the images is much less sensible than in one double prism only, which gives an equal divergence; for this, we might easily give a reason founded in theory. But great care must be taken that the superposition of the prisms be exactly according to the principal sections, in order that the images may not be multiplied beyond two; and we must also use certain precautions in cutting the prisms, that they may not be coloured. These particulars, taken together, render the process in question almost impracticable, at least for astronomical purposes.

137. In what precedes, we have supposed the images FF', ff', formed by the object glass at the focus, to be observed by the naked eye; whereas, in general, we look at these images through a magnifying glass, or a system of glasses disposed in

such a manner as to enlarge them indefinitely without rendering them indistinct. This is called the *eye-glass*, because it is situated in the part of the telescope next the eye, in the same manner as the former is called the *object-glass*, because it is situated towards the object. But as the action of the eye-glass is subsequent to the formation of the double images, it can evidently have no influence on the existence or non-existence of their contact; but only enables us to judge of it with greater precision. Thus all we have said on the supposition that we use the naked eye, is equally true in the case where the eye is provided with a glass; accordingly we have not had reference to this modification, in the account we have given.

The celebrity of Rochon's micrometer, and the numerous applications which are made of the process of division of images, on which the instrument depends, have rendered it necessary to go into this minute description of it. Still it is not exempt from inconvenience in practice, especially when we would apply it to cases of great magnifying power, as is required in astronomical observations. Accordingly, M. Arago has devised another and a decidedly preferable way of employing double refraction, which will be made known hereafter.

Of certain Singular Appearances occasioned by Double Refraction.

137. When we look at small objects through a rhomboid of Iceland spar, the disposition of the two images presents certain peculiarities, which are so many consequences of the theory. These did not escape the notice of Huygens. As they might embarrass persons unaccustomed to considerations of this kind, it may be worth while to take a brief notice of them.

Let L be the radiating point, and S the eye, which, for greater simplicity, we will suppose situated in the plane of the principal section $ABA'B'$ of the rhomboid. Let us inquire in what manner vision will take place under these circumstances. Fig. 97.

Here, as in the case of refraction through any other transparent body with parallel faces, there will be an ordinary ray which can reach the eye. Let LII' be this ray. As it enters at *I*, by the first face of the rhomboid, there will be an extraordinary ray II'_1; but this cannot come to the eye. To be assured of this, we have only to draw through the point of incidence *I*, the line *IA*, parallel to the axis of the rhomboid. The force emanating from this axis will repel the luminous particles, and those which yield to it, must evidently be removed beyond the ordinary ray, taking, for instance, the course II'_1. But having arrived at I'_1, in the second surface, they will emerge parallel to their primitive direction *LI*. Hence there results a ray I'_1S', which being parallel to *I'S*, cannot pass through *S*, where the eye is placed. For a still stronger reason, the same may be said of every other extraordinary ray proceeding from the incident rays, which approach nearer to the obtuse angle *A'*.

The extraordinary image will therefore be given by the incident rays which diverge from *LI* in the contrary direction, that is, which approach the solid angle *B'*. Among these, there will be found one, as *Li*, whose ordinary branch cannot come to the eye, but whose extraordinary branch *i i'*, may come to it after its emergence; so that the eye will receive two images, the one ordinary in the direction *SI'*, the other extraordinary in the direction *S i'*; and the first will always appear nearer the small solid angle *B'*, than the other. If the position of the eye and that of the radiating point, with respect to the rhomboid, be given, we can easily calculate the directions of these two rays, by taking as unknown quantities the angles of incidence and emergence, which they form with the two faces of entrance and emergence. Because, for each ray, ordinary or extrordinary, these angles must be equal to each other. But the result being one of mere curiosity, it is sufficient to have indicated the general course. We can prove the crossing of these rays in the interior of the rhomboid, by a very simple expedient devised by Monge. Things being disposed as in the figure, we pass a card slowly over the surface *A'B'*, situated towards the radiating point. When it has come to *i*, it will intercept the incident ray *L i*, which gives the extraordinary image. We shall then see the emergent ray *S i'* disappear, although from the direction in which it seems to come, we should expect to see the ray *SI'* disappear first. In order that this priority may be accurately observed, we must place the eye very near the rhomboid, which will enlarge the angle *S*: and in order to have a luminous object of a small diameter, we may look at the light of the sky, through a small hole pierced in a card, or what is still better, at a black dot made on a white sheet of paper. In each case, the luminous object must be placed at a considerable distance. For the more the radiating point *L*, approaches the surface of the rhomboid, the more will the point *K*, where the rays cross each other, approach also this surface. But however near we suppose *L*, provided it be without the crystal, so that the formulas of Huygens may be applied to the luminous rays which emanate from it, the intersection of the two rays will take place in the interior of the rhomboid; and consequently, the phenomenon we have described, will continue to occur, although with different divergences. Figure 98 represents the case where the radiating point and the eye are situated in the same straight line perpendicular to the faces of the rhomboid.

138. When we look thus, through a rhomboid, at the two images of a luminous point, whatever position we otherwise give it, the ordinary image will always appear nearer the eye than the extraordinary image; this likewise is a consequence of the theory.

In order to understand the reason of it, let *L* represent the radiating point, *S* the centre of the eye, and *SI′* the ordinary ray refracted in passing through the medium *ABA′B′*, whose opposite faces are parallel. If the eye were only a mathematical point, the ray *I′S*, would be the only one of its kind which could come to it; but as the pupil has a certain extent, it will be seen that it must receive also a certain number of other ordinary rays near to *SI′*, and consequently derived from incident rays near to *LI*. These rays departing from *L*, form a cone which has its vertex in *L*; they form another cone when refracted by the medium, and another still when they emerge from the second face *AB*. Now if we calculate the distance *SL″*, from the eye to the vertex of this last cone, whatever be the nature of the intervening medium, whether it be crystallized or not, we shall find that the point *L″* is always nearer the eye, than the point *L*; and the more so, according as the medium has a greater refracting power; because its refraction increases the divergence of the emergent rays. Now it is precisely this divergence, which enables us, in general, to judge of the distance of the luminous points and their images; so that the refracted image Fig. 99.

always appears nearer than the direct image. But in the rhomboid of Iceland spar, there are two refractions, of which the ordinary one is always the strongest, because the velocity thus impressed upon the light, is diminished in the other by the repulsive force. The ordinary images must therefore always appear nearer the eye, at least while the refraction takes place through plane faces, as we have here supposed. The contrary would be observed in a rhomboid formed of an attractive crystal, because the extraordinary refraction would then be more powerful than the ordinary.

139. In attending to the course of the two refractions in the plane of the principal section of a rhomboid of Iceland spar, we have seen that the extraordinary ray is always carried towards the small solid angle of the second face. In this case, if the
100. incident ray *SI*, is also directed towards this angle, there exists an incidence, under which the repulsive force emanating from the axis *IA′*, exactly counteracts the effect which the ordinary forces tend to produce, and the direction of the extraordinary ray is not changed by refraction. The incidence at which this phenomenon takes place has been calculated and found to be 16° 45′. It only takes place on the side of the normal which we have considered; since, on the other, the repulsive force acts in the same direction with the ordinary refracting forces, and increases the ordinary deviation of the ray. But this last would be the side for the occurrence of the phenomenon, if the crystal were attractive.

Of Double Refraction in Compressed Glass.

140. Some substances which do not naturally possess the property of double refraction, acquire this property when their state of aggregation is artificially modified by external pressure. This important fact, already rendered very probable by certain physical characteristics presented by luminous rays, after having traversed such bodies, has been completely proved by M. Fresnel, in the following manner.

On the same straight line, parallel to each other, are arranged four glass prisms, whose refracting angle is a right angle, and which are designated by *A*, *A*, *A*, *A*. These prisms having each one of its lateral faces *A*, placed upon the same plane, and being brought into contact by their longitudinal edges, we apply to their extremities a strip of pasteboard, so as to cover them, and on this one of wood, and then one of steel; after which, we introduce the whole into a sort of vice, the jaws of which, acting against the exterior plates of steel, press them against the opposite ends of the prisms, and consequently exert on these prisms, in the direction of their axes a nearly uniform longitudinal pressure. This being done, we introduce between these prisms thus compressed, four other prisms *B*, *B*, *B*, *B*, of the same refracting angle and the same kind of glass, but in their natural state of aggregation; and we cement them to the first by a stratum of oil of turpentine thickened by heating, which, by establishing the contiguity of their surfaces, renders possible the passage of light through the system of eight prisms, notwithstanding the largeness of their refracting angle. Thus, when a very slender luminous ray is introduced perpendicularly through the first surface of this apparatus, it emerges freely and nearly perpendicularly through the last; and moreover (which is the main point to be attended to) it emerges divided into two portions, whose sensible divergence increases for the same distance, according as the exterior pressure becomes more considerable. In some cases, the divergence amounts to 0,06 of an inch for 40 inches distance, so that it is impossible to mistake it; nor can we suppose that it is produced by a doubling of the path or trajectory, analogous to that of mirage, which takes place in media of variable density; for, in consequence of the number of prisms, there would in this case be formed more than two images. But the circumstance which removes all doubt on the subject, is, that the two emergent portions exhibit, either in their absolute properties or in their relations to each other, the physical modifications which characterize the two portions emerging from a doubly refracting crystal. Fig. 100.

141. This experiment of M. Fresnel, therefore, completely proves that exterior pressure imparts to glass the property of double refraction; but it does not teach us in what manner the pressure developes this property; nor can we conclude from it, that the double refraction of bodies naturally crystallized, is

owing to the same cause. It is evident, that this property results from the mode of aggregation of the particles, because it ceases to exist, when we change this mode, or when we destroy it, as may be done in many crystals by fusion or dissolution in liquids. But we are ignorant how these two things are connected together. We cannot even say with certainty what takes place in a flexible and elastic body, such as glass compressed, nor what particular kind of modification is introduced into the system of aggregation of its parts. It is, however, probable that they are not only compressed individually, in the direction in which the exterior compressing force is exerted; but that the pressure thus exerted, acts with more or less energy in all directions, by virtue of the flexibility of the medium, together with its cohesion. But then the individual state of the particles becomes very complicated, and it is extremely difficult, if not impossible, in the present state of our knowledge, to foresee what it will be. This, however, far from diminishing the merit of the experiments which apply to them in the aggregate, rather enhances their value.

ANALYSIS OF LIGHT.

Of the Dispersion of Light produced by Refraction.

142. Hitherto we have only considered, as it were, the axes of the refracted rays, and have paid no regard to the *dispersion* which they experience in traversing refracting substances. We come now to consider this phenomenon, the exact and complete analysis of which constitutes one of the finest monuments of Newton's genius.

When we look at an object through a refracting prism, we know that its image does not appear in its true place; but is shifted towards the vertex of the refracting angle of the prism. Let *O* represent the eye of the observer, *ACB* the section of the prism made by a plane perpendicular to its edges, and passing through the point *O*, and *SS'* the object situated in this plane. The image *s s'* will be elevated towards the angle *C*, because it is seen in the direction *OR*, *OR'*, according to which the luminous rays emerge from the prism, after having suffered the two successive refractions at the faces *AC*, *BC*. We have sufficiently examined the laws of this deviation in the preceding sections, and have explained the means of calculating the course of the ray, both within and without the prism. Fig. 101.

But we have since remarked, that the borders of the image are not well defined and sharp, like those of the object. The image is lengthened in the direction *s s*, perpendicular to the edges of the prism, and painted with the colours of the rainbow. This circumstance of colour is what we are now to examine.

143. For the sake of distinctness, let us suppose that the edges of the prism are horizontal, which renders the line *S s* vertical, and that the vertex of the refracting angle is directed towards the zenith, as represented in the figure. Then let a piece of black cloth be placed beyond the object to prevent the rays

it emits from being mixed with foreign light. In this case, the lower border of the image will invariably appear painted red, and the upper one blue and violet.

If we observe in this manner a very thin white object, for example, a white pin, a silver wire, a thread of white silk, or a narrow Fig. 102. strip of white paper, placed on a dark ground, parallel to the edges of a prism, of a sufficiently refracting power, we shall perceive no white at all in the image *s s′*; but this image will appear entirely made up of parallel zones of different colours, among which we distinguish particularly, three distinct hues, the red at the bottom, the green in the middle, and the blue at the top.* Whatever be the nature of the substance whose image is thus observed, provided the colour be white, it exhibits, when seen through the prism, exactly the same series of colours; and if the dimensions of the several substances thus seen, are equal, it is absolutely impossible to distinguish one from the other.

144. Let us now endeavour to analyze this phenomenon and see the consequences to which it leads. The first circumstance we have to remark, is the dilatation of the image in the direction of its height. Indeed, if the object *SS′* were a mathematical line, parallel to the edges of the prism, and if all the rays which emanate from it were refracted according to the same ratio of the sine of refraction to the sine of incidence, the figure of the refracted image *s s′*, must also be a straight line without breadth; and though we cannot rigorously reduce an object to this limit, it is yet easy to see that, when the object is very narrow, as a pin, any further reduction is idle, since it does not sensibly change the size of the observed image. Moreover, Fig. 101. whatever be the length *SS′* of the object, if all the rays coming from it are refracted through the prism according to the same ratio of the sines, it is evident from calculation, that we can always find such a position, that the angle *ROR′*, contained be-

---

* To perform this experiment well, we should make use of a prism of flint glass, of a refracting angle equal at least to 60°. Then the zones are perfectly obvious and distinct. If we have not such a prism, we can construct one with two glass plates, inclined to each other at an angle of about 60° or more, the space between them being filled with pure water, or water saturated with the acetate of lead, in order to increase its refracting power.

tween the two emergent rays coming from its extremities, will be precisely equal to the angle SKS', contained between the two incident rays from which they are derived, that is, equal to the apparent diameter itself; at least if we suppose the object so far distant, that the distances $R'K$ and RO, of the point K and the observer from the prism, may be regarded as nothing compared with SI and $s\,I'$. The position of which we speak, is that in which the angle SIA is equal to the angle $OR'B$. Now this equality is far from what we actually observe; for whatever position we give to the prism, and whatever inclination it has with respect to the incident rays, we can never render the refracted image equal in extent to the direct image; and the difference is more sensible, according as the object is smaller. This result, which is confirmed and rendered general by all the experiments which follow, obliges us to conclude, *that the luminous rays which come from terrestrial objects, do not all observe, on being refracted, the same ratio of the sine of incidence to the sine of refraction.*

146. We have, moreover, remarked that the image of the pin, silk thread, &c., is composed of parallel portions of different colours. Hence, we must infer that the refraction is different for the rays which produce the sensation of the different colours; so that the dilatation of the image arises from the different refrangibility of the rays which cause these colours.

Again, since the light of the pin which produces all these colours, appears white when the eye receives it directly, before it is separated by the prism, it is manifest that what we call whiteness, is merely the union of a certain number of rays, which, considered separately, produce the sensation of the different colours, but which, united, produce the sensation of white. This may be easily verified. We have only to turn the pin, or strip of paper, in such a manner that its direction shall be perpendicular to the edges of the refracting prism, instead of being parallel to them. Then the upper extremity of the image is violet and the other extremity red. But if the object be of the same size throughout its length, these are the only portions of the image which appear coloured, and all the intermediate part is white, as if seen directly. Now it is manifest, that in turning the pin or strip of paper on itself, we do not change the manner in which light emanates from it. The rays, therefore, which proceed from each point, still undergo, in the prism, the same modifications as before; that is, the red rays are least refracted, and the violet most. If then we do not distinguish these decompositions in the middle of the image, it is a proof that the rays, coming from the several consecutive points of the object, recompose white by falling upon each other; and thus reproduce, by their union, the white which refraction had destroyed in each of them. It is also for this reason, that the pin, or strip of paper, must be throughout of the same diameter, in order that the experiment may succeed.

147. Thus far we have confined our observations to reflected light. That which emanates directly from burning bodies presents similar phenomena. Of this we shall be readily convinced by looking at the flame of a candle through a prism. The image of this flame, like that of every other object, is tinged with red and violet at its two opposite extremities. But in order to observe these phenomena in all their beauty, it is necessary to analyze by the prism the light of the sun itself.

To do this in a manner at once the most convenient and the most exact, we cause the light of this luminary to be reflected from the mirror of a heliostat, and to enter, by a very small aperture, into a perfectly dark room, twenty or thirty feet in length, where it is to be received upon a screen of white pasteboard, perpendicular to the direction of the reflected beam. This image, perfectly stationary and composed of the most pure and brilliant light, offers for our experiments, the most favourable conditions. If we observe it attentively, we shall perceive, in the first place, that the light is not equally brilliant throughout; its intensity is greatest in the interior of the image, and thence it gradually decreases towards the borders, where it grows weaker and weaker till it becomes altogether insensible. It is necessary to note all these circumstances critically, before Fig. 103. we subject the light to refraction. Let then *SS′* represent the diameter of the sun's disc, supposed to be spherical, *FF′* the diameter of the aperture, supposed to be circular, and *TT′* the plane of the pasteboard or screen, on which the luminous image is received. We are obliged to alter very much the true proportion of these lines in order to render them sensible in the figure; but this does not affect the reasoning we are to employ. Now, in order to see how the image of the sun, transmitted through the aperture is formed and disposed, we conceive two straight lines drawn through the extremities *S*, *S′*, of the diameter, so as to be tangents to the sides *F*, *F′*, of the opening. We shall thus form the two cones *TVT′*, *t v t′*, one of which will have its vertex without the room and the other within. The first of these cones limits the space through which the solar rays can enter, and consequently its intersection with the plane of the pasteboard, limits the magnitude of the image, whose total extent is thus equal to *TT′*. But this whole extent will not be equally illuminated. Indeed, if we consider any point *M*, comprehended between *t* and *t′*, that is, within the interior cone, an eye placed at this point will see the image of the aperture projected wholly upon the sun's disc; since, if we conceive two tangents to the sides *F*, *F′*, drawn from the point *M*, these lines will meet the straight line *SS′*, between the points *S* and *S′*. But if the eye were placed without the interior cone, at *N* or *N′*, for instance, it would see only a portion of the aperture projected upon the sun's disc; and this portion would diminish according as the eye approached the extremities *T*, *T′*, till at length, if it were placed at *T* or *T′*, no part of the sun would be visible, except its extreme edge. Whence we see that the pure image, illuminated by the whole of the sun's disc, is comprehended between the points *t* and *t′*, and is surrounded by an annular penumbra, the width of which is *T t* or *T′ t′*. It is easy to calculate the exact dimensions of all these elements, when we know the diameter *FF′*, of the aperture, its distance *CC′* from the pasteboard, and the angle *SVS′*, subtended by the sun's disc, that is, its apparent diameter, which is the same for the points *V*, *v*, *F*, *F′*, without any appreciable difference, on account of the almost infinite distance of this luminary, compared with the distance *CC′*. We thus find that the total diameter *TT,′* of the illuminated space, is equal to the diameter *T′ t*, or *T t′*, of the solar image, seen from the aperture, in the pasteboard, plus the diameter *T t*, or *T′ t′*, of the aperture; it being supposed always that the luminous rays arrive freely at the pasteboard, without deviating from a straight line, and without traversing any other medium than the air.

148. This being determined, let us proceed to examine the modifications which the image in question undergoes, from the interposition of a prism. For this purpose, we make use of a prism *ABC* of very clear glass, whose refracting angle is at least Fig. 104.

60°, and which is mounted in such a manner, that it can be turned about its axis in any direction. Let us suppose, for the sake of distinctness, that the solar beam *FS* is horizontal; place the prism very near the aperture, in order that it may receive all the light introduced, and give its edges a vertical position, that the rays may deviate in a plane parallel to the horizon. In the prolongation of the refracted light, we place the screen, which should be made of fine white paper, and supported on a moveable stand that it may be placed nearer or more distant, higher or lower as occasion may require. In order that the spectrum, formed by the prism, may be distinct, and the colours well separated, the screen should be placed at the distance of fifteen or twenty feet. The whole being thus disposed, we turn the prism slowly about its axis, alternately from left to right, and from right to left. We thus find that, after we have caused the image *VR* to move on further and further in the same direction, we arrive at a point from which, if we continue still to turn the prism, the image, after remaining stationary for a moment, begins to return in the contrary direction. Having found this point of equilibrium, we fix the prism in its position; it appears by calculation, that this position is very nearly the same with that indicated in figure 101; in other words, the sum of the angles of incidence is, in this case, equal to the sum of the angles of emergence. This position, therefore, offers very great advantages; for we not only know directly, by calculation, the angles of incidence and emergence which correspond to it, and can thus dispense with taking the measure; but we know also, that if all the rays which compose the incident light, observe the same ratio of the sine of refraction to the sine of incidence, they must, after refraction, form a refracted image of the object, equal to its direct image; that is, in the case in question, they must form a round image, equal in breadth to the apparent diameter of the sun's disc, seen from the point *F*, and enlarged by a penumbra equal to the diameter *FF′*, of the aperture, where the rays cross each other.

149. By performing the experiment in the manner supposed, the refracted image *VR*, is very far from being circular; it forms, on the contrary, upon the white screen, an oblong coloured spectrum, terminated laterally by two horizontal straight lines, and at the ends by two semicircles. The extremity *R*.

which deviates least, is tinged with a deep red; the extremity *V*, where the refraction is greatest, is tinged with a dark violet; between these two extremes, we observe an infinity of shades. This elongation of the image is not owing to any imperfection of the prism, as impurities in its substance or inequalities in its polish; for all transparent prisms, of whatever substance they are formed, even those which are first made hollow and then filled with the purest liquids, produce similar effects; the only difference is in the absolute length of the spectrum, which is greater or less, according to the refracting power of the substances, and the magnitude of the refracting angle. The elongation of the image is, therefore, a constant phenomenon, which belongs to the nature of refraction; and we are consequently obliged to conclude that the rays which compose the light of the sun, are not all equally refrangible; for if they were, the image would not be elongated but circular, in the supposed position of the prism.

150. Newton varied this experiment. He looked through the prism, at the image of the sun obtained from the aperture, as we have before done with respect to the image of a pin; and he found the same effect as to colour, and the same disposition of the colours.

He endeavoured moreover to verify this important result, in all the consequences which might be deduced from it. If the dilatation produced by the prism, proceeds from the unequal refrangibility of the luminous rays, then those rays which are found to deviate equally by refraction, are equally refrangible. Thus, if we suppose *S* to represent the centre of the direct solar image, and *VR* the oblong, coloured image, produced by the vertical refraction of a prism whose edges are horizontal; and f we trace on this image the horizontal lines *II*, *BB*, *GG*, these ines will designate the limits of equal refrangibility. Consequently, if we make the image *RV* undergo a second refraction, and in a lateral direction, by a prism whose edges are placed vertically, the rays contained in each of these lines *II*, *BB*, &c., being equally refrangible, ought not to be separated by refraction. If, moreover, the second prism is in all respects equal to the first, and is so placed as to have the same inclination to the incident light, its effect upon each of the rays contained in these

Fig. 105

lines, will be also equal to that of the first.* Consequently, the inferior extremity *R′*, of the second image will be removed as far from the inferior extremity *R* of the first, as this last is from the direct image *S*, by the first refraction. And, as the same may be said of all the other points of the first image *VR*, it follows that if we prolong the horizontal lines *II*, *BB*, *GG*, we shall, after the second refraction, find *RR′* equal to *SR*, *VV′* equal to *SV*, *BB′* equal to *SB*, and so on; whence we conclude that the second image must be contained between the same horizontal lines with the first, without being at all dilated in length. The lower part of the first image, which suffers the least refraction, and is of a red colour, must also form the red part of the second image, and suffer also the least refraction. The same ratio must hold true for the opposite extremity, which is violet. Moreover, on account of the equality of *SR* and *RR′*, the new image must be terminated laterally by two straight lines, inclined to the vertical *VS*, at an angle of 45°; and finally its axis *V′R′*, being prolonged, must pass through the centre *S* of the direct image. Now, by performing this experiment, we find that all these consequences are exactly realized, as Newton had assured himself. This agreement, therefore, confirms in the fullest manner, the unequal refrangibility of light, and it shows that this property of the rays is not accidental, but that it is inherent in their nature; since each of these preserves it invariably, after the first refraction, and even after the second and third, as Newton in like manner ascertained.

151. We have already remarked, that the size of the aperture through which the rays are admitted, must produce, even in the direct image of the sun, a penumbra or gradation of brightness, by which the borders of this image pass insensibly from the most vivid light to the most perfect obscurity. Now the feeble light which forms this penumbra, being absolutely of the same nature as the rest of the image, must be modified in the same manner by refraction, and consequently will be found on the rectilinear sides of the oblong image, whose distinctness

---

* We suppose the second prism, as well as the first, so placed as to make the angle of incidence equal to the angle of emergence. For it is only in this position, that an image formed of rays equally refrangible, is neither dilated nor contracted by refraction.

it thus affects. This is confirmed by experiment. In order to avoid this defect, Newton placed before the aperture in the window, a glass lens, which, by its refraction, concentrated nearly in one focus, all the rays sent from the several points of the sun's disc; and hence resulted an image of this luminary, white, circular, and perfectly free from penumbra. This being done, he received this collection of rays upon a prism placed behind the lens, and the oblong image formed by refraction, was found, in like manner free from every thing like penumbra on its rectilinear sides; so that these sides appeared as distinctly terminated as the direct image given by the lens.

152. In the experiment relative to lateral refraction above described, Newton found the means of operating separately upon each of the several rays of equal refrangibility. The following is another method employed by him of effecting the same purpose.

Let *SF* represent a beam of solar light introduced into the dark room by the aperture *F*. Near this aperture we place a prism *ABC*, which, refracting unequally the differently refrangible rays contained in this beam, will form on the screen *TT* an oblong, coloured image, as we have seen in the preceeding experiments. In order to examine separately the differently refrangible rays, of which this image is composed, we make in the screen *TT*, a very small circular aperture *O*, which answers to one of the points of its length. Then there will pass through this aperture a small cone of rays sensibly homogeneous, which, falling upon another screen *TT′*, parallel to the first, will form there a small circular image of the aperture *O*. This image will be of one single colour, as red, for instance, if the aperture *O* answers to the rays of the first image, which produce the sensation of red; and green, if it answers to the green rays, and so on. Now, when this homogeneous light is separated from the rest of the spectrum, we can examine it at pleasure. For this purpose we make at *O′* in the second screen, a small aperture, in such a manner as to admit only a small portion of the homogeneous light, and cause this portion after passing the aperture to pass through a second prism *a b c*. Then if the unequal refrangibility of the rays, contained in a natural beam, be the only cause of the elongation and colour of the image, nothing of this kind will here take place. The homogeneous portion, refracted by the second prism, will not have its colour changed; Fig. 106.

and if the inclination of this prism be such as to make the incidences and emergences equal, the image formed by this portion on the wall, will be round like the aperture itself. This is completely verified by the experiments of Newton and all succeeding philosophers.

153. If now we turn the first prism slowly about its axis, the rays which produce the sensation of the different colours, will pass successively through the aperture *O*, and will also arrive successively at the second prism *a b c*, exactly in the same direction *OO′*, and consequently with the same incidence. We shall thus see formed successively on the wall, the image of red, yellow, green, &c., according as the rays which then pass through the aperture *O*, are found to be those which produce the sensation of red, yellow, green, &c. If then it is true that these rays are unequally refrangible, and that this property is inherent in them, they will experience in the second prism, unequal refractions, that is greatest for the violet, least for the red, and intermediate degrees for the rays which produce the intermediate colours; which may be easily perceived by the unequal height of the images, and by their rising or descending as we turn the first prism. All these consequences are perfectly confirmed by facts.

154. This unequal refrangibility of the luminous rays must necessarily produce its effect, when light is refracted by spherical lenses. For a lens is nothing but a circular assemblage of an infinite number of prisms of unequal refracting angles. Hence there must result for different rays, different focal distances, the rays most refrangible forming their focus nearest the lens, and those least refrangible, having their focus most distant. This is but too well confirmed by experiment; for the dispersion of the foci, thus occasioned, has been for a long time, a great obstacle to the perfection of telescopes.

155. Hitherto we have examined only those properties of rays which depend upon their unequal refrangibility. We come now to consider another property, discovered also by Newton; namely, the facility of the rays for interior reflection by refraction, is also unequal, and the more so, according as they are more refrangible. He conducted the experiment in the following manner. Fig. 107. Having introduced through the aperture *F*, into a dark room. a beam of light *FM*, he caused it to fall upon the side *AC*, of a

prism *ABC*, whose two angles *B*, *C*, measured each 45°, *A* being a right angle. The refracted ray emerging at *M*, proceeded to form below the prism, a coloured image *VR*, as in the preceding experiments. By turning this prism slowly about its axis, in the direction *ABC*, the refracted rays are made to fall more and more obliquely upon the base *BC*, and the emergent rays also take a direction more and more oblique to this base. Hence, if we continue to increase the obliquity, the refracted rays will at length be no longer capable of emerging, but will be wholly carried back into the prism by refraction, as we have already explained. In common glass, this phenomenon takes place when the rays, in their interior incidence make with the base *BC*, an angle of about 48°. Then if the refracting angle *C* is 45°, as Newton made it, the incident rays will be nearly perpendicular to the first surface *AC* of the prism, and consequently the dispersion, caused by their first refraction, is nearly nothing; so that their interior incidences upon *BC* are sensibly equal. But notwithstanding this equality, their interior reflection is progressive; for if, during the motion of the prism, we observe attentively the coloured image *VR*, we shall see that the violet first disappears from this image, while the other colours remain. After the violet, the blue disappears, then the green, then the yellow, and finally the red, which disappears last. It is then only, that the interior reflection is total. These rays, successively reflected, emerge through the side *BA* of the prism, to which they are also nearly perpendicular. If they were the only ones which followed this direction, we should thus have a very simple means of separating them wholly from the rest. But this is not the case; for the refracted beam experiences always at *M* a partial reflection, which, acting without distinction upon all the incident light, sends a certain proportion of it directly towards the face *AB* of the prism; so that the coloured parts of the portion *VR*, which come next to be reflected, only add to those already reflected. Still we can here recognise their influence. For this purpose we first place the prism *ABC*, in such a position, that the refracted beam *VR* shall emerge entirely; then in the direction *MN* of the returning beam, produced by partial reflection, we place a second prism *A′B′C′*, which, refracting it, forms upon the screen *TT*, another oblong and coloured image *R′V′*. We then observe attentively the intensity of this image; and if we turn the first prism *ABC*, slowly about its axis, in such a manner as to increase gradually the obliquity of the rays refracted at its base, we shall see that at the instant when the violet rays can no longer emerge through *BC*, the violet part of the image *R'V'*, will receive a very sensible increase of intensity, compared with the other tints which compose it. Next the blue will increase, then the green, the yellow, and finally the red, when, by the continuing the motion of the prism, the reflection becomes total at *M*. Newton varied the experiment in several ways, and the result of the whole was what I have above stated.

156. In all this variety of experiments upon light transmitted through natural bodies, upon light reflected from specular surfaces, and lastly, upon refracted light, we always find rays which, at equal incidences, (the medium being the same) suffer unequal refractions, although no dispersion takes place in each simple ray. We find, moreover, that this effect is not produced accidentally by imperfections in the refracting substance; but that it is governed by regular laws, depending upon the position of the refracting prism, the refracting angle, and the nature of the substance. From all this, it follows incontestibly, *that the light of the sun, like other kinds of light, which can be subjected to the same experiments, is a mixture of heterogeneous rays, of which some are constantly more refrangible than others, and which, considered separately, are capable of producing upon our organs the sensation of different colours.* Moreover, since the violet rays return into the prism, under interior incidences at which the others emerge, we may add, *that these rays differ also in reflectibility, and that those which are most refrangible are also most susceptible of being reflected inward by refraction.* According to the theory of attractive forces, this is a consequence of unequal refrangibility.

Another consequence of this inequality is, that the solar spectrum, as given by a prism, is really nothing but the succession of an infinite number of small circles, partly superposed upon each other, and having each a simple colour, from the violet end to the extreme red. This is represented in figure 108, in which it is only necessary to suppose an infinite number of successive circles, from the violet to the extreme red, instead of the small number here represented.

If the image thus dispersed be that of the sun, whose apparent diameter is about half a degree, each of these several circles, seen from the centre of the circular aperture, supposed to be infinitely small, subtends also an angle of about half a degree. For each kind of simple rays, coming from the opposite limbs of the sun, forms, in passing through the aperture, a cone whose angle at the centre is equal to the apparent diameter of the sun's disc; and when refracted in the prism, with the angles of incidence equal to the angles of emergence, the refracted cone has sensibly the same angular opening as the direct cone. The size of these consecutive images, causing them necessarily to lap upon each other, it follows that, strictly speaking, light is nowhere absolutely homogeneous, except on the rectilinear sides of the image where the circles are detached from each other.

The means of simplifying the effect, is then to diminish the diameters of these circles, in order to separate them more from each other, the distance between their centres still remaining the same. This we do by greatly contracting the aperture by which the rays enter the dark room, or by giving it the form of very narrow slit, having its small diameter in the direction of the refraction. Among the many precautions employed by Newton to obtain this separation, one of the most important, is to concentrate the beam by a lens, before it falls upon the prism. We should take care also to place without the aperture, in the direction of the solar beam, a narrow and black tube, in order to exclude the foreign light, which, coming from the lateral parts of the heavens at a great distance, would form in the dark room large cones, and consequently, present after refraction, large circles, which mixing with each other and with the principal image *VR*, would affect the distinctness of its colours.

157. It is upon light thus rendered pure, that the decisive experiment on the immutability of colour, illustrated by figure 106, should be made. This remarkable property is then preserved, not only after one refraction, but after all the successive refractions which the homogeneous beam is made to undergo. It is not destroyed even by reflection; for if insects or other small objects, be placed in this light, and examined through a prism, even of a great angle, we see them just as distinctly as if we viewed them directly, and with the single colour that illuminates them; whereas we could not distinguish them at all, if we looked at

them in this manner, when they were illuminated by the compound light of the sun, because the image of their different parts would be elongated and deranged in consequence of the unequal refrangibility of the different rays. We might make the same experiment with a printed book. If it is illuminated by homogeneous light, the characters can be read with perfect distinctness, however fine the impression is; while they cannot be distinguished at all after reflection, if they are illuminated with compound light.

If we consider the course of luminous rays, thus purified, through the same refracting medium under different incidences, we find, as before remarked, that the sine of refraction is always in a constant ratio to the sine of incidence. We shall hereafter have occasion to note the most delicate proofs of this fact. For the present, we only observe that, according to the theory of the emission of light, it becomes a necessary consequence of the affinity existing between luminous particles and refracting bodies.

158. The reason why refraction causes certain luminous particles to deviate more than others, cannot be assigned with any degree of certainty. We might be induced to believe that this inequality is owing to a difference as to their mass or velocity; but in this case bodies which refract the one class of rays equally, ought also to refract all the other classes equally, which is not the case, as will soon be shown by direct experiment. For instance, there are some bodies, which refract the green rays as little and even less than other bodies, while they refract the violet rays more. We are obliged, therefore, to believe that the chemical nature of the particles of light, and perhaps their form, contributes to this phenomenon, and causes the affinities of these particles, while traversing different bodies, not to preserve among themselves the same constant relations.

159. We have seen that certain crystallized bodies have the singular property of dividing into two portions the luminous rays which traverse them, and have called this phenomenon double refraction. In this case, each of the two refracted portions becomes also dispersed. In crystals of one axis, where the velocity of the ordinary ray is constant, the general law of its dispersion is the same as in bodies which produce only simple refraction. The law of dispersion of the extraordinary ray is more complicated, because it is affected at the same time by the

ordinary refracting forces, and the repulsive or attractive forces which emanate from the axis of the crystal. In crystals of two axes, this complexity extends to the two velocities. We shall examine the combined effects thence resulting, when we have completely established by experiment, the phenomena exhibited in ordinary dispersion. For the present, it is sufficient to remark, that the refrangibility and colour, peculiar to each ray, are not altered or changed by double refraction.

160. Since each species of rays of a particular refrangibility, retains thus invariably its own peculiar colour, we may, for the sake of brevity, designate each ray by the species of colour of which it produces the sensation. Thus we denominate red rays, those which are least refrangible and which produce the sensation of red; yellow rays, those which produce the sensation of yellow; and violet rays, those which produce the sensation of violet; not because we suppose that these rays are really red, yellow, and violet, or that they have colour in themselves, any more than sonorous bodies contain sound; but only to express the sensation which they are capable of producing, and which they do constantly produce in eyes properly organized.

Nevertheless it is impossible to assign, in this manner, particular denominations to all the rays; for since each ray endued with a different refrangibility, produces on our organs the sensation of a peculiar colour, the number of shades following each other in the spectrum, must be infinite like that of the rays which produce them. But since the most accurate eye cannot have a distinct sensation of so many shades, differing so little from each other, it is sufficient to establish among them, a certain number of divisions, which, comprehending all the shades in their several intervals, will enable us to fix their place and character with a precision corresponding to that of our senses. This was done by Newton; he drew seven principal lines of separation in the spectrum between the extremes of the greatest and least refrangible rays. The divisions thus formed, being designated by the colours belonging to them respectively, are the following; *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, *red.*

The rays comprehended in each of these divisions, may be regarded as homogeneous, as to refrangibility and colour; and each portion retains its peculiar refrangibility and colour, whatever refraction it is made to undergo; and, what is no less remarkable, this colour is not altered by the radiant reflection which takes place in natural bodies. "For," says Newton, "every body, whether white, grey, red, green, blue, or violet; as paper, ashes, red lead, orpiment, indigo, blue embers, gold, silver, copper, grass, blue flowers, violets, soap bubbles tinged with different colours, peacock's feathers, brazil wood, and the like; all these, when exposed to a homogeneous red light, appear entirely red; to a homogeneous blue light, entirely blue; to a homogeneous green light, entirely green, and so on." In a word, in homogeneous light of whatever colour, all bodies appear entirely of this colour; with the single difference, that some reflect it more strongly than others. "But," adds Newton, "I have never seen a body, which could sensibly change the colour of homogeneous light by reflecting it." It is evident that this applies only to light perfectly homogeneous; and consequently, in order to perform these experiments, we must take the utmost care to purify the solar image by the methods already described. For among natural bodies which reflect the most vivid colours, there is no one, which sends, by radiant reflection, colours absolutely simple and homogeneous, as these experiments themselves prove, and as we may assure ourselves by the consideration, that all these colours are capable of being decomposed by a prism, and resolved into a spectrum where we can recognise several different shades. If, therefore, different bodies illuminated by the same white light, appear to us to have determinate colours, it is only because they emit in greater abundance, the rays which produce the sensation of one particular colour, than those which produce any other. Accordingly, when we expose them in perfectly homogeneous light, where they have only one species of rays to reflect, they will all appear to possess the colour belonging to these rays, and there will be no difference among them, except in the quantity of light which they are capable of emitting. But if we expose them in light imperfectly decomposed, among the rays which fall upon them, they will separate those which they are best fitted to reflect; and these rays, existing thus in greater proportion in the reflected than in the incident light, will sensibly alter the colour of the latter; whereas such an effect never happens when we expose them in perfectly homogeneous light.

The above experiment may be in a manner reversed. If we expose two bodies of different colours in the same natural light, deprived of these colours, they will both appear to have the same colour. For instance, gold is nearly a pure yellow, and silver is white, which comprehends all colours; and if we expose both in the same natural light deprived of the yellow rays, they will appear of the same colour.

161. In order to define these phenomena accurately, it is necessary to fix the comparative spaces taken up by these several colours in the spectrum. Newton has investigated this point, but the proportions he obtained are necessarily particular, and refer to the kind of glass he used; so that they will afford only a general indication of the mode of distribution of the colours, with respect to other substances. Being thus restricted in the use of these results, let us suppose the whole spectrum divided into 360 equal parts; then, according to Newton, the comparative spaces occupied by the different colours are as follows.

Violet	80
Indigo	40
Blue	60
Green	60
Yellow	48
Orange	27
Red	45

We shall hereafter furnish a very certain method of comparing these intervals in different refracting bodies. Newton believed that the proportion was always constant; and this error, resting upon such high authority, retarded for a long time the invention of achromatic telescopes.

Recomposition of Colours.

162. We have seen that the refrangibility and colour of the several homogeneous rays, are not changed either by refraction or reflection. We come now to show that these properties are not affected by compounding rays of different colours; but that they are preserved in the mixture, and transferred without varia-

tion. This important truth is established by the following experiments of Newton.

We introduce a beam of solar light into a dark room, and after ig. 109. it is refracted by a prism, we receive its spectrum *VR*, on a lens *L*, placed vertically at a distance from the prism equal to double its focal distance. Then this lens will concentrate the rays in a focus situated on the other side of its surface, at a distance equal to that of the point *P*, where the refracted rays begin to diverge. This we have already demonstrated, in treating of the effects of lenses. Beyond the focus *TT*, the rays diverge anew, and form on the white screen *F*, another spectrum *V′R′*, inverted with respect to the first *VR*, because the interposition of the lens causes the rays to cross each other. Things being thus disposed, let us examine the different modifications which these rays undergo. We see that they are dispersed in *RV*, by the first refraction; this dispersion continues also, after they have traversed the lens, and the colours belonging to each ray are not changed; for if we place a piece of white paper near the posterior surface of the lens *L*, it will present a spectrum similar to *RV*, and in which the order of the colours is the same. As we move the paper from the lens, the spectrum becomes smaller, because the rays which the lens has rendered convergent, constantly approach each other; till they meet, at length, in the focus *F*, and there form on the paper a white circular image. But, after this concentration, they separate anew with their original properties; for they retain these properties in the formation of the image *V′R′*; and we may perform on this image precisely the same experiments we performed on the spectrum, which had not traversed the lens *L*, and whose rays had not crossed each other at the focus *F*. Hence it is manifest that their concentration at *F*, has not altered their refrangibility or their colour, as would have been the case, if, in crossing, they had exerted any action upon each other.

163. Let us now return to the image formed at the focus; it is the result of the simultaneous impression made on the eye, by all the different rays which are concentrated in this point. This union, therefore, is the necessary condition for producing the sensation of white; and the rays are still capable of producing it after having been dispersed by the prism *P*, and united by the lens *L*, just as well as when they were first united in the incident beam.

It has already been remarked that the several coloured rays are not strictly united at the same focus by the lens, but that they form, at their departure from its second surface, cones of unequal lengths, which must render the mixture in this case less perfect than it is in the incident beam. But this union, which is not effected by the concentration of the rays themselves, is completed by the action of the white screen upon which they fall; for this screen, reflecting them every way, like a direct radiation, mixes them as completely as they were mixed in the incident beam. Accordingly, we perceive no colour, except on the exterior borders of the image, where the cones, formed by the most refrangible rays, always project a little.

164. From the result of the above experiment, respecting the cause of whiteness, we may presume that this sensation would be modified by taking away some of the rays which conspire to produce it. This is in fact the case; for if we fix the screen at the focus *F*, where the white circular image is formed, and intercept some of the rays which fall upon the lens, by covering the space on which they fall, with a black rule, placed transversely with respect to the spectrum, the image formed at the focus immediately ceases to be white, and the colour thus produced, is that which ought to result from a combination of the remaining colours. But if we take the rule away and suffer the intercepted colours again to pass, so as to fall upon this compound colour, they reproduce the white. If, for instance, the violet, blue, and green are intercepted, the yellow, orange, and red which remain, will compose a sort of orange on the screen; but if, after this, we suffer the intercepted rays again to pass, they mix with this orange and restore the white. In the same manner, if the red and violet are intercepted, the yellow, green, and blue which remain, will compose a sort of green on the screen, after which, if we suffer the red and violet to combine with this green, they will reproduce the white. We can repeat these decompositions and recompositions at pleasure; they will always take place in the same manner, and always produce similar alternations, provided the eye has time to appreciate them separately. But if we move the rule very rapidly over the spectrum *RV*, the eye is incapable of distinguishing the variety of tints which succeed each other, and which are interrupted by intervals of white. In this case we always see white. The experiment succeeds still better,

if we substitute, in the place of the rule, a piece of black pasteboard *CC*, in which several slits are cut parallel to each other.

165. It has been demonstrated that the colorific properties of homogeneous rays are not changed by the forces which produce reflection, whether radiant or specular. We may, therefore, presume that these forces will not destroy the property by which the rays form white, when united together after having been separated; and this presumption is confirmed by experiment. Let *SI* represent a beam of white light introduced into a dark Fig. 110. room. Let this beam be refracted and dispersed by the prism *ABC*, and let the spectrum *RV*, thus produced, be received upon a screen placed at the distance of fifteen or twenty feet. Suppose now that an observer, stationed at *O*, very near the first prism, holds before his eyes a similar prism *A'B'C'*, formed of the same substance, and having the same refracting angle. If the observer places this second prism parallel to the first, the rays of different colours, emanating from the different points of the spectrum *RV*, will meet the prism *A'B'C'* almost exactly under the same angles, which they formed at their emergence, with the surfaces of the first prism; and since we have proved, in another place, that their refrangibility is not altered by radiant reflection, the rays will be affected here as they were in the first prism, undergoing the same refractions in an inverted order, and will form at their emergence in *O*, a collection of emergent parallel rays, such as they formed at their incidence in *I*. The observer, therefore, receiving all these rays together, will still experience the sensation of white, provided their power of producing this sensation is not changed by the radiant reflection from the screen; and experiment proves that it is not. But in order to obtain an accurate result, a number of precautions are necessary, of which a detail is given in the *Traité de Physique.*

This recomposition of colours by means of equal and inverted refractions, may likewise be obtained with a single prism, by means of interior reflections. It is to this that we are to attribute the numerous white images seen to emerge after several reflections, from the different faces of a prism exposed to a solar beam; images whose light is often an obstruction to the accuracy of our experiments. The manner in which these recompositions take place, is described in the *Traité de Physique.*

166. It is proper in this connexion to notice an experiment, which seems, at first view, to indicate that the colorific properties of rays may be changed by reflection; though, in reality, it proves the contrary. For this purpose it is necessary to recur to the apparatus represented in figure 109, in which the refracted rays were united by a lens. If we place at the focus *F*, a piece of white paper perpendicular to the axis of the luminous pencil, the light reflected from this paper will appear white; but if we substitute instead of the paper a polished metallic plate, the light reflected in some points of the room will appear coloured. It would seem from this, that the properties of the incident rays were changed by reflection from the mirror; but an attentive examination will lead us to alter this opinion. The luminous rays fall, indeed, very nearly upon the same point of the metallic plate, but not under the same inclinations. As they experience the specular reflection in forming the angle of reflection equal to the angle of incidence, this reflection must necessarily separate them; so that when they leave the mirror, they only continue the divergence which they had when they came to it; and this is the divergence they would have had beyond the focus *F*, if they had been left to pursue their course without interruption. When the rays fell together perpendicularly upon the paper, their reflection did not take place in this manner, because the paper is not a polished body; each of the rays was then diffused indifferently in all directions, at their departure from the point of incidence, and as each point of the paper diffused thus, at the same time, all the rays which fell upon it, these would of course arrive together at the eye. But even in these circumstances, the separation of the colours becomes perceptible, when the paper is very considerably inclined to the rays, because, though it is not polished, yet it is not entirely of an opposite nature, and accordingly a part of the incident light is reflected specularly, especially when the incident rays make small angles with its surface. This property is common to all bodies, and belongs even to glass from which we have taken pains to remove the polish, by rubbing its surface with sand, as has already been remarked in treating of reflection.

In the preceding experiments we have seen that white is produced by combining the heterogeneous rays which the refractive power of the first prism had separated; but we can also produce it by uniting the different colours obtained from different prisms; and this explains the progressive decomposition which the same beam, if it be of sensible magnitude, experiences in its different parts, according to its distance from the prism where it has been refracted.*

Compound Colours produced by the Mixture of Simple ones.

167. By the preceding experiments we have proved, in the most rigorous manner, that each ray carries in itself its colorific property, which, like its refrangibility, cannot be destroyed or changed. But what is very remarkable, this property is not peculiar to them; for we can compose artificial mixtures of colours, which produce on our organs the same sensation. This may be done in several ways; one of the most simple, consists in employing the apparatus represented in figure 109, where the rays, after being dispersed by a prism, are concentrated by a lens in one uncoloured focus. Suppose a piece of black pasteboard *TT*, placed before the lens. In the middle of this pasteboard, let several rectangular openings *VV*, *II*, be made, and let slides be adjusted in such a manner as to admit of their being covered or uncovered at pleasure. If the pasteboard be so placed as to cause the spectrum to fall upon these openings, we can allow such rays as we choose to pass, and in such proportions as we choose; and these, uniting at the focus of the lens, will paint there, on the white screen, the particular colour which results from their mixture. We find in this way, that each colour may be imitated by the mixture of the two colours which border upon it. Thus the mixture of orange and greenish yellow, produces yellow; the mixture of greenish yellow and bluish green, produces deep green; which may be imitated also, but not so well, by blue and yellow. Blue is imitated by the mixture of bluish green and indigo; and, what is rather remarkable, violet itself may be imitated by the mixture of blue and

ig. 111.

---

* For further particulars on this subject see Biot's *Traité de Physique.*

red. But all these mixtures, when observed through a prism, are resolved again into their elements, they being separated by refraction; whereas the colours produced by homogeneous rays are not changed by the prism, and this forms their essential characteristic.

168. We may also imitate the simple colours more or less perfectly, by mixing together powders of different colours. This is the way in which painters compose their colours; but these can never equal, in vividness, the colours obtained by the analysis of light, and the prism immediately separates them, and makes known their composition.

We may even imitate white by such mixtures, but yet in a very imperfect manner, for the coloured powders, made use of for this purpose, always absorb a great part of the light which falls upon them; and it is because they thus absorb a part, that they appear of the colour supplementary to this part. In general, therefore, we can compose, with such mixtures, only a grayish and dirty white, such as results from the mixture of a powder perfectly white with a black powder. But if we increase the intensity of the light by which they are illuminated, we are enabled to add to their lustre and to render it equal to that of the whitest bodies, as paper, for example, but paper faintly illuminated and placed in an unfavourable light.

169. These mixtures, varied continually, would present an infinite variety of shades, which make no part of the simple colours of the spectrum; but there is no one of these shades which might not be produced by a proper mixture of simple colours, with the advantage of a far greater lustre. By considering the different portions of simple light which compose the spectrum, as so many forces acting upon the organ, and measuring the influence of these forces, in the various mixtures which may be formed of the prismatic colours, collected at the focus of a lens, according to the process indicated above, Newton arrived at an empirical rule, by means of which we are enabled to calculate beforehand the kind of colour that will result from a mixture of simple rays, when the kinds and proportions are given.

With the centre C, and a radius equal to unity, describe a Fig. 112. circle, and divide its circumference into seven parts proportional to the numbers $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$; so that when these parts are expressed in degrees,* we shall have;

$$\begin{array}{rl} RO = & 60^\circ\ 45'\ 34'' \\ OY = & 34\ \ 10\ \ 38 \\ YG = & 54\ \ 41\ \ \ 1 \\ GB = & 60\ \ 45\ \ 34 \\ BI = & 54\ \ 41\ \ \ 1 \\ IV = & 34\ \ 10\ \ 38 \\ VR = & 60\ \ 45\ \ 34. \end{array}$$

Let us now consider these different arcs in the order in which they follow each other, as representing the seven principal colours of simple light, which compose the spectrum; so that the entire circumference will represent the whole series of shades through which this light passes, from the first rays of red to the last of violet. Then, having determined the centres of gravity r, o, y, g, b, i, v, of all these successive arcs, suppose in each of them a weight proportional to the corresponding arc; and conceive these weights to be so many forces tending to draw to themselves the centre C, and the eye supposed to be placed there. According to this supposition, it is evident that the eye will remain at rest, being placed at the centre of gravity of all the weights; and this rest will correspond to the perfect white produced by the simultaneous sensation of all the shades of simple light, when they are mixed together according to the proportions in which they exist naturally in the spectrum. But suppose these proportions changed, as they always are in a coloured mixture which differs from white; then it will be necessary to place in each partial centre of gravity, not the total weight of the corresponding arc, but the half or third, or generally the n part of this weight, according as the given mixture contains the half, the third, or generally the nth part of all the light which composes this colour in the spectrum. This being done, if we find the common centre of gravity of all the partial weights, it will, in general, no longer coincide with the centre of the circle. While it does coincide, the colour of the mixture will still be white; but when, on the contrary, it approaches considerably one of the partial centres, the mixture will present, as the predominant colour, that which belongs to this centre. Finally, wherever it falls, as in G', for example, we have only to draw from the centre to this point the line CG'; then the direction of this line will indicate the predominant colour of the mixture, and its length, or the distance from the point G' to the centre C, will indicate its vividness. If, for example, CG' is situated exactly midway between CY and CG, the compound colour will be the most perfect yellow; but if CG' approaches more to CG or CY, this yellow will incline more to the orange or the green. On the first supposition, if the point G' falls very near the circumference, the colour will be strong and vivid in the highest degree; if it falls half way between the circumference and the centre, the intensity of the colour will be diminished one half, and it will be that of a mixture of the liveliest yellow with an equal quantity of white. In general, if we represent the distance CG' by Δ, the radius of the circle being 1, $1 - \Delta$ will express very nearly the proportion of white which it will be necessary to employ, in order to imitate to the eye the colour in question; this white being mixed with the proportion Δ, of the single colour towards which CG' is directed. In this manner, then, the nature and intensity of the colour are ascertained. It is proper, however, to remark, that if the point G' falls upon the line CR, or very near this line, the red and violet being then the principal elements of the compound colour, it will no longer correspond to any of the prismatic colours, but will be a purple, inclining to the red or violet, according as the point G' varies from the line CR, towards the one or the other of these colours; and in these two cases, the purple or mixed violet, will be more dazzling and brilliant than the simple violet. In general, we observe the greatest analogy between the effects of the violet and the red; so much so that we can form with blue and red, mixtures which produce on the eye the sensation of a fine violet; and it is probably this kind of return of tints upon themselves, perfectly analogous to the consonance of octaves, which led Newton to compare, as he has frequently done, the impressions produced by the different colours to those produced by the musical intervals.

---

* This division requires only the simple rule of fellowship. As the sum of all the fractions is to 360°, so is one of the fractions to the arc which corresponds to it. The sum of the fractions is $\frac{3}{9} + \frac{2}{16} + \frac{2}{10}$ or $\frac{79}{120}$. If, for example, we would find the arc corresponding to $\frac{1}{10}$, its value will be $\frac{\frac{1}{10} \cdot 360^\circ}{\frac{79}{120}}$, or 54° 41′ 1″.

170. According to the extent assigned by this rule to the arcs which represent the seven principal shades of the spectrum, there are no two whose centres of gravity are diametrically opposite to each other. Accordingly, in whatever proportion two colours are mixed, their common centre of gravity can never fall upon the centre of the circle; and, therefore, we infer that they can never form a perfect white, but only a pale tint, nearly approaching to white. This inference is confirmed by experiment. "For," says Newton, "I have never yet been able to obtain a pure white, from mixing only two primitive colours. I do not know that it can be done even with three colours. But these are points of curiosity, which have little or nothing to do with the understanding of natural phenomena; for, in all natural whites, there is ordinarily a mixture of all the primitive colours, and consequently these whites are compositions of all these colours, which circumstance confirms the propriety of the mode of composition just described."

171. We may make use of this rule of Newton, to produce the sensation of white by a rapid succession of the different colours. We have only to divide a pasteboard circle according to the proportions above indicated, paint the several divisions with the purest colours that can be obtained, and then turn it rapidly about its centre. But as the colours thus employed are never simple, we must not expect to obtain by this process a perfect white.

Influence of the Unequal Refrangibility of Rays of Light upon Vision through Refracting Surfaces.

172. From the time that the phenomenon of dispersion presented itself in our experiments on refraction, we have foreseen the general effects which it must have upon vision through triangular prisms. The knowledge we have now acquired of the individual properties of simple rays, and the constancy of their colorific powers, affords a full confirmation of what was thus indicated, and enables us to state it with more precision.

When an infinitely small luminous point is seen by refraction through a triangular prism, each kind of simple rays which proceeds from this point gives an image tinged with its peculiar colour. The unequal refrangibility of the rays of different kinds causes the images to appear separate and placed by the side of each other in the order which their colours take in the spectrum, those formed of the most refrangible rays being most deflected. But, if instead of a single radiating point, infinitely small, we look at several points placed by the side of each other, each of them will be found to produce a similar spectrum; but these spectrums admit, to a certain extent, of being placed one upon the other, in such a manner as to produce white in the place where they unite. This usually happens when we thus look at objects of a sensible magnitude. If the surface of the objects is white and equally luminous, the interior portions of the refracted image appear white, and the colouring of the image is perceptible only at the extremities, in the general direction of the refraction.

Newton states a fact, that is easily observed, which, at first sight, seems to be inconsistent with this theory, but is in reality a consequence of it. Having placed horizontally, before an open window, the base of a prism *ABC*, bring the eye to *O* so as to receive the light of the sky reflected inward from its base. Then, when the reflected rays form with this base an angle of about 50°, there will be seen an arc of a blue colour of a certain width *SS′*, which will extend the whole length of the prism, upon turning its concavity towards the eye; and the portion of the base *SB*, situated beyond this arc, will appear much more brilliant than the portion *S′A*, which is on this side of it. Fig. 113.

In order to explain this singular phenomenon, let us conceive that the angle of reflection *NSI*, which corresponds to the convex part of the arc, is the smallest angle at which total reflection commences for the red rays, which are the least refrangible; they cannot then be wholly reflected under an incidence approaching nearer to the normal *SN*. Let us suppose also that *N′S′I′* is the smallest angle of total reflection for the last violet rays; then from *B* to *S* there may be rays of all kinds which will undergo a total reflection and arrive afterward at the eye; but on this side *S*, the interior incidence becoming too near to the perpendicular, none of the extreme red rays can be perceived; thus, as the point of interior incidence approaches *A*, the possibility of total reflection towards the eye will cease successively for the different rays, one after another, to *S′*, when this possibility will cease, even for the last rays of the violet colour; so that from *S′* to *A* total reflection will not take place for any of the rays. If we now examine these results, we shall see that from *B* to *S* the reflection will be vivid and brilliant, because it will be total, and will comprehend the rays of all the colours. From *S* to *S′* the total reflection will go on diminishing, and will be limited to the more refrangible rays, which must produce a zone where the colours of these rays are predominant; finally, from *S′* through every other point nearer to the eye, the reflection will not be total for any of the rays. The eye will then receive from this space only the feeble portion of light which arises from radiant reflection; it will of course appear obscure in comparison with the intense brightness of *BS*. The limits of all these phenomena may be fixed by the aid of the Calculus.*

We see, also, that the remainder of the rays which fall on the space *SS′*, passing out below the prism, must produce, after emergence, an arc in which the red rays are predominant, 155. very nearly, as in the experiment of Newton, already mentioned.

173. This theory furnishes also an explanation of the phenomena of the rainbow, and enables us to calculate these phenomena. It is well known, that this meteor presents one, and sometimes two arcs, tinged with all the colours of the spectrum. It is never produced, except when there is a fall of rain accompanied with sunshine; but the union of these two circumstances is not sufficient of themselves to cause this appearance; we require also a particular position of the cloud, of the observer, and of the sun, which must always be directly opposite to the centre of the bow. These circumstances long since led to the belief that the rainbow was produced by the refraction of light in the drops of rain; indeed a similar appearance is exhibited in that kind of artificial rain which is caused by jets and cacades, especially when agitated by the wind. In order to conceive in what manner the light is capable of being thus dispersed by refraction, let us sup- Fig. 114. pose a single ray of light *SI*, infinitely small, coming from the sun, to fall upon a single spherical globule of water, and let us follow the course of this ray. It will first undergo a refraction at *I*, which will direct it to *I′*, where part of the ray will be a

---

* See Biot's *Traité de Physique.*

second time refracted, and will emerge into the air in the direction $I'R'$, but the rest of the ray will be reflected into the interior of the globule toward I'', where a similar effect will be produced, that is, a part of the ray will emerge into the air, while the other portion will be again reflected inward to I''', at which point the same division will take place, and so on indefinitely. This explanation may be verified by introducing a small solar ray into a dark room by means of a heliostat, and directing it through a glass cylinder filled with water. The course of the ray in the interior of the water will become visible by the partial reflection of light, occasioned by the particles of the water, and the successive emergences will also become sensible by placing the eye, or unpolished glass in the direction of the emergent rays. In this manner it will be seen that the rays are dispersed, presenting the series of prismatic colours; their intensity is diminished according as they have undergone a greater number of divisions, and at last the ray will become so feeble as to be insensible.

174. The phenomenon of the rainbow is caused by coloured spectrums, which emerge from the different drops of water after two refractions, separated by one or two intermediate reflections. Let us now inquire how the superposition of these partial spectrums produces the colours of the rainbow and determines its magnitude. In order to do this in a simple manner, let us take one simple incident ray, as red, for example. If this ray, after being refracted in the globule, is reflected once or several times at the second surface, and then emerges into the air, it will make, after emerging, a certain angle, with its primitive direction. This angle will be constant for all the rays of the same nature, which enter the globule under the same incidence; but it will vary with the incidence. In order to comprehend these variations, let us first consider the case in which the ray suffers only one interior reflection before emerging. Then, if we calculate numerically Fig. 115 the quantity of deviation for several incident parallel rays on the surface of the globule, the deviation at first is nothing, when the incidence, being perpendicular, the ray passes through the centre of the globule; then, the deviation gradually increases until the angle of incidence has reached a certain limit, which is about 54°½ for the red rays. Accordingly, a small parallel beam of these rays entering the globule at I, under this incidence, and being reflected once at the bottom, emerges also a par-

allel beam at I'', although the general direction of the ray will have deviated 42°. But for greater incidences the deviation diminishes as it before increased, and this diminution continues to the last rays, which are tangents to the globule. Now, if we intercept the rays at so great a distance from the globule that it may be considered as a point, it is evident, that all those which correspond to the unequal deviations, will depart more and more from each other, according as their distance from the globule is increased, so that they will at last be too feeble to produce the sensation of the globule in an eye placed in their direction; while the eye may be affected, and indeed is affected at any distance whatever, by the emerging rays which answer to the maximum of deviation, because, being parallel to each other they may pass to any distance without being separated. The effect will even be so much the more vivid according as, the density being uniform at their incidence, they are compressed and condensed, as the calculus shows that they are when they emerge in the manner above stated. Let us then suppose that a number of such globules are arranged circularly in such a manner that the refracted rays which come from them, and which are supposed to be of a uniform colour, shall arrive at the eye of the spectator. They will give the sensation of a luminous line; and several such rows, placed side by side, will produce, on account of the sensible opening of the pupil, a coloured space which will be equal to it in width.

The same remarks apply equally well where the reflections and refractions are more numerous. There is, in every case, a certain limit of incidence, at which the rays coming from the same beam and emerging near to each other, will leave the globule sensibly parallel, so that they may be transmitted to a great distance without becoming less vivid.

175. In order then to develope the consequences of what we have now established, we will suppose a person, placed at O, to look at a vast cloud, consisting of a multitude of spherical globules of water. Through the eye of this observer we draw the line SOC to the centre of the sun, to designate the direction of the incident rays, which we will first suppose to be parallel, or, which amounts to the same thing, to proceed from a point at an infinite distance. This being done, there will take place at the first surface of the globules, a partial reflection of all the colours

ig. 116.

composing the radiant light, which will impart a whitish tinge more or less deep, to the whole extent of the cloud. Besides this, if the cloud is sufficiently extended, two concentric arcs will be seen, tinged with all the colours of the spectrum. For if, through the eye at *O*, we draw the line *OV*, forming with *OC* an angle of 40° 17′, and cause it to revolve round *OC*, describing a conic surface, all the globules in the prolongation of this surface will be exactly in the situation necessary, in order that the most refrangible violet rays, which have undergone two refractions and one intermediate reflection, may emerge parallel and arrive at the eye in *O*, and this will not take place in the same manner on any other part of the cloud; so that by means of this species of rays, the spectator will see on the cloud a violet coloured arc, of which *OC* will be the axis, and *C* the centre. He will, moreover, behold an infinity of other concentric arcs exterior to the above, each of which will be formed of a single species of rays; and, according as these rays are less refrangible, their arcs will have a greater diameter, so that the largest, composed of extreme red, will subtend an angle *ROC* of 42° 2′. Thus the whole width of the coloured bow will be 42° 2′ — 40° 17′, or 1° 45′. The red will be on the outside and the violet within.

The contrary order will be observed in the case of two reflections. If we draw through the eye, the lines *OR′*, *OV′*, making with *OC* angles of 50° 59′, and 54° 9′, and with these respective inclinations cause them both to turn round *OC*, as an axis, the first will meet with all the globules, which, after having twice refracted and twice reflected the extreme red rays, can transmit them to the eye parallel to each other. The second will give the corresponding limit for the extreme violet rays. Between these two arcs there will be others tinged with all the intermediate prismatic colours. The whole taken together will form another bow, which will have for its width 54° 9′ — 50° 59′, or 3° 10′. This bow will have its colours disposed in the inverse order of those of the first, that is, the red will be on the inside, and the violet without. The distance of the two red arcs will be 50° 59′ — 42° 2′, or 8° 57′.

176. Such, therefore, would be the dimensions and distance of the rainbows, if the sun were but a point. But the sun has a sensible apparent diameter, which may be supposed, at a mean,

to be about 30′. Hence, if we consider the arcs we have determined, as produced by rays coming from the centre of his disc, the rays which come from the borders and from the interior, will form so many similar arcs of the same size, but which will have each for its axis the line drawn from the observer's eye to the point on the disc of the sun from which they emanate.

Consequently, if we describe from the point C the circumference of a circle $C'C''C'''$, equal to the apparent diameter of the sun, as seen from the point O, there will not only be formed around the centre of this circle an interior violet arc at the distance of 40° 17′; but there will also be as many of these arcs as there are points in the circle $C'C''C'''$, which are capable of becoming centres in their turn; that is, there will be formed a circular violet arc of a width equal to the apparent diameter of of the sun, and which will have its interior radius equal to 40° 17′ — 15′, or 40° 2′, and its exterior 40° 17′ + 15′, or 40° 32′. In like manner, the red arc which we found to be 42° 2′ from OC, will become a red bow, whose exterior border will have for its radius 41° 47′, and its exterior border 42° 17′; thus the whole width of the rainbow, comprised between these two extremes, will be 42° 17′ — 40° 2′, or 2° 15′, greater by 30′ than if the sun had been only a point. In like manner, the width of the exterior iris, which we have fixed at 3° 10′, will become 3° 40′; its interior semidiameter which was 50° 59′, will become 50° 44′, and the exterior, which was 54° 9′, will become 54° 24′. Finally, the distance between the two bows which was found to be 8° 57′, will be reduced to 8° 27′. But, owing to the width and the superposition of the partial arcs which compose them, the colours will be much less distinct than in the former case. Still these dimensions, determined by calculation, are exactly conformable to those furnished by observation, at least when the colours are the most vivid and the most strongly marked. For Newton, having on a certain day measured a rainbow, by means of instruments which he then possessed, found that the exterior semidiameter of the interior iris was about 42°, and that the red, the yellow, and the green, of this bow, taken together, were about 63′ or 64′, besides three or four minutes that may be added on account of the exterior red rays which were enfeebled and obscured by the brightness of the neighbouring clouds. The width of the blue was about 40′, at the least, without reckoning the violet, which was so obscured by the light from the clouds, that its width could not be measured. But upon the supposition that the width of the blue and violet, taken together, was equal to that of the red, yellow, and green, united, the whole width of the interior iris would be about 2° 15′, as above stated. The smallest distance between the two bows was 8° 30′. The width of the exterior iris was greater than that of the interior; but its hues were so feeble, especially on the blue side, that it could not be measured with accuracy. On another occasion, when the two bows were distinct, Newton found that the width of the interior iris was 2° 10′, and that, in the exterior, the width of the red, yellow, and green, was to that of the same colours in the interior, as 3 to 2.

177. In that part of Newton's optics where this admirable theory is unfolded, the first idea of the explanation of the rainbow is attributed to Antonio de Dominis, archbishop of Spalatro, who, he remarks, confirmed it by observations made upon a glass ball, filled with water, and placed in different situations with respect to the sun and the eye of the observer. Newton adds, that Descartes corrected the observations of the exterior rainbow. But he had probably never read the work of De Dominis, for if he had, he would have perceived that this prelate had formed a vague notion that the rainbow might be the effect of refraction in the drops of water, without attempting to confirm this idea by the experiments of which Newton speaks; and his mode of explaining the formation of the meteor is without reference to the true theory. The experiments really belong to Descartes, and to Descartes alone. This philosopher did every thing that could be done, at a time when the unequal refrangibility of light was unknown. He determined, by numerical calculation, the course of the rays which enter the drop of water, and emerge thence after one or more reflections. By this calculation, he discovered that of all the rays that enter the drop, those only which fall upon it at a certain inclination can come to the spectator without diverging from one another, and being consequently enfeebled. He thus ascertained the true circumstances that are necessary to the existence of the rainbow, and he proved by experiment, that they were conformable to actual observation. It only remained to assign the cause of the colours. Descartes, unacquainted with this cause, reduced it to

another more simple case, that of the decomposition of light by the prism; and he proved that the part of the drop of water in which the light is refracted, must disperse this light, in the same manner as a prism of water, with plain faces, whose angle of refraction is equal to that formed between two planes tangent to the points of the drop where the rays enter and emerge. He confirmed this theory by a very circumstanstial experiment, which is given in the *Traité de Physique.*

178. Besides the rainbow, other meteors are frequently observed in the atmosphere, as the large whitish circles or haloes, that are seen to surround the sun and moon, commonly at the distance of 45°, and parhelia and paraselenes, false suns and false moons, attended with other remarkable appearances. A particular description of these phenomena, with the hypothetical explanation of Huygens, may be found in the *Traité de Physique.*

Achromatic Combinations.

179. When two or more prisms are so constituted and disposed as to render colourless the images of objects seen through them, the combination is called *achromatic.* If these prisms were all of the same substance, the opposite refractions would balance each other, when their angles were equal and their directions contrary, so that they would together form a plate with parallel faces. Rays traversing such a system of prisms would continue on in their primitive direction. But if substances could be found capable of producing equal dispersions, with unequal refractions, a system of prisms might be formed which would give images of objects free from colour, the rays being deflected at the same time. Accordingly, by transferring this principle of compensation to the curved prisms which form the borders of spherical lenses, we might obtain compound object-glasses, which, with a limited focal distance, would produce colourless images of objects; and we might employ them in the construction of telescopes incomparably superior to those provided with single object-glasses. Are there then in nature substances which thus compensate each other? If not, can their place be supplied by an approximate compensation? And in this event, upon what prin-

ciples is this approximation to be established? How can it be produced with given substances? Is it more easily produced by some substances than by others? These are the questions which the subject presents for our examination.

180. Accordingly, let us first consider a perfectly homogeneous ray *SI*, which, after traversing a certain number of prisms, of whatever substance and angles, arrives at last at the eye of the observer situated in *O*. From this point let us draw a line *OS* to the object from which the ray proceeds, and which, for greater simplicity, is supposed to be removed to an infinite distance, so that the line *OS* may be considered parallel to *SI*. It is evident, that if this direction is given, together with the positions of the prisms, their substance, their angles, and the place of the observer, the angle of deviation *ROS*, comprehended at the eye between the primitive direction *OS* and the emergent ray *OR*, may be determined by calculation. Let us suppose now, that the ray, instead of being simple, is compound, like the white ray. Then, after traversing the system of prisms, it would be dispersed into an infinity of different rays, having different directions *v v′*, *o o′*, *r r′*, from the violet to the red; so that the observer, placed in the direction of one of these rays, would perceive only that single ray. But if we conceive an infinite number of white pencils of rays, all proceeding from the same point, to arrive together at the first surface of the prism, the observer, stationed at *O*, would receive all these different species of simple rays, through different points of emergence; which will be found by drawing from *O* the lines *OR′*, *OO′*, *OV′*, respectively parallel to the lines *r r′*, *o o′*, *vv′*, and terminating in the same manner upon the surface of the last prism; for these rays, being turned back through the successive prisms, each with the proper degree of refraction, will emerge in a direction parallel to the incident rays, and consequently, will make a part of the incident light. Then, in order to take from the observer the sensation of these separate rays, we have only to dispose the system of prisms in such a manner, that the rays shall all emerge parallel to each other; for then the angles *O′OV′*, *O′OR′*, &c., with which they cross each other at the eye, becoming nothing, the points of incidence *V′*, *O′*, *R′*, will coincide, and the pencil will consequently enter the eye free from colour.

Fig. 117.

Fig. 118

181. From this analysis it will be perceived, that the rigorous determination of the achromatic effect involves for each ray a particular condition which must be satisfied by the system of prisms. Hence it follows, that as there is an infinite number of rays of unequal refrangibility, so there are, strictly speaking, an infinite number of conditions which cannot all be fulfilled, because the number of prisms must necessarily be limited. The problem does not therefore admit of a rigorous solution. But a solution will be obtained with sufficient accuracy, if instead of attempting to render all the different rays parallel, we only render a few of them parallel to each other, which may easily be accomplished by a small number of prisms. Indeed, if we bring together in this manner the two extreme rays, the violet and the red, or two extreme rays and one mean, as the red, the violet, and the green, the removal of colour for the intermediate rays will manifestly be so far accomplished, that no sensible effect will be produced upon the eye.

This is in fact the course to be pursued; and it will be easily perceived that two rays may be united by two prisms, and in general, as many rays as there are prisms employed. If but two prisms are employed, they should be joined base to vertex, as in figure 119, so that their refractions may be in opposite directions; indeed, if the two prisms are of the same substance, we have seen, that in order mutually to compensate each other, they should be placed in this position with equal refracting angles.

182. But when the substances are not of the same nature, how are we to determine the ratios of the angles of refraction with which a compensation must take place? We can manifestly approximate to this determination, by considering the magnitude of the spectrum furnished by prisms of these two substances formed with equal angles. We can then effect the proposed object still more nearly by gradually diminishing the power of that which causes the greater degree of dispersion, and which, when the two prisms are combined, colours the image in the direction of its own dispersive power. When the compensation is thus in part effected, we can easily discover the best of all possible compensations by the following process, which was applied, in a great number of instances, with entire success, by M. Cauchoix and myself.

At the extremity of a good achromatic telescope, and parallel
g. 120. to its axis, are placed two metallic rods *T*, *T*, diametrically oppo-

site. These are pierced transversely with two holes *A*, *A*, through which pass the axes of two copper frames C, C, that turn freely round the straight lines *A*, *A*, and perpendicular to the axis of the telescope, the motion being measured by graduated circles. To these frames are attached the two prisms that are to be compensated, and the apparatus will then be disposed, as represented in the figure. In order that the experiment may succeed completely, the angles of the prism must be such that when they are turned in opposite directions, they will nearly form an achromatic system; this can be accomplished by repeated trials, as mentioned above. It has been already shown, that the dispersion does not depend simply on the angular opening and chemical nature of the substance; it varies also with the incidence of the rays; and this incidence enters as an element into the analytical conditions to be fulfilled, in order that the rays of different kinds may be parallel after their emergence. We can avail ourselves of this variation, in seeking, by experiment the precise situation in which the compensation is most perfect. Accordingly, let us suppose, that the telescope and the two prisms are so disposed, that a white object at a great distance may be seen through the system. If the image of this object is found to be equally white throughout, the prisms are properly placed; and this is the position which renders the achromatic effect complete; but it is not to be expected that a colourless image will be obtained on the first trial. A very sensible colour almost always presents itself after the refracting angles of the prisms are opposed to each other. Then we turn slowly one of the prisms in the direction necessary to diminish the colour. If a motion of one of the prisms is not sufficient, both must be moved successively, and thus we shall at length find the position in which the compensation is the most perfect. The prisms are to be fixed in this position, and it will only be necessary to measure the angles they make with each other, and with the rays that traverse them. Hence the design of the graduated circles that serve to measure the angles passed over by the frames. There are some precautions to be used for determining the precise point of each division which renders the frames perpendicular, respectively, to the axis of the telescope, and also for arranging the prisms in such a manner, that their refracting angles shall be exactly comprehended in the same plane. In the experiments made by M. Cauchoix and myself, the object selected as a mark, was a strip of white paper pasted to a sheet of black pasteboard; its width was about four inches, and its length about three and a half feet. We fixed the pasteboard in a vertical position, at a distance of about twenty rods, so as to present the longer sides of the piece of paper in a horizontal direction; and we placed the prisms in such a manner, that the planes of their refracting angles, in which the dispersion took place, should be vertical, which was effected by directing them towards the edge of some edifice or tower at a distance, turning them on their frames, till they ceased to exhibit any lateral deviation. When a proper position was found, they were made fast, and the foot of the telescope was also secured by a screw, preventing any horizontal motion. These preparations are of great importance in facilitating the achromatic process. For, when the image of the object is held in the telescope, it cannot be displaced, except in a vertical direction. Hence, while, by gently turning the prism with one hand for the purpose of destroying the colour, the image is displaced, it may easily be brought back by raising or depressing the telescope with the other. If the refracting angles are so disproportionate that the effect of colour cannot be corrected by a mere change of incidence, it will be easily perceived; for, if we suppose, for example, that the second prism is too powerful, and that the vertex of its refracting angle is directed towards the earth, then this prism, if it acts alone upon the light, will give a coloured image of the object, in which the red rays, being least refrangible, will appear highest, and the violet rays, which are most refrangible, lowest. Consequently, if the image of the object, seen through the two prisms, presents constantly this arrangement of the colours, in every variety of situation, it is a certain proof that the dispersion of the first prism, which acts in a direction contrary to the other, can never be sufficient to compensate it, much less to exceed it. It follows therefore, that the diminution of colour that can be produced by this system, is far from the greatest possible. In such cases it will be necessary to diminish somewhat the refracting angle of the second prism, which has thus been found to be too great, and then to replace it. If it has been sufficiently diminished, upon moving the two prisms alternately, we shall obtain images of the object in which the red fringes are turned upwards, and other images in which they are turned downwards. In the first case, the posterior prism prevails, in the last, the anterior. Between these two opposite states there will be found one or more situations, in which the colours of the borders of the image will be the least possible. Such combinations afford the most favourable compensations. We preserve that combination which seems best, that is, which gives the shortest and especially the dullest fringes. The prism and the telescope are then made fast in their places by means of screws. Special care should be taken to avoid the red and yellow fringes; for these two colours being brighter than the others, have a much more sensible effect when these results are applied to the construction of telescopes, for which they are principally designed. For a contrary reason, we should give the preference to those positions which produce fringes of obscure and deep colours, those, for example, of a brick red, or of a greenish blue; for these colours being less vivid, their effect will be less conspicuous in telescopes, and it will be altogether insensible in observations made at night.

183. Having found the most favourable disposition, we observe the positions of the two prisms by means of the graduated circles; from these we deduce the angles of incidence and emergence of the mean rays, and these elements, introduced into formulas, determine the ratios of the refracting angles under which the compensation of the two prisms will be effected, when placed upon each other, and exposed perpendicularly to the luminous rays. In our first experiments upon dispersion, we remarked that each prism had a certain position in which the refracted image becomes stationary; and that before and after this limit, the refraction and dispersion increase simultaneously. Hence in experiments upon this subject we ought to find for each prism several positions, in which the same effect is produced upon the image transmitted. This is indeed the case. But upon calculating the course of the rays, for all the various positions, on the supposition of a constant ratio of refraction for the different angles of incidence, we arrive finally at the same ratio of compensation, and that too with surprising accuracy, as may be seen in the *Traité de Physique.* This agreement is a very delicate and certain proof of the constancy of the ratio of refraction for each simply ray.

184. Whatever care we use in these experiments, if we employ a telescope which magnifies fifty or sixty times, and prisms whose refracting angle is at least fifteen degrees, conditions which are necessary in order to render the coloured fringes sensible, it will soon be perceived that an entire removal of colour is altogether impossible, except in the single case in which the two prisms are composed of one and the same substance. For, when the prisms have those refracting angles that are very near the proportion, which produces the best achromatic effect, each colour may be made to pass successively from one side of the image to the other without an intermediate position being found in which the coloured fringes disappear entirely. From this impossibility we infer with the utmost certainty, that the dispersion of the rays does not by any means take place according to the same laws in substances, which differ in their chemical composition; we infer also, that when two homogeneous rays, of whatever kind, have been refracted in such a manner as to be parallel at their emergence from the two prisms, the homogeneous rays of different species will be inclined to each other, and will form fringes on the borders of the object. Hence it is evident, that in order to obtain more perfect compensations, more than two prisms must be employed. Three are used in some achromatic telescopes; a greater number would diminish the light too much by their successive reflections; besides, when the achromatic effect of these instruments is determined with care, by the methods already pointed out, their imperfections are no longer the consequence of unequal refrangibility in the rays of light, but of the diffusion of the foci, in which spherical lenses, having any sensible aperture, concentrate the several incident pencils, which cover their whole surface.

185. Finally, the absolute quantity of dispersion, measured between two determinate colours, is as variable as the law itself, according to which the different rays arrange themselves in each spectrum. This is plainly indicated in the above process, by the varieties of coloured fringes obtained when two different substances are compensated by each other.

If substances are compared together, which have very unequal refracting powers, the dispersive powers will, in general, be found to be so likewise, and in the same direction, the most refractive substances being the most dispersive. For example, the oxide of lead introduced into the composition of glass, augments to a considerable degree, its dispersive power; it augments likewise its refractive power, although in a smaller proportion. Of all the substances which were tried by M. Cauchoix and myself, the liquid known in chemistry under the name of sulphuret of carbon, appeared to have the greatest dispersive power. The dispersion produced by it was ten times that of water, under similar circumstances; the refractive powers of sulphur, and of solid carbon are also very considerable. Still this correspondence between the increments of refraction and dispersion is very far from being general, especially when the ratios of refraction differ but little. The essential oils of citron and turpentine, muriatic acid, pure or saturated with ammonio-muriate of mercury, disperse more than crown glass, but refract less, as has been well ascertained. The same is true in a number of other instances; so that the relation of the dispersive forces of bodies to their chemical composition, is even more difficult to be foreseen than that of the refractive forces.

John Dollond, a celebrated English optician, was the first who proved, by experiment, the error of Newton, respecting the possibility of obtaining an achromatic compensation, preserving at the same time an excess of refraction. Euler supposed this to be possible, because it was realized, at least very nearly, in the construction of the eye, which is achromatic when well adjusted, and sensibly unites all the refracted rays in the image upon the retina, and paints the objects in their proper colours, as may be seen by taking the eye of an animal recently killed, and removing the outer coating from the back part, and examining the images formed there. But this remark, upon an organ thus composed, was not sufficient to develope the true principles upon which the compensation is founded. Euler proposed several hypothetical laws, which might produce this effect. It was in trying these laws that Dollond was lead to repeat some of the experiments of Newton, upon compensation by prisms of different refracting substances; and being led by chance, or a happy conjecture, to try some of a nature very different from each other, like crown glass and flint glass, he discovered that the refraction produced by crown glass remained predominant, when the angles of the prisms were such that the compensation was sensibly complete. By preserving the same relation between the prismatic borders of concave and convex lenses, made of these two substances, Dollond obtained achromatic object-glasses, which enabled him greatly to enlarge the apertures hitherto employed. Although great use has been made of this discovery, but little pains have been taken to perfect it, and opticians are still compelled to use the rules of compensation given by Dollond, even in instances where they are no longer applicable; and when under the necessity of deviating from them, on account of great difference in the substances, they have resorted to expensive and imperfect trials. These are the considerations which led M. Cauchoix and myself to seek an exact process, such as has here been given.

Of the Dispersion which accompanies Extraordinary Refraction, and the Separation of the Axes of Double Refraction with respect to the different Simple Rays.

186. It has been mentioned, that in crystals endued with double refraction, the extraordinary image is generally coloured, as well as the ordinary. This phenomenon, therefore, proves that, at equal incidences, the extraordinary velocities are unequal for the different simple rays, as analogy alone would indicate; but it may be easily shown that the mode of dispersion resulting from it must generally be more complicated than that for images where the velocity is constant. Indeed, confining ourselves, for the sake of simplicity, to crystals of one axis, if we call the ordinary velocity v, and the angle formed by the extraordinary ray with the single axis U; we have seen that the extraordinary velocity v', is given by the formula,

$$v'^2 = v^2 + k \sin^2 U,$$

in which k is a constant coefficient peculiar to each crystal, and which is positive in some and negative in others. Now, experiments show, that generally the values of this coefficient are sensibly different for the different simple rays, and increase with the refrangibility. Their variations, or rather those of the product $k \sin^2 U$, which result from them, combine therefore with those of the square of the ordinary velocity to form v', and as the values of v increase also with the refrangibility of the rays it is manifest that if k is positive, as is the case in attractive crystals, the two causes of variation conspire in the extraordinary velocity; while on the contrary, if k is negative, as it in fact is in repulsive crystals, they counteract each other. In this last case, therefore, it is possible that they may nearly or entirely destroy each other; and then the extraordinary image may be sensibly white, although very powerfully refracted. Indeed, this singular phenomenon, is exhibited in the rhomboids of Iceland spar, when the incident ray SI, is directed towards the small solid Fig. 78. angle B'; because then the ordinary refraction tends to disperse the spectrum, by causing the most refrangible rays to approach the normal to the point of incidence; while the repulsive force, emanating from the axis IA', tends to disperse them in the opposite direction. Malus found that these two opposite causes counterbalance each other when the incidence is about 40°. Then the ordinary image only is dispersed; and the extraordinary image, although strongly refracted, is sensibly white.

187. It has likewise been remarked that, according to a very 118. ingenious discovery made by the son of the celebrated Sir William Herschel, the axes of double refraction in crystals, are not always the same for all kinds of simple rays, but have different positions, and different inclinations to each other, for these different rays. This dispersion has as yet been observed only in crystals of two axes. Indeed, it is easy to see that it is not possible in others, simply from the condition of symmetry of faces, which is necessary for the existence of a single axis. For the 120. same reason, in crystals with dispersed axes, all the pairs of axes are comprehended in the same plane, and have the same intermediate line common. But the direction of the dispersion is subject to no rule. In some crystals, such as the sulphate of barytes, the nitrate of potash, aragonite, sugar, and hyposulphate of strontian, the axes of the red rays are less inclined to each other than those which correspond to the violet rays. The reverse takes place in borax, siberian mica, sulphate of magnesia, white topaz, and tartrate of potash and soda. The phenomenon is particularly excessive in the last salt. According to Mr Herschel, the inclination of the axes in this, is 55° 14′ for the violet rays, and 75° 42′ for the red rays, each being taken at the extremity of the spectrum. It is obvious that these differences as to position and angles must affect the course of the refracted rays, calculated

eye were furnished with a magnifier A_2, placed in such a manner that the image $f_1\,\varphi_1$, may be a little within the principal focal distance; for there would result a second image $f_2\,\varphi_2$, likewise inverted, but greater than $f_1\varphi_1$. There will then be no need of a ground glass, which would absorb a great portion of the light; for the particular foci, which united, form the image $f_1\,\varphi_1$, will have with respect to the lens the same effect as so many radiant points, only much more luminous. Such is the manner in which the eye-glass A_2 is disposed in the compound microscope. The two lenses are fixed in the ends of a tube, the parts of which slide in each other for the purpose of varying the interval which separates the glasses.

191. Here, as in simple microscopes, the magnifying power is in the ratio of the absolute dimensions of the object $S\Sigma$ to those of the last image $f_2\,\varphi_2$. Hence, other things being the same, it magnifies objects the more according as the focal distances of the lenses are less. The focal distance A_1F_1 of the object-glass being diminished, the image $f_1\,\varphi_1$, formed behind this glass, becomes greater at the same distance. By shortening the focal distance A_2F_2 of the eye-glass, the image $f_2\,\varphi_2$, resulting from $f_1\,\varphi_1$, is enlarged, the distance of distinct vision being the same. The first kind of variation is limited by the difficulty of accurately constructing very small lenses, the injurious effect of increasing the angles of incidence and emergence of the rays at the surfaces of these lenses, and lastly, the very small quantity of light which they are capable of receiving and transmitting. But the increase of power derived from contracting the eye-glass, is still more limited, on account of the necessity of having it of considerable magnitude. Indeed, the surface of the object-glass being small, the pencil of refracted light that comes from any part of the object, is very minute; so that when it is concentrated in one of the points of the image $f_1\,\varphi_1$, and radiates anew from this point, it will not cover the whole of the eye-glass A_2, but will fall upon only a small portion of it; hence, if the eye-glass is so contracted in its dimensions that the pencil from a given point of the image shall not meet this glass, it will be wholly lost to the eye. The *field of view* of the instrument, that is, the space embraced by the eye as it looks through the lenses that compose it, will be limited also by the last rays which meet the extreme border of the eye-glass; this is another reason why it cannot be indefinitely diminished.

no rays but those which come nearly in the direction of the axis, common to all the lenses, can be of use in vision. Hence, in order to insulate these rays completely, a number of transverse partitions with circular openings, called *diaphragms*, are placed in the interior of the tube, being coloured black, in order to arrest by their opacity the rays which are too oblique.

In general, all instruments of this kind may properly be considered as cameræ obscuræ, or dark rooms, of small dimensions and a field of view of little extent. This last limitation is required in order to the enlargement to which it is to be subjected in the image; for it would appear distorted if it were not reduced to very small dimensions.

189. Each kind of instrument is appropriated to a particular purpose; some are employed in examining very minute objects at a short distance by enlarging their image; these are called *microscopes;* others are used in viewing distant objects, under a greater angle than they present when seen by the naked eye, they being seen with equal distinctness at the same time; these are called *telescopes*. In both these kinds of instruments, the same principles are employed; and they are adapted to their respective objects by introducing some peculiarities in the construction. This will be seen as we proceed to examine successively those which are the most common and most useful.

Compound Microscope.

190. The object-glass of this instrument is a small lens A_1 of Fig. 12 a very short focus, before which are placed minute objects $S\Sigma$, at a distance $A_1 P$ or Δ, which exceeds very little the distance $A_1 F_1$ of the principle focus. Behind this lens is formed an inverted image $f_1\,\varphi_1$, at a distance $A_1 P_1$ or S, much greater than Δ; and if the magnitude of $S\Sigma$ is expressed by I, that of $f_1\,\varphi_1$, will be expressed by $\frac{IS}{\Delta}$; hence this image is likewise much greater than the object $S\Sigma$. If this degree of magnifying power were sufficient, the image might be received upon ground glass, placed at $f_1\,\varphi_1$, and viewed in this situation by the naked eye. But it is evident, that the effect would be increased still more if the

eye were furnished with a magnifier A_2, placed in such a manner that the image $f_1\ \varphi_1$, may be a little within the principal focal distance; for there would result a second image $f_2\ \varphi_2$, likewise inverted, but greater than $f_1 \varphi_1$. There will then be no need of a ground glass, which would absorb a great portion of the light; for the particular foci, which united, form the image $f_1\ \varphi_1$, will have with respect to the lens the same effect as so many radiant points, only much more luminous. Such is the manner in which the eye-glass A_2 is disposed in the compound microscope. The two lenses are fixed in the ends of a tube, the parts of which slide in each other for the purpose of varying the interval which separates the glasses.

191. Here, as in simple microscopes, the magnifying power is in the ratio of the absolute dimensions of the object $S\Sigma$ to those of the last image $f_2\ \varphi_2$. Hence, other things being the same, it magnifies objects the more according as the focal distances of the lenses are less. The focal distance A_1F_1 of the object-glass being diminished, the image $f_1\ \varphi_1$, formed behind this glass, becomes greater at the same distance. By shortening the focal distance A_2F_2 of the eye-glass, the image $f_2\ \varphi_2$, resulting from $f_1\ \varphi_1$, is enlarged, the distance of distinct vision being the same. The first kind of variation is limited by the difficulty of accurately constructing very small lenses, the injurious effect of increasing the angles of incidence and emergence of the rays at the surfaces of these lenses, and lastly, the very small quantity of light which they are capable of receiving and transmitting. But the increase of power derived from contracting the eye-glass, is still more limited, on account of the necessity of having it of considerable magnitude. Indeed, the surface of the object-glass being small, the pencil of refracted light that comes from any part of the object, is very minute; so that when it is concentrated in one of the points of the image $f_1\ \varphi_1$, and radiates anew from this point, it will not cover the whole of the eye-glass A_2, but will fall upon only a small portion of it; hence, if the eye-glass is so contracted in its dimensions that the pencil from a given point of the image shall not meet this glass, it will be wholly lost to the eye. The *field of view* of the instrument, that is, the space embraced by the eye as it looks through the lenses that compose it, will be limited also by the last rays which meet the extreme border of the eye-glass; this is another reason why it cannot be indefinitely diminished.

192. From the smallness of the refracted pencils the field of view may be considered as limited by the pencils whose axes $A_1 f_1$, $A_1\varphi_1$, graze the borders of the glass. No notice need now be taken of the more oblique pencils, of which only a part falls upon the eye-glass, on account of the feebleness of the light; they are moreover excluded by a diaphragm, placed between the glasses, where the image $f_1\ \varphi_1$ is formed. Hence, if SA_1, ΣA_1, represent the axes of the extreme pencils, the extent of the field of view will be equal to the angle $SA_1\Sigma$, or $I_2A_1\ i_2$; it may then be easily calculated; for if we call h the interval A_1A_2 between the two lenses, and z_2 the semidiameter I_2A_2 of the second, half of this angle, or $I_2A_1A_2$, will have for its trigonometrical tangent the ratio $\frac{z^2}{h}$; from this expression the angle may be obtained by the tables. In order that the eye may embrace the whole of this extent, it must be placed at the point O of the axis, where are collected all the extreme emerging rays, and generally all the most minute pencils which proceed from the several points of the last image $f_2\ \varphi_2$. Hence the distance of this image from the point O should be equal to the distance at which distinct vision takes place. Each observer can fulfil this condition by varying, according to the reach of his eye, the interval h between the two lenses, or the distance Δ of the object, which varies also the magnifying power and the extent of the field of view. The eye being thus placed at the proper point O, will perceive the whole of the field of view, and would perceive it even if the opening of the pupil were infinitely small. But when removed only a small distance from this point, it would scarcely receive any rays, except those in the axis, and the field would vanish, were it not for the sensible opening of the pupil, which enables the eye to take in the whole field when it is not situated exactly at the point O.

193. By introducing the several particulars of this construction into the general formulas which express the course of the rays of light through several spherical lenses, arranged on the axis, we obtain the exact measure of all the phenomena above indicated. But they may be obtained also by experiment.

The extent of the field of view is determined immediately by tracing the limits of the space visible at a given distance A_1P. The magnifying power requires certain details which we proceed to make known.

ig. 122. The body of a compound microscope generally consists of three tubes that slide within each other. The upper part *GE*, in which the eye glass is fixed, and which for this reason is called the eye piece, is fitted to slide firmly in the tube *FC*; this in its turn slides firmly, by means of friction, in the lower tube BA_1, in the bottom of which the object-glass is screwed. This being premised, the eye piece *GE* is taken out, and at *DD* a circular diaphragm is introduced, its diameter having been previously measured; then, replacing the eye-piece, it is made to slide down till the borders of the diaphragm are distinctly seen, which will then be found precisely at the point where the image of an object, formed by the object-glass, must be brought, in order to be distinctly seen through the eye-glass. The diaphragm *DD* is usually in the piece *FC*; and the proper distance between the eye-glass and the diaphragm, for different eyes, is obtained by drawing out or pushing in the piece *GE*.

Before the object-glass A_1 there is a double circular ring of metal *SS*, in which are placed the objects which are to be examined with the microscope; or rather these objects are fixed on plates of glass or mica, which are made to slide between the rings. The instrument is mounted in such a manner that the rings *SS* may be made to approach to, or recede from, the object lens, for the purpose of placing the object at a proper distance from this lens. Then, when the magnifying power is to be measured, instead of an object, a piece of glass is made to slide in the rings. On this glass are drawn, with a diamond, a number of parallel lines, the distances of which from each other are accurately known; let them be thousandths of an inch, for example. A glass thus divided is called a *micrometer*. When this is placed, the middle piece *FC* is let down to a certain fixed point. Then, without touching the eye-glass, the whole body of the microscope *GFC* is raised or lowered till the lines on the glass micrometer are distinctly seen. The image will thus be found to be exactly in the diaphragm *DD*, since it is only in this situation that objects can be distinctly seen through the eye-glass. The divisions, contained in the diaphragm, are thus counted; let us suppose the number to be equal to m, and that the real diameter of the diaphragm is M; it follows therefore that m divisions of the micrometer, thus magnified, are equal in extent to M. Hence the magnifying power of the object-glass will be represented by $\frac{M}{m}$. Suppose the micrometer to be divided into thousandhts of an inch, and the diameter of the diaphragm into 5 tenths of an inch. The instrument being brought to the proper point of view, I will suppose that 20 parallel divisions are contained in the diameter *DD*; there are then 20 thousandths of an inch, which, by the power of the object-glass, become equal to 5 tenths, or 500 thousandths of an inch. Thus the magnifying power produced by the object-glass, at this distance, is found to be $\frac{500}{20}$ or 25; hence we conclude, in general, that the first image $f_1\,\varphi_1$, formed behind the object-glass, is twenty-five times as large as the object.

194. Now the enlargement of this image $f_1\,\varphi_1$, produced by the object-glass, may be calculated upon the principles made use of in the case of a simple magnifying glass, from its focal distance and the distance of distinct vision, the rule heretofore given 62. being reversed. Thus the total enlargement produced by the compound microscope, will be the product of these two partial enlargements. If, for example, in the situation where the object is placed, it is magnified twenty-five times by the object-glass alone, and its image is magnified ten times by the eye-glass, the total effect will be equal to the product of 25 by 10 or 250.

The proportions thus determined belong to the distance Δ, at Fig. 121 which we place the object from the object-glass. The effect is increased if we bring it nearer the principal focus *F*, and diminished if we remove it farther off; for, in the first case, the image $f_1\,\varphi_1$, is thrown further from the object-glass A_1, and, in the second, it is brought nearer. Accordingly, if we do not wish to alter the eye-glass, it would be necessary to move the intermediate piece *FC*, in order to lengthen or shorten the tube, in such a manner that the new image may always be formed within the borders of the diaphragm *DD*, and thus be found at the constant distance from the eye-glass which is adapted to distinct vision. We might repeat, in each of these new positions, the process for determining the enlargement produced by the object-glass, but this is not necessary; for it is shown by calculation, that, for the same distance of distinct vision, the values of total enlargement are sensibly proportional to the distance A_1E, or the interval between the object-glass and the eye-glass, diminished by the focal distance of the eye-glass. It is sufficient then to measure these distances in the first experiment, when the intermediate piece *FC*, is pushed down to its limit; and by tracing on the tube a scale of equal parts, which will indicate, for all other cases, the quantity by which it is lengthened, we can deduce the magnifying power of the microscope, by the proportion we have stated. Or, if we wish, we can mark these enlargements on the tube itself, against a certain number of divisions, sufficiently near together, to allow the intermediate ones to be calculated by simple means. But these values, being compounded of the effect of the object-glass and that of the eye-glass, will still vary for different eyes, according to the distance of distinct vision; because it will be necessary, in order to adapt the distance of distinct vision to different eyes, to increase or diminish the distance of the eye-glass from the diaphragm; so that the same instrument will always magnify more for longsighted than for shortsighted persons, as we have shown in the case of simple microscopes.

195. When the microscope is once prepared in the manner above described, it may be employed for the purpose of obtaining the absolute dimensions of small objects, and the result will not be far from the truth. We might place one of these objects immediately on the glass micrometer, and see with the microscope how many of its divisions are covered by it, were it not that the thickness of the object, however small we suppose it, has an important influence on the place of the image, on account of its proximity to the principal focus of the object-glass; so that we can never see it distinctly through the eye-glass, and the divisions of the micrometer at the same time. To remedy this inconvenience, we place another micrometer in the interior of the microscope, exactly in the place where the first image is formed, that is, on the diaphragm *DD*, which answers to the focus of the eye-glass. Then we easily observe *how many divisions the image of the object takes up on this interior micrometer.* This number, divided by that which expresses the enlargement produced by the object-glass alone, for its actual distance from the image $f_1 \varphi_1$, or the micrometer, gives the absolute size of the object.

196. The above method was long since made use of by M. Charles. The limits of this treatise will not allow a detail of all the improvements introduced by this skillful observer. But I

will mention some particulars to be attended to without which the microscope would be altogether useless.

In the first place, it is absolutely necessary to illuminate strongly the objects we wish to observe. These objects being hardly ever luminous of themselves, emit directly very few rays; and only a small number of these are admitted into the microscope, on account of the very small opening which must necessarily be given to the object-lens. If then we confine ourselves to this small degree of light, the image will be so faint, that we can hardly ever perceive it, however little it may be magnified. On this account we illuminate the object strongly, by throwing upon it, from a slightly concave mirror, the ordinary light of the sky, or that of a lamp condensed by means of a converging glass. If the object is opaque, we illuminate it in this manner by light from above; but if it is transparent, we generally receive the light from below. I say, generally, because there are some cases in which it is best to have the light directed otherwise. When, for instance, we wish to observe the divisions of the micrometer, in order to determine the magnifying power, we never see them better than by an oblique reflection; then they are delineated in black on the glass plate. There are some of these micrometers which contain 900 visible marks in the space of a French line or $\frac{8}{90}$ of an English inch.

197. Another indispensable precaution is to place diaphragms in the interior of the instrument, to limit the field of view, and to exclude all that part of the image which is ill defined. For in all the preceding considerations, we have supposed the incidences and emergences infinitely small. They are not so in reality, and they are the less so, according as we give to lenses a greater opening. Hence the concentration of the rays in a single focus, the regular formation of the image, its perfect similarity to the object, and all the other properties which exist under very small inclinations, are only approximations from which we deviate more and more, in proportion as the glass used has a greater aperture. Now we must deviate only just so much as to prevent vision from becoming too defective; and this we do, by limiting the field of view by diaphragms, differing in extent according as the case may require; and a very simple method, is to retrench from the image whatever might affect the distinctness of its outlines.

198. Dr Brewster has contrived a very ingenious method of employing the microscope for measuring the ratios of refraction in liquids, and a great number of substances partially solid, such as wax, gum elastic, &c., which are opaque in the mass, but which become translucid, or even transparent, when made sufficiently thin. For this purpose, we fix before the object lens, and nearly in contact with it, a very thin plate of glass with parallel faces; then we bring the microscope to the point of distance proper for seeing distinctly, through this system, the marks of the object-glass micrometer. This being done, we insert between the glass plate and the lens, a drop of the liquid whose refraction we wish to examine; or if it is a solid substance, we detach a very thin lamina and press it strongly between the glass plate and the lens, until it is moulded, as it were, upon the surface of the lens. This operation forms, with the substance pressed, a true divergent meniscus, whose anterior surface is plane, and whose posterior surface has the same curvature as the anterior surface of the object lens. The interposition of this meniscus necessarily increases the distance at which the image of the object is formed behind the object lens; so that if we keep the object-glass micrometer at the same anterior distance, we must lengthen the body of the microscope in order to see clearly the image in the diaphragm, or what leads to the same result, we may leave the body of the microscope unaltered, and increase the distance of the object from the lens. The amount of this alteration depends upon the curvature of the object-lens and upon the nature of the substance interposed. If the curvature is known, we deduce from this amount, the ratio of refraction of the substance. If it is unknown, and it is always best to make this supposition, then we begin by forming the meniscus with pure water, whose ratio of refraction is known, being, according to Newton, equal to $\frac{529}{396}$ or 1,33586; and by comparing the alteration of distance necessary in this case, with that required for the particular substance afterwards subjected to experiment, we deduce the ratio of refraction of this substance compared with that of water. The application of this process requires much skill and address, because the accuracy of it depends upon the more or less perfect exactness with which we bring the image into the diaphragm, in doing which, our only guide is the condition of greatest distinctness of vision.

Still it admits of many very useful applications, both for substances which only become transparent when reduced to their laminæ, and for liquids of which we possess only very small quantities, a single drop being sufficient for the observation.

Amplifying Glass and Achromatic Eye-Glasses.

199. The imperfections of the microscope arise in a great degree from the want of achromatic lenses, which is the more felt according as we attempt to make use of higher degrees of magnifing power. Unfortunately it is impossible to remedy this defect entirely, since it is in vain to think of forming achromatic lenses so small as those which the microscope requires. But this evil may be, to a considerable degree, removed by a method which suggested itself to practical men, before any theory had been devised to explain its effect, or to show how it might be employed in the most advantageous manner. This method consists in placing, in the interior of the microscope, behind or before the first image $f_1\ \varphi_1$, a third converging glass, of a focus properly determined. Then the course of the rays will be such as is represented in figures 123, 124. The first disposition, figure 123, was invented by Campani; the other, figure 124, by Ramsden.

The employment of this glass, called the *amplifying glass*, is common to all dioptric instruments. Its evident use is so to collect the pencils separated by the object-glass, to concentrate them in a smaller space, and thus to render the image more distinct, smaller, and consequently to make a greater part of the object visible by a given eye-glass. But there is another less obvious use, which consists in the achromatic influence which this glass exerts.

200. When the rays coming from an object, have been refracted by any system whatever of spherical lenses, which form images of it about the axis AX, owing to the unequal refrangibility of light, the foci of the rays of different colours are not, in general, formed at the same distance; so that if φ_1 is the focus of the violet rays, φ_2 will be that of the indigo, φ_3 that of the blue, and finally φ_7 that of the red; and as the same property Fig. 125.

belongs to the radiating points situated without the axis, there will be formed in φ_1, a violet image VV, in φ_2 a blue image BB, and in φ_7 a red image RR; and the same cause which distributes them at different distances will give them also different dimensions. Now, if the eye be placed at some point O, in the axis AX, for the purpose of looking at these images, it will, in the first place, suffer the inconvenience of their unequal distance, which will prevent them from being seen together at the exact distance necessary for distinct vision. Moreover, it will be disagreebly affected by the inequality in their size; for, as they project beyond one another, they will exhibit the outlines of objects bordered with coloured fringes of red, violet, or the intermediate colours, according as one or the other may predominate, in the course of the refractions they severally undergo. A great advantage, therefore, will be obtained if we can regulate the size of these images, in such a manner as to make them exactly proportional to their distances from the eye, as represented in figure 126; for then the eye seeing all their borders in the same straight line VRO, will receive at once, from these borders, the sensation of all these species of rays, and consequently the coloured fringes will disappear. Now this disposition, which would seem at first to be very complicated and difficult, is found in fact to be extremely simple; and is precisely the effect produced by the amplifying glass, when its focal distance, and its position with respect to the other glasses, are properly determined. This can be done only by calculation, and consequently, I cannot here state the conditions. I will simply remark, that this arrangement, in order to be possible, requires that there should be in the instrument, at least two glasses A_2, A_3, besides the object-glass A_1; for, with the object-glass and only one other, we should not be able to render the borders of the last image colourless, except in one particular situation of the object. Accordingly, in all the applications that follow, *the achromatic eye-glass must always be composed of at least two lenses.*

Fig. 123. 201. The achromatic eye-glass of Campani, is the one always employed in compound microscopes, and generally, in instruments where we do not wish to extend fixed wires over the image given by the object-glass. But when these wires become necessary, as in astronomical instruments destined for measuring, where we are obliged to fix precisely the direction of the rays which come from the heavenly body to the eye, at a known instant, we cannot employ this arrangement; because, in drawing out or pushing in the eye-glass to accommodate it to different eyes, we should necessarily move the wires, and if this movement were not exactly in the axis of the telescope (and we cannot suppose it would be), the successive passages of the body would not be comparable with each other. In this case, the eye-glass of Ramsden is particularly applicable; because, Fig. 124. being situated completely beyond the first image $f_1\,\varphi_1$ it can be drawn out or pushed in, without moving the wires extended at the place where the image is formed. Accordingly, we always employ it under these circumstances, and it was for this purpose that that celebrated artist contrived it. By examining the effect of the eye-glass of Campani, in microscopes, M. Cauchoix found it advantageous to give to the amplifying glass the form of a meniscus convex towards the object-glass. As to the enlargement produced by this apparatus, we shall explain hereafter the means of determining it according to a process devised by M. Arago, and which is applicable to all optical instruments. We repeat this observation for two different distances of the object from the object-glass; which will cause the image to advance or recede, and make it necessary to move the two lenses of the compound eye-glass, in order to bring it to the true point for distinct vision. We shall thus have two known enlargements, for a known lengthening of the tube; their difference, distributed uniformly among all the intermediate lengthenings, will give the corresponding intermediate enlargements, which may be traced on the tube.

A bare inspection of figures 123, 124, will show that these eye-glasses do not change the inversion of the objects produced by the object-glass; but their erect position may be restored by employing more than two glasses, as we shall see hereafter. This is done, in certain cases, in the telescope, which is next to be considered.

Dioptric or Refracting Telescope.

202. If we enlarge the object-glass of the microscope and remove the object to a great distance, we shall have a refracting telescope, which, like the microscope, may be composed of two, three, or a greater number of glasses. But the first lens A_1, being no longer of very small dimensions, may be formed of an achromatic assemblage of several glasses in contact with each other, which will give the same focal distance for all the coloured images produced by it. These images will, in reality, be separated from each other, in traversing the eye-glasses to arrive at the eye. But besides that this separation will be very small, in consequence of the short distance through which they move, the effect will become altogether insensible, if the eye-glasses are combined in conformity to the principles laid down in a preceding section; for then, the coloured images which are presented to the eye, being very close to each other, will, at the same time, have dimensions proportional to their distances, so that the achromatic effect will appear perfect. Accordingly, this arrangement is generally practised.

203. The first and most simple kind of telescope is that called *astronomical.* It is represented in figure 127. The object-glass A_1, is a converging lens, and it must always be such, in order to throw the image on the posterior side towards the eye. The eye-glass A_2 is also a converging lens, and the last image $f_2\ \varphi_2$ is inverted.

This disposition is exactly similar to that of the microscope with two glasses, except with respect to the diameter of the object-glass A_1. Hence result larger pencils and a more consider- Fig. 121. able accumulation of light. But if we consider the axes of these pencils, which enter through the centre of the object glass, their course is absolutely the same, and accordingly the conditions which they must fulfil, in order to produce in the eye distinct vision, will be the same as before; only, as the object is now very remote from the object-glass, the image is formed behind it at nearly the same invariable distance, which is that of its principal focus. Moreover, since its distance from us is so great that we cannot have a distinct idea of it, while the distance of the last image from the eye is so small as to admit of no comparison with the other, the enlargement is no longer measured by the actual ratio that exists between the size of the object and that of its image, but by the ratio of the visual angles $SA_1\Sigma, f_2\,O\,\varphi'_2$, which the object and the image respectively subtend at the eye. If the instrument is composed of only two glasses, this ratio is sensibly equal to the focal distance of the object-glass, divided by the focal distance of the eye-glass, at least when this last may be considered as very small compared with the distance of distinct vision. Such accordingly will be the value of the magnifying power. But here, as in the microscope, we hardly ever employ the simple eye-glass, on account of the colour which it produces on the borders of the last image $f_2\,\varphi_2$, even when the object glass is achromatic. If the instrument is intended for astronomical observations, where distinctness of image and abundance of light, are the only essential conditions, we employ the compound eye-glass of Ramsden, or that of Campani. These do not give the image erect; but this is not important in astronomical observations.

204. The same cannot be said with respect to telescopes intended for terrestrial objects. Here it is essential that the last image, situated near the eye, should represent objects erect. This purpose is effected by making the eye-glass to consist of four separate lenses, of which the two first A_2, A_3, nearest the object-glass A_1, are intended simply to render the image erect, while the two last A_4, A_5, situated near the eye, complete the achromatic effect upon the borders, and have also the same ratio to each other, as those in the eye-glass of Ramsden or Campani. The magnifying power of the telescope depends upon the foci of these five glasses and their intervals asunder. Now, if we give to the two last, situated next the eye, the distances which are necessary for rendering the borders colourless, we can still vary the positions of the rest within certain limits, without the instrument failing to perform well. But the magnifying power will vary, and we can, by this single movement, cause it to pass through all its degrees. M. Cauchoix has effected this in his terrestrial telescopes, which he has called for this reason *polyalde.*. They thus give at pleasure, a feeble or a great magnifying power, which is often very advantageous, the first being more convenient in case of a fog, and the second in clear weather. Fig. 128.

Telescopes thus disposed and intended to be portable, vary their magnifying power from twenty to forty times, or from thirty to fifty times. The magnifying power of the greatest astronomical telescope hitherto known, amounts to 1000 or 1200 times, this estimate, like the preceding, being applied to the diameters of the objects. But we do not adapt to these instruments the polyalde apparatus, because the multiplicity of reflections from the glasses would weaken the light too much; and when we wish to alter the magnifying power, we do it by changing the eye-glass.

205. There are also telescopes in which the object-glass, always converging, is combined with a simple eye-glass, but Fig. 129. diverging. This arrangement, invented by Galileo, is still made use of in opera-glasses. It causes the objects to be seen erect.

In this case, the first image f_1 φ_1, given by the object-glass A_1, is not actually formed, although in estimating the results, we must consider it as really existing. Before the focus P_1, where it ought to be produced, we place the diverging eye-glass A_2, at such a distance that the convergent pencils, tending towards the points f_1, φ_1, may be changed to divergent pencils, departing from other points f_2, φ_2, situated before the eye-glass, and at a distance equal to that of distinct vision. These points form then the last image, or that which the eye perceives. The deviation produced by the eye-glass in the axes of the pencils which compose this image, makes it erect, because their directions cut each other before reaching the eye. But for this very reason, the eye cannot be placed at the point of meeting O, which falls in the interior of the tube; but, foregoing this favourable position, it must be placed somewhere without, in the axis of the glasses, at O', for example, where it receives only the divergent portion of each pencil, which passes, in this place, sufficiently near the axis A_1X, to be able to enter the pupil of the eye. It follows from this disposition, that in proportion as the eye is removed from the point of meeting O, a greater number of pencils, diverging beyond the space embraced by the pupil, escape it entirely; and as this disappearance necessarily commences with those which are most distant from the axis, and which form the borders of the image, it follows that the field of view is diminished in proportion as the eye is removed. Accordingly, the position of the eye which is the nearest possible to the eye-glass, is that which gives the greatest field of view. Notwithstanding these inconveniences, the use of diverging eye lenses in opera-glasses has two advantages; one is, that it makes the object appear erect; and the other, that it shortens the total length of the instrument, by being placed within the focus of the object-glass, whereas converging eye-glasses lengthen it, by being placed without. We accordingly employ in these glasses, only a simple eye-lens, although it inevitably produces colour, even when the object-glass is achromatic; because, being intended to be used in the evening, in places where the degree of illumination is less than daylight, the colours which are developed, when the instrument is well made, are not very vivid, especially if we take care to place the pupil in the axis; we should, moreover, weaken the light too much, if we made use of eye-glasses composed of several lenses.

Instruments which consist of a Combination of Mirrors and Spherical Lenses.

206. Catoptric instruments of whatever kind consist of mirrors either concave or convex, arranged in such a manner as to give, by reflection, distinct images of objects, which are observed by means of a simple or compound eye-glass. Both telescopes and microscopes may be made in this way; but we shall here consider only the former, the latter being no longer in use; and we shall confine ourselves to the illustration of the most common case, in which the distance of the object may be considered as infinite. It will easily be perceived that the method must be the same in all other cases.

The simplest of all reflecting telescopes is that which is represented in figure 130. It is formed with a single concave mirror *MM*, which, receiving the rays from a distant object $S\Sigma$, forms with them, at its focus, an image $f_1\ \varphi_1$, which we look at through an eye-glass for the purpose of magnifying it. But in this case, the observer being interposed between the mirror and the object, necessarily interrupts a part of the incident rays; on this account, we cannot employ such an arrangement, except with very large mirrors; and to prevent as much as possible the loss of light, we direct the axis of the instrument a little obliquely with respect to the object, in order that the image may be formed without the axis, and the top of the head only intervene. Then, if the mirror be very large, the loss of light will be small, compared with that which more multiplied reflections and refractions would occasion. Sir William Herschel constructed, in this manner, a large telescope of forty feet focus, with which he made a number of discoveries. There was a similar one at the observatory of Lilienthal, in the hands of M. Schroeter. The trouble of moving such large instruments makes it very difficult to use them.

207. Next to the form above described, the most simple is the Fig. 131. one invented by Newton. It consists in placing in the interior of the telescope, and near its principal focus F_1, a small plane mirror *m m*, inclined to the axis at an angle of 45°, and of dimensions just sufficient to receive all the reflected rays. This mirror changes the direction of the rays, but in changing them, does not at all alter their convergence. It only throws the focus in a direction perpendicular to the axis, and at the same distance from the point where the mirror cuts the axis, at which it would be if formed in its prolongation. Opposite to this new direction, we make a lateral opening in the tube of the telescope, in order that the rays may emerge, and we look at the image through a simple or compound eye-glass. This arrangement prevents the direct interposition of the observer, and allows us to employ mirrors of all dimensions. But it occasions a considerable loss of light by the second reflection which it requires, especially since the reflection takes place from a metallic surface, whose absorbing power is always very great. Accordingly, in telescopes of this kind constructed by Newton himself, in order to change the direction of the image, he made use of total interior reflection, at the second surface of a rectangular glass prism, so situated in the telescope, that one of the sides containing the right angle, should be perpendicular to the axis of the telescope, as represented in figure 132.

208. The position of the observer at the side of the telescope is inconvenient, when he is seeking, with the telescope, any of the heavenly bodies. It is much better when in the direction of the axis of the instrument. This purpose is effected by the construction of Gregory, represented in figure 133. It consists in substi-

tuting for the plane mirror, a small concave mirror $m\,m$, which reflects the rays coming from the large mirror, and sends them towards its centre, where an opening is made for their passage. Then a second image of the object is formed behind this opening, at the compound focus of the two mirrors, which is viewed through an eye-glass placed in the axis. If we suppose the incident rays parallel, the first image will be formed at F_1, the principal focus of the large mirror, and will take the place of an object with respect to the second. Accordingly, from what has been laid 15. down, it follows that the first image must be situated between the centre of curvature and the principal focus of the second mirror, in order that the second image may be thrown beyond the first, towards the observer.

209. Cassegrain further modified this construction, by substituting in place of the small concave mirror, a small convex mirror $m\,m$, in order that the aberrations of sphericity produced Fig. 134. by the two mirrors, might mutually compensate each other. In this case, that the second image may be formed on the side of the observer, it is necessary that the first should not be actually formed; but that its imaginary place should fall beyond the small mirror, between its surface and the principal focus. This result which was not considered in article 15, because it supposes the incident rays convergent towards the mirror, is easily demonstrated by the general method of article 13.

210. In figures 130, 131, 133, and 134, for the sake of simplicity, we have only represented a simple eye-glass. But, it is in fact necessary, in order to render the last image colourless, that the eye-glass should consist of two lenses, arranged according to the principles laid down in articles 200, 201. We have also been obliged, in the figures, to increase very much the true dimensions of the eye-glass, compared with those of the mirror MM, that we might be able to represent the mass of rays and succession of images, in a conspicuous manner.

It is hardly necessary to remark, that in these telescopes, the mirrors are firmly fixed in the axis of a tube, sufficiently long to permit only those rays which are nearly perpendicular, to fall upon them. Indeed, we often contract this opening by means of diaphragms, for the express purpose of interrupting those rays which would fall on the borders of the mirror, this being never so well executed as the centre. The tubes ought to be blackened on the inside, like those of refracting telescopes, for the better absorption of the light irregularly reflected from their sides. In fine, they must be mounted in such a manner that they may be directed at pleasure, towards the different points of space.

Method of M. Arago for determining the Magnifying Power of Optical Instruments.

211. It has already been observed, that in instruments intended for viewing distant objects, the magnifying power is equal to the ratio of the visual angles, under which the same object is seen with the naked eye, and through the system of glasses of which the instrument is composed. If the object is sufficiently near the eye to allow its distance, in these two cases, to be compared, it is necessary to combine the ratio of the visual angles with the ratio of the real and apparent distances of the object, in order to deduce the ratio of its real and apparent magnitudes, both taken at the distance of distinct vision.

The process of M. Arago gives immediately the ratio of the visual angles. For this purpose, we take a double prism of rock crystal, similar to those whose construction is represented in figure 92, and which serve for double-image micrometers. We measure the angle OcE or C, at which it divides the light. This can be done in a very simple manner, as we shall soon see. We next place the double prism behind the eye-glass of the instrument under examination, which we first suppose to be either a reflecting or refracting telescope. If we view, through this system, a distant circular object, of a known diameter, we shall see it double, and in general its two images will be separated from each other. Then we remove it further or bring it nearer until these two images touch each other by their opposite edges. When this takes place, we know that the rays proceeding from the borders of this object, after having traversed the instrument, emerge from it, making with each other an angle precisely equal to C. Now, since we know the diameter M of the object, and its distance which we call Δ, we can easily calculate the visual angle a, under which the same rays cross each other at their incidence upon the first glass; for $\frac{M}{\Delta}$ expresses its trigonometrical tangent. The ratio of these angles or $\frac{C}{a}$, will therefore express the magnifying power of the instrument. Trig.

Now, in order to determine exactly the angle C, we can view the object through the double prism alone, and carry it farther off or bring it nearer, until its two images appear to touch each other. Then, from the known diameter of the object and its distance, we can calculate the visual angle which it subtends, and this will be the value of C. But M. Arago rendered this observation more exact by viewing the two images through a small telescope, before which, in contact with the object-glass, was placed the double prism which gave more distinctness without altering the coincidence of the images. Moreover, instead of a single object, he substituted several of different diameters, at the same distance, and even gave them a triangular form, in order to be able, without displacing them, to choose, in each experiment, the visual angle suited to the double prism, whose amplitude he wished to determine. Fig. 1

212. The method of M. Arago may be applied also to the microscope, under all the different forms of the eye-glass. Only the observation through the instrument, must be made upon an object very near and divided into small portions, like the object-glass micrometer, for example, already described. When we have placed this micrometer before the object lens, at a convenient distance for seeing distinctly through the eye-glass, the image the dimensions of which are traced upon it, we place the double prism between the eye-glass and the eye; and directing the double refraction perpendicularly to the series RR of marks traced upon the glass, we count the number RR', of divisions, embraced by the divergence of the two rays. Suppose it equal to m lines. This then will be the real magnitude of the object, which, being seen through the instrument, and brought by it to the distance D, of distinct vision, subtends at the eye the constant angle C; its apparent magnitude, as it is seen through the instrument, will therefore be equal to the distance D, multiplied by the trigonometrical tangent of the angle C; that is, equal to D tang. C; an expression which may be reduced to $\frac{DC}{206265}$, if we sup- 193. Fig. 13 Top.14

59. pose the angle *A* converted into seconds. It only remains, then, to divide this apparent magnitude by the real magnitude *m* of the 61, 62. object, as in the case of a simple magnifying glass, and the quotient $\frac{DC}{206265\,m}$ will express the magnifying power. It is hardly necessary to remark, that the distances *D* and *m* must be expressed in units of the same kind.

Instruments employed in Optical Experiments.

213. After having described the instruments which serve to enlarge the power of vision, it is proper to say a word concerning certain other kinds of optical apparatus, remarkable for the beauty or singularity of their effects.

Camera Obscura.

A converging object-glass adjusted to the shutter of a dark room, will concentrate the rays which come from external objects; and if these objects are very distant, compared with the focal distance of the glass, and situated nearly in the direction of its axis, it will give distinct images which may be received upon a white screen. These images are inverted; but in order to render them erect, it is sufficient to bring to the object-glass, instead of the direct light of the object, that of the image, already reflected and inverted by a metallic mirror *MM*. This apparatus is called a *camera obscura*. We may substitute for the screen a plate of ground glass; and for the room, a box fitted by means of a curtain to receive the head. It can then be transported with ease, for the purpose of landscape painting.

g. 137. g. 138.

Dr Wollaston has remarked, that the best form for the object-glass of a camera obscura, is that of a meniscus convex towards the image, and concave towards the object, as represented in the figure. And some fortunate experiments, made by Cauchoix, seem to indicate that the ratio of curvatures the most favourable, is that of 5 to 8. The shortest of the two curvatures belongs to the surface turned towards the image, because the lens must be converging.

Megascope.

214. Here, as in the preceding case, an object-glass is adjusted to a window shutter; but, instead of causing it to produce the images of distant objects, we place without the room, at a small distance, in the direction of the axis an object strongly illuminated by the light of the sun, either directly or by reflection from several mirrors. If this object is not one of too great dimensions, a distinct image will be formed in the room, the distance and magnitude of which will depend upon the focal distance of the object-glass, and the distance at which the object is placed before it. In proportion, therefore, as we bring the object nearer to the principal focus, we can obtain larger images; but as these will also be thrown at a greater distance, we must be guided in this respect by the dimensions of the room, and must limit ourselves to such distances as will give images sufficiently magnified and at the same time well defined. The images will appear inverted, but may be made erect by inverting the object. Such is the *megascope.* Instead of a single object-glass, we may employ several combin- Fig. 13
ed, so as to become achromatic. Then the limits within which the images are distinct, are sufficiently great to enable us to form, in this manner, magnified or reduced representations of pictures and statues, or even of natural figures. M. Charles, who invented this instrument, contrived to magnify objects from 2 to 20 times. In this state, it may be very usefully employed in numerous researches relating to natural philosophy and natural history, where it is necessary to determine with precision the forms and outlines of objects whose smallness or delicate texture prevents their being measured directly. In this case, we receive the image on a plate of ground glass, and sketch the outlines on the opposite surface of the glass or on transparent paper applied to this surface. This process, although graphic, is susceptible of very great accuracy.

215. The *magic lantern* is simply a portable megascope, in which transparent objects are illuminated by the light of one or several lamps. The term *phantasmagoria*, or the raising of spectres, has been given to the exhibition of an optical apparatus, similar to the magic lantern, in which the distance of the object from the con-

verging glass is made to vary, in order to present the appearance of an object which approaches or recedes. But to make the illusion complete, it is necessary to vary also the light with which the object is illuminated, in order that it may appear to diminish with the distance. Now the reverse actually takes place in instruments of this kind, in which we leave to the image the natural concentration of light produced by the glass.

Solar Microscope.

ig. 140. 216. Instead of a very large object-glass, we adapt to the window-shutter, a tube containing a magnifying glass A_2, of a short focus, and place before it a small object $s\sigma$, a little beyond its principal focus. The rays coming from this object, pass through the glass, and form behind it a magnified image, whose dimensions and distance increase according as we place the object nearer the principal focus. Accordingly, if we receive this enlarged image upon a white screen placed in the dark room, we can examine it in all its details. But in order that this examination may be easily made, the image must continue to be sufficiently illuminated notwithstanding its dilatation. It is neces-
ig. 141. sary, therefore, to illuminate the object very strongly. For this purpose we place beyond it, on the outside, another glass $A_1 A_1$, at such a distance as to concentrate upon it, a large portion of the solar rays directed by a heliostat. Then, if the object is transparent, the image thrown upon the screen, will be very luminous. In this manner we can magnify small objects, such as microscopic animalculæ, or the organs of insects, to a surprising degree. But the most beautiful subject for this instrument is a small drop of some saline solution. When placed upon a glass plate and exposed to view, in a few moments, the liquid being evaporated by the heat of the solar rays, crystallization takes place, and we can observe it on the screen in all its details, as well as the configuration of the crystals that are thus formed. The muriate of ammonia and the sulphate of soda, exhibit this effect most readily. It is likewise to M. Charles that we are indebted for the perfecting of this instrument.

Camera Lucida.

217. Suppose a quadrangular prism cut in such a manner that the rays coming from neighbouring objects, and falling nearly perpendicularly upon its first surface, shall twice experience total reflection at its interior faces, and then, emerging perpendicularly through its last surface, arrive at the eye situated in *O*. We shall see an erect and horizontal image of the object, which will appear to come through the prism. But suppose the pupil of the eye to be so placed, that the rays thus reflected shall occupy only half of it; the other half jutting a little over the edge of the prism so as to admit the rays coming directly from a piece of pasteboard *AB*, placed below. It is evident, that in this manner the spectator will see at once, with the same eye, the image and the screen upon which it appears to be thrown. If then he wishes to sketch the outlines with a fine-pointed pencil, he will see at the same time the point of this pencil and the image, and there will be nothing to prevent his delineating it. For this purpose, if his eye require it, he may be aided by a converging or diverging glass placed before the prism. This ingenious instrument was invented by Dr Wollaston. We can extend the use of it to distant objects, by fixing behind the eye-glass of a telescope, a metallic mirror inclined and very thin; then, if we place the eye against the edge of it and look with half the pupil, as before, we shall be enabled to delineate the objects in question. Fig. 142.

Organ of Vision.

218. The organ of vision, in man, is an optical instrument, composed of different transparent media, whose curvatures and refracting forces are so adjusted, that the aberrations of sphericity and refrangibility are rendered insensible. This apparatus concentrates the luminous rays coming from objects, and throws their images upon an expansion of the optic nerve, where the sensation is produced, which is afterwards transmitted to the brain. To give a general idea of this admirable mechanism, I

shall avail myself principally of the *Traité de Physiologie* of M. Magendie, who has described it with great clearness and precision.

As an optical instrument, however perfect, always concentrates to most advantage the rays which are near its axis, the eye is provided with muscles, subjected to the will, which direct it towards the object we wish to see. The same foresight which has so admirably constructed it, has protected it in a thousand ways. It is seated in the front of the face to guide the motions of our limbs, and placed in a cavity surrounded by walls of bone which jut over and shelter it. It is likewise embedded in a soft, unctuous mass, by which it readily yields to the action of the directing muscles without being liable to injury from its motions. In order to protect it from the sudden contact of bodies which might prove hurtful, it is supplied with movable coverings which shut over it with the rapidity of thought. These coverings, called *eyelids*, are bordered with *eyelashes*, which prevent the accidental introduction of any minute particles of dust, which might wound or tarnish its surface; and if by chance any such substance should gain admission, it is washed out by a fluid, the *tears*, secreted by glands provided expressly for this purpose. These same glands in their habitual state, serve to maintain upon the globe of the eye, a slight humidity, which prevents it from becoming parched by the contact of the air, and preserves the polish of its surface, so essential to the regularity of its refractions. Finally, in order that the moisture which collects upon the forehead, in consequence of bodily exercise, may not descend into the eye to irritate it, over the cavity in which it is embedded is extended an arch of stiff hair, called the *eyebrows*, that turn aside the descending fluid and continually repel it by means of an oily matter that is secreted at the roots.

219. Such are the external guards of the eye. The organ itself, considered in a detached state, presents nearly the form of a globe, to which are attached veins and arteries to administer to its nourishment, and muscles to cause and direct its motions. If it were cut by a plane passing from the fore to the back part, in the direction of its axis, on the supposition of its parts becoming solid by exposure to cold, the section would be represented in figure 143. We distinguish three media or *humours*, varying in form and refracting power. Beginning with the fore part, the first is a

concavo-convex lens or meniscus *AA*, filled with a transparent liquid, in appearance resembling water, and called for this reason the *aqueous humour*. Next is a solid transparent substance *CC*, which has the form of a converging lens, and which is called the *crystalline*. It is more flattened on the anterior than on the posterior side, and grows flat with age. Beyond this, the whole posterior cavity is filled with a viscous liquid resembling melted glass, and hence called the *vitreous humour*. The envelope which contains the whole system, may be considered as formed by the prolongation and extension of the teguments of the optic nerve *NN*. The exterior tegument gives rise to the exterior covering *NSA*, which is hard and opaque, but yet flexible like horn, and called the *sclerotica*. But extending round to *A*, before the eye, this membrane becomes thin and transparent, like the crystal of a watch, a necessary condition, in order that there may be a free passage for the light; here it is called the *cornea* from its resemblance to horn. In this part it is covered by the skin, rendered extremely thin. The second envelope of the optic nerve, spreads out below the preceding, and forms a stratum called the *choroid coat*, which is covered with a black pigment, for the same reason that we blacken the interior of instruments of our own construction, namely, to prevent the confusion that would result from the multiplicity of reflections. Finally, the interior and medullary portion of the optic nerve expands in its turn, like the preceding, and forms a nervous membrane *RR*, of a whitish grey colour, which is spread over the interior surface of the choroid coat and called the *retina*. Here the sensation is supposed to take place.

220. The general mode of action of this organ is manifest from the preceding description. The rays, coming from distant objects, fall upon the transparent cornea, traverse the aqueous humour, the crystalline and the vitreous, and are concentrated upon the retina, at the focus of the combined media, where they form a small inverted image. This may be verified by means of the human eye or that of an ox, it being removed a short time after death. For if we render thin the posterior part of the sclerotica, and place at some distance before the cornea, a luminous object, the flame of a lamp, for instance, and look at it from behind, we shall see a small but very distinct image formed in the interior of the eye, having the same colours as the object,

and increasing or diminishing according as the object is made to approach or recede, as has been shown by M. Magendie. This physiologist has also pointed out a method of rendering the observation much more easy, by operating upon the eyes of albinos, as white rabbits or white mice; for in this case the black lining of the choroid does not exist, the posterior part of the sclerotica is transparent, and we can perceive directly the images painted on the retina.

Not only are the images formed, but every thing is disposed in such a manner as to give them the requisite distinctness and regularity. We know that all our optical instruments are subject to two imperfections which limit extremely the field and power of vision. One arises from the circumstance that the borders of lenses do not concentrate the rays exactly at the same focus as the centre; this is called *the aberration of sphericity.* The other proceeds from the circumstance that the same lens concentrates at a greater or less distance from the axis the rays of unequal refrangibility, and is called *the aberration of refrangibility.* As to this latter defect, we have seen that if it cannot be entirely removed, it can at least be rendered insensible in telescopes, by making each glass to consist of several lenses of different kinds, whose dispersive forces are unequal and opposite. It appears that the aqueous humour, the crystalline lens, and the vitreous humour are combined in such a manner as to produce nearly a similar compensation, since the images painted on the retina, appear ordinarily in the proper colours of the objects from which they are derived; and if we sometimes perceive a sensible colour, it is only when particular pains are taken to place the eye in circumstances different from its habitual state. The other species of aberration, that of sphericity, appears also to be compensated, or at least rendered very small, by the construction of the eye; for we can still see objects with sufficient distinctness, although placed at a considerable angular distance from the axis, and even when we look at them quite awry, in which case the rays fall very obliquely upon this organ. It is possible that this compensation results in part from the composition of the crystalline, which is found to consist of an infinite number of distinct lamina, whose refracting forces are probably unequal. Moreover, we find several particulars in the structure of the eye, which manifestly tend to effect the same purpose; such, for example, as the existence of a diaphragm *PP*, placed just before the crystalline, whose opaque contour interrupts those rays which would make too great an angle with the axis. This diaphragm, when seen from without, exhibits the appearance of a coloured ring, of a variable tint, even in man, and called the *iris*. The circular aperture through its centre, by which the rays enter into the interior of the eye, is called the *pupil*. The construction and disposition of this diaphragm are no less wonderful than those of the other parts of the organ. In the first place, the posterior surface, covered by the prolongation of the choroid, is tinged with the finest black; so that it absorbs all the rays which, being reflected from the interior surface of the crystalline or even the back part of the eye, would be thrown back and thus impair the obscurity of the dark chamber where the image is formed. Moreover, the membrane of which this diaphragm is composed, has a striking advantage over all our artificial diaphragms, in its susceptibility of contraction and dilatation, thus admitting more or less light, as occasion may require; and that too, from the mere irritation produced by the light, without any care on our part. Finally, the very place of this diaphragm in the interior of the aqueous humour, is a circumstance that perfectly corresponds with every thing else. For, in vision, each point of the visible object sends to the eye a cone of luminous rays, which has its vertex at this point, and for its base all the exterior surface of the transparent cornea. This cone, being refracted in the aqueous humour, changes into a shorter cone, which has its vertex in the interior of the eye. Then, being obliged to traverse a small circular aperture, concentric with the axis of this organ, it loses all those rays whose Fig. 14 primitive obliquity of incidence upon the cornea, would have produced too great an aberration of sphericity; and this favourable exclusion, taking place thus near the point of the interior cones, operates with equal success whatever be their direction of incidence, as is shown by the figure. Instead of such a position, suppose that the diaphragm, having the same opening, had been placed without the eye, near the surface of the transparent cornea. Then it would still exert its favourable influence on the incident Fig. 14 cones very near the axis; but with respect to the oblique cones it would be very defective, since it would admit the rays most distant from the axis and most oblique to the surface of the cornea; whereas these are the very rays which ought to be excluded, and which the interior diaphragm does in fact exclude; so that in order to effect the same exclusion, when situated without, there would be no other expedient than to make the diameter of the aperture extremely small. The situation, therefore, of the diaphragm, in the interior of the first medium by which the rays have been refracted, is a means of admitting a greater quantity of light, with a less aberration of sphericity. This is undoubtedly the reason why it is thus situated with respect to our organ, in which all the parts are so perfectly adapted to the purpose they are designed to answer. And even human art, rude as it is, when compared with the skill displayed in nature's handiwork, has had recourse to the same expedient, probably without the analogy being observed; for such precisely is the construction of the periscopic glasses, invented by the celebrated Dr Wollaston, which consist of two segments of spherical plano-convex lenses, adjusted to each other at their plane sides, and separated by a diaphragm. This construction is found to possess indisputable advantages, both in regard to the quantity of light, and the distance from the axis to which vision may be extended. Fig. 146.

221. There are also two circumstances in the construction of the eye, which appear to concur in diminishing the aberration of sphericity for the pencils oblique to the axis. The first is the peculiar form of the posterior surface of the crystalline, which, if we may judge from the representations of anatomists, is not strictly spherical, but more flattened at the centre than toward the borders. This causes the oblique pencils to meet it under smaller incidences than they otherwise would have. The other is the concavity of the retina by which it is made to present itself, as it were, to the proper focus of each pencil.

222. But, with all we know of the astonishing wisdom of this mechanism, its operation still offers peculiarities which we are unable to explain. Our optical instruments have different focal lengths for different distances of objects, and we cannot adapt them to these different distances, without lengthening or shortening the intervals between the glasses which compose them. There must, therefore, exist in the eye the means of some analogous adjustment, since vision takes place with great distinctness at very different distances; for example, at several inches and

at several feet; and even for remote objects, the indistinctness of vision must necessarily be similar to that which belongs to glasses fitted for small distances. We can satisfy ourselves that an actual effort of the eye is necessary, in order to vary thus its extent of vision; for if we place at a short distance from one of our eyes a small object, such as a hair, which is projected on a larger and more distant object, we can never see both of them distinctly at the same time; but an actual effort, and even a short interval of time, is necessary to pass from the one to the other. Yet we are not able, by the most careful dissection, to discover in the eye any cause capable of modifying its operations in this respect. Some have supposed that the anterior part of the cornea is susceptible of becoming, at will, a little more concave or convex; but it has been shown by the very accurate experiments of Dr Thomas Young, that no sensible change takes place in this respect for the greatest difference as to distance. Some have had recourse to a motion of the crystalline forward and backward, and to a supposed change of curvature; but we find no muscle which could produce this effect. The crystalline does not adhere to any substance capable of moving it; it is suspended freely at the centre of a ring formed of radiating fibres, which are called *ciliary processes*, because they resemble the eyelashes. We do not know that they are not susceptible of contracting, to a certain degree, so as to allow the crystalline to move in a direction perpendicular to the axis of the eye, but this motion does not seem to be of a nature to vary the focal distance of this organ. The safest and most prudent course is to admit, that as yet we are ignorant how the eye can modify its action according to the distance of the object. May it not be that the small diameter of this organ renders imperceptible the changes of curvature and elongation which it undergoes, or that the aberration of the focus for different distances is compensated, as in other cases, by some special contrivance.

223. As the last instance of the wisdom with which this wonderful organ is constructed, it is to be observed, that in all our optical instruments, if we take away or derange one of the glasses, the whole effect is instantly destroyed; but the eye, like our other organs of sense, is not so easily deranged. If we pierce the eye of a living man, in order to draw out part of the aqueous or vitreous humour, as is necessary in some diseases,

these humours are in a short time replaced; if the pupil becomes closed up, we can make an incision in the iris and thus form an artificial pupil. Finally, if the crystalline itself becomes opaque, and it is necessary to remove it in order to open a passage for the light, we can still see, without its assistance; only the distance of distinct vision is greatly increased, as we should naturally expect, after taking away from the instrument a converging lens; but we supply the defect by placing before the eye a converging glass of a suitable curvature; and then we can see almost as distinctly as with the natural crystalline unimpaired. This stability seems to be a general characteristic of all our organs of sense.

224. Hitherto we have considered only the instrument of vision. Its connection with the sensation, presents a mystery still more inexplicable. All we know is, that the impression produced upon the retina is transmitted to the optic nerve, and by that to the brain. The circumstance that we have the sensation of objects erect, although we receive the image of them inverted, is not surprising. The image, which is the cause of the sensation, must not be confounded with the sensation itself. The rays refracted by the humours of the eye, have, when they arrive at the retina, directions quite different from those which they had when they reached the cornea; yet, in thought, we always refer the object to the prolongation of this primitive direction. We do this, because the experience of our whole life has taught us that the object is found in this direction. This result of experience becomes associated with the sensation, as an invariable consequence, and the mind draws the inference instantaneously. Accordingly, a deception is practised, when the same artificial indications are exhibited, without the intervention of real objects. Thus objects, seen by reflection from a plane mirror, appear to be situated beyond its surface, although reason and experience, but an experience subsequent to the first lessons of our senses, apprise us of the error. This principle offers a simple and natural explanation of all the optical illusions, which are produced by glasses and mirrors.

225. When we look at objects with only one eye, and are not previously apprised of their distance, we suppose them at the vertex of the luminous cone, which has the opening of the pupil for its base, and for its vertex some point of the object. Ac-

cordingly, this estimate is only so far just, as the objects are sufficiently near to allow the angle of divergence of the rays which form the cone, to be sensible. When we look with both eyes at once, the same principle leads us to suppose the object placed at the point of meeting of the pencils which arrive at the two eyes, and which is the common vertex of the cones of which we have been speaking. Then the basis of measurement is the space between the eyes. It is therefore greater than in the preceding case. Accordingly, this estimate is much more correct than the other, as we may easily assure ourselves by observing the difficulty of threading a needle when we look at it with only one eye, while nothing is easier when we look at it with both. But in order to see images single, it is necessary that they should fall upon corresponding parts of the retina, where we are accustomed to see them when they come from the same object. If we press the eye a little sideways with the finger, we immediately see two images, one at the same place with the former, and the other on one side of it in the direction in which the pressure has carried the eye. But if we keep up the pressure for some time, this secondary image becomes faint and finally disappears, and we again see the image single at the same place as before.

226. Experience teaches us that such pressures, when sudden and violent, produce tremors in the optic nerve from which results the sensation of light. This sensation may also be excited or destroyed by contrast. If, for example, the eye has been fixed for some time upon an extended space of a uniform colour, it seems afterward to abstract this colour, when it is directed to any other object; and we thus see a spot on this object, the colour of which is *complementary* of that upon which the eye had been resting; that is, it is composed of those rays from the object, which did not make a part of the first colour. These appearances, produced by contrast, are designated by the name of *accidental colours*.

227. Sometimes also, we observe in luminous objects, certain deceptions of another kind; namely, stripes tinged with the colours of the rainbow, or bright halos of divergent beams. These effects are produced by the decomposition of light in the small humid drops, which are accidentally lodged between the

eye-lashes; and likewise by the diffraction which the eyelashes, in consequence of their fineness, exert upon the luminous rays.

228. The instrument of vision, so wonderful in man, is far from being confined to him. In bestowing it upon the different orders of animals, nature has varied the structure in certain particulars, either by suppressing some parts which would be useless in the situations in which the subject is placed, or by adding others more suitable to its condition. At other times, she has pursued a plan entirely new, the reasons of which we are totally unable to explain. In connexion with this topic, I will cite a few general results extracted from the lectures of M. Cuvier, on comparative anatomy.

229. All red-blooded animals have eyes constructed on the general plan of the human eye, but with more or less considerable modifications.

Among quadrupeds, many have the pupil during the day, in the form of an oblong opening; but, during the night, the iris contracts and the pupil enlarges into a circular form, having a much greater aperture. This is to be observed in the ox, the horse, the cat. Consequently, these animals, being able to concentrate a proportionably larger number of luminous rays than man, can see much better in the dark.

The same is true of those birds which seek their prey in the night, as owls, &c. In general, the eye of birds occupies a considerable portion of the bulk of the head. It is distinguished, likewise, by several other important particulars. The crystalline, almost gelatinous, is much flatter than that of man, and to make amends for this the cornea is much more convex; it is also proportionably smaller; the reason of which undoubtedly is, to prevent the increase of the aberration of sphericity, that would be produced by an increase of its curvature. This small cornea, very convex, is adjusted to the extremity of a very short cone, which projects out before the globe of the eye, like the tube of a small telescope. Here, as in man, we are ignorant how the eye is adapted to different distances. Yet what extent must this variation have in the eye of a bird of prey, which, singling out from its aerial height the small animal which is to be its victim, encloses it in the vast circles of its flight, and gradually approaching, at length pounces upon it with the rapidity of lightning, and transfixes it with its

Fig. 147.

talons and its eye, almost at the same instant. The interior of the eye of birds contains a peculiar organ, which may be destined for this purpose. It is represented in figure 148. It is a kind of black veil, formed by a membrane of the same nature as the choroid, and plaited like a fan, except that the folds, instead of radiating from the same centre, come from different points of a common stem, which is the continuation of the optic nerve itself, prolonged to the interior of the eye, and applied along its concavity. The direction of this membrane is oblique to the axis of the eye, and floats in the interior of the vitreous humour, extending nearly to the back part of the crystalline, to which it sometimes appears attached as by a thread, which connects the extremities of its folds. Naturalists have given it the name of *comb*. From its vascular structure, and the number of blood vessels which chequer it, may it not be designed, by its swelling and collapsing, to push out and draw in the posterior cavity of the eye, in such a manner as to change, by pressure, the curvature of the crystalline, or its distance from the rétina, in order to accommodate it to the different distances of objects?

230. The eye of fishes, represented in figure 149, exhibits different peculiarities. Destined to live in a medium, the refracting power of which is nearly equal to that of the aqueous humour in the eye of man, this fluid would not produce any refraction. Accordingly its place is supplied by a viscid liquid, probably of a greater refracting power than pure water, the quantity being very small at the same time. By a necessary consequence, the pupil is extremely near the cornea, and this circumstance deprives it in part of the properties which it possesses in man, but which are the less necessary here on account of the small refraction which the rays experience in entering the humours of the eye from a watery medium. Moreover the pupil of fishes is not susceptible of dilatation, and the iris is usually tinged on the outside with the most lively colours, although on the inside it is always black. The crystalline, applied almost immediately to the back part of the cornea, is very nearly spherical. We are absolutely ignorant of the reason why nature has given it this form; but it appears to be adapted to vision in the liquid medium in which fishes live; since, in most birds which dive in the water, we find the crystalline in like manner spherical. In other respects, it is composed of concentric laminæ like that of other animals.

231. Lastly, to confound our penetration, there are some animals whose eyes, more or less in number, are formed entirely of a transparent and lenticular cornea, behind which the nervous filaments expand. We have an example of this kind in insects, whose eyes are sometimes smooth and continuous; at others they appear to be cut with facets. But it appears on dissection, that, in this last case, each facet is a real eye which has its cornea and particular nervous filament, proceeding from one common stock. Accordingly, this last sort of eyes has been denominated compound, and the other smooth; it would be more natural to divide them into *multiple* and *simple*. Each is always fixed in insects, and makes visible only the objects which are brought of themselves within its field of view. It is impossible to conceive how such a system can produce distinct images; but still it is very evident that it serves the essential purposes of vision; for if we cover the eyes of insects with a black envelope, but of such a nature as not to injure the eyes, they become entirely blind, and cannot avoid any of the obstacles which are in their way. While there are some which have multiple eyes, there are, on the other hand, whole classes of animals which have no eyes at all; such are the moluscæ acephalæ, zoophites, and worms.

232. We close this account by remarking that there are many animals which see when it is too dark for us to perceive any thing. There can be no doubt that cats and screech-owls discover their prey in the night, since it is then that they seek and secure it. Fishes, which live at the depth of two or three thousand feet in the sea, are in a still greater degree of darkness; for through the immense body of water that covers them, they can receive only the most feeble portion of light. Yet it is certain that they see their prey and pursue it with impetuosity and effect. These, accordingly, as well as cats and owls, have very large eyes. Is it not possible that, in these animals, vision is produced by those rays, which, to our eyes, are only rays of heat? From the relations of light and heat, it would seem that this supposition is not without probability.

Reflections, Refractions, and Colours of thin transparent Bodies.

233. In all the phenomena hitherto examined, the thickness of bodies which act upon light, whether in the case of refraction, dispersion, or reflection, has been considered as infinite, compared with the distance to which this action sensibly extends. The refraction of rays, for example, would have taken place under no larger angle, if we had used thicker prisms; and the reflection produced at the surfaces of metallic mirrors or reflecting glasses, would have been exactly the same, if we had increased their thickness. But when the bodies are reduced to very thin plates, the results are different; they reflect less light at the first surface, and at the second, they reflect or transmit certain colours in preference to others, according to their chemical nature, and their degree of tenuity. This phenomenon, exceedingly important in its consequences, was analysed by Newton, in a course of experiments which are now to be described.

When we take two prisms of polished glass, and place one of them upon the other without pressing them, the small plate of air, which naturally adheres to their surface, ordinarily possesses all the thickness necessary to its complete action upon light; for the phenomena of reflection and refraction through these two prisms and through this plate of air, obey exactly the laws which we have established in the preceding sections; and if we separate further the two contiguous surfaces, the increased thickness of the plate of air will not lead to a different result. But if we rub the two prisms against each other, and press them together with so much force as to exclude a part of the air which separates them, we shall at once perceive an adhesion, which is generally closer in some parts than in others, either because their surfaces are almost always slightly curved, or because the powerful pressure exerted, bends them. We thus obtain a plate of air thinner than the preceding, and whose thickness goes on increasing in all directions from the point where the surfaces applied touch each other or come nearest to touching, to the points where they are farthest apart. Then if we present the prisms to full daylight, and turn them in such manner, that the eye may receive the light partially reflected at the plate of air which separates them, we shall perceive a greater or less number of coloured rings, which, when the prisms are sufficiently pressed, will surround a dark spot corresponding to the point of contact.

234. In order to observe well the order of these rings and the succession of their colours, it is necessary to employ prisms of a small angle, so that the light which traverses them to form the rings, may not experience a sensible dispersion before it arrives at the plate of air. It is well to place these prisms over a dark body, in order that no foreign light from external objects may mingle, by transmission, with that of the rings which we wish to examine. We should, therefore, take our position before Fig. 150. an open window, turning the edge *B* of the upper prism inward, and placing the eye above this prism, in such a manner as to receive only the light reflected from the inclined surface *BC*, and avoid that which is reflected from the upper surface *AB*. The most advantageous position is that in which rays reflected at the plate of air between the two prisms, traverses perpendicularly the surface *AB*; while the incident light *RI*, which is partially reflected at the same surface, is directed towards *R'*, without coming to the eye of the observer, which we suppose so far elevated above the plane *AB*, as not to receive any of the rays which it reflects obliquely. Moreover, if the inferior prism is placed over a black body, as mentioned above, the rings will reflect the most vivid colours; and the central spot, entirely dark, will appear like a hole pierced through the middle of them. But we shall distinguish them more clearly still, if we dilate them, as we may do, by viewing them through a simple lens, or small compound magnifier of a short focus.

The rings thus formed are not produced by the light reflected from the superior surface of the prisms, since this light does not come to the eye of the observer in the position we have supposed. Neither are they formed, for the same reason, at the inferior surface *CA'*, of the second prism; moreover, we might blacken this surface with India ink, or with any other substance capable of absorbing the light, and the rings would not suffer any diminution. It necessarily follows, therefore, that they are formed by the reflection of light between the contiguous surfaces of the two prisms.

235. The light which is transmitted through the plate of air also produces coloured rings, as may be easily ascertained by looking at the sky through the system of prisms placed at a little distance from the eye. This second set of rings, presents the same contours as the preceding, but their colours are different and their tints much more feeble. We shall examine them further, when we have attended to those which are given by reflection.

236. We can form coloured rings, by pressing together, in this manner, two prisms of whatever substance. We can form them also by placing a surface of glass on a plate of resin, metal, metallic glass, or any other polished body. The rings continue to be formed, likewise, in the most perfect vacuum that can be produced by the air-pump. They continue, too, when we heat the glasses to such a degree as to soften them and solder them together.• If we could in this way exclude the air entirely, we should infer directly, that the rings were not produced by the proper action of the interposed plate of this fluid; but, although it is physically impossible to attain this limit, yet the permanence of the phenomena, after the nearest approaches to it, evidently indicates that they would remain, even when the air was entirely expelled from between the two surfaces. Indeed the laws established by Newton, show that in this extreme case, the dimensions of the rings would experience only a very small increase, almost inappreciable by the most delicate experiments.

It is not indeed necessary that the thin plate should be air, or that it should be contained between two solid bodies. A plate of water, of alcohol, of ether, or of any other evaporable liquid, being extended over a black glass, produces similar colours, when evaporation has rendered it sufficiently thin; only, the thickness of such a plate always varying in an irregular manner, the tints thus developed, will exhibit all the irregularity of its undulations. We obtain the same results also with sheets of mica and of glass when they are sufficiently thin.*

---

* Thin sheets of mica are obtained, by separating with a knife, the leaves of a thick sheet. In order to obtain thin sheets of glass, we melt with a lamp the extremity of a hollow tube of this substance; we then blow into it while the extremity is red hot, and force it to develope itself in a ball, which will expand until it bursts on account of its thinness.

237. In general, whatever be the substances of which the rings are formed, and in whatever manner we combine them, the colours which they present are always the same, and arranged in an order precisely similar, from the least to the greatest thicknesses. The only difference we find is in the absolute space which the colours of the several rings occupy, which changes with the nature of the substance and with the more or less rapid diminution of the thickness. Accordingly, the most natural method of examining these phenomena, seems to be, first, to determine the order and succession of the colours of the rings, in a lamina of any substance whatever, but whose thickness varies regularly according to some known law; secondly, to find, for this substance, the relation which the thickness bears to the colour; and lastly, to compare these relations in different substances, and see what they have in common or how they differ.

The order in which the colours are arranged in the different rings, may be conveniently investigated, by observing those which are formed between two prisms, especially if we select those cases in which the rings are large, and if we dilate them still more by looking through a magnifying glass. Nevertheless this process is not susceptible of very great precision; for the adhesion of the prisms being commonly effected by a violent and irregular pressure, the rings which result from it are likewise almost always irregular. Sometimes there are several points of pressure, and consequently, several black spots, from which the colours go on diminishing continually, and compose sets of rings which intermingle with each other. Thus, in order to extend our observations beyond what these first phenomena make known, it is necessary to employ some other apparatus, which will exhibit the same phenomena with more regularity. This we do by placing, one upon the other, two spherical glasses of unequal curvatures. If the surfaces of these glasses are well formed, they can have only one point of contact, from which the space comprehended between them, goes on constantly increasing in thickness, according to the laws of geometry resulting from the difference of their radii. Moreover, the small curvature generally given to these glasses, causes the light which traverses them to arrive at the plate of air, without being sensibly dispersed. This disposition is, therefore, eminently fitted for taking exact measurements of the dimensions of the rings, and for comparing their tints with the corresponding thicknesses; and this is the disposition which Newton employed. Among all the possible combinations of curvature, he always had one of the glasses plane, and the other of the same convexity on both its surfaces, because this disposition afforded the greatest facility for determining the curvatures with exactness; for we can easily ascertain by reflection, when a glass is plane; and we can likewise determine the radius of a glass, equally convex, by measuring the distance at which it concentrates the simple rays, which fall parallel upon one of its surfaces.

238. When a convex glass of a large radius is thus placed upon a plane glass, and gently pressed against it in order to establish the contact more perfectly, the coloured rings present themselves immediately, and appear regular and distinct, with a black spot at their centre. Then each circular zone of the plate of air, reflects the colour which belongs to its thickness; and consequently, if we gradually augment this thickness, by gently raising the superior glass, the colours reflected at the same distance from the centre of the rings must change; and those which at first formed rings distant from the centre, must gradually approach it, until the thickness of the air, even at this point, becoming too great for them, they disappear and give place to others which were at first still farther removed. But since, at this last place where they disappear, the distance between the glasses can only vary by degrees almost insensible, on account of their being tangents respectively to each other, it follows that each colour, when brought there, must extend itself very widely and thus become easily distinguished. This succession, which can be rendered as gradual as we please, affords, therefore, a simple and exact method of determining the species and order of the colours which compose the series of rings.

Newton performed this experiment with a convex glass, the two surfaces of which were wrought upon the same sphere, the radius of which was 51 feet; and by means of the results thus obtained, as well as by observations which he had before made with the prisms, he fixed, in the following terms, the order and series of tints of which the circular rings were composed.

"Next the pellucid central spot, made by the contact of the glasses, succeeded blue, white, yellow, and red. The blue was so little in quantity that I could not discern it in the circles made by the prisms, nor could I well distinguish any violet in it, but the yellow and red were pretty copious, and seemed about as much in extent as the white, and four or five times more than the blue. The next circuit in order of colours, immediately encompassing these, were violet, blue, green, yellow, and red; and these were all of them copious and vivid, excepting the green, which was very little in quantity, and seemed much more faint and dilute than the other colours. Of the other four, the violet was the least in extent, and the blue less than the yellow or red. The third circuit or order was purple, blue, green, yellow, and red; in which the purple seemed more reddish than the violet in the former circuit, and the green was much more conspicuous, being as brisk and copious as any of the other colours, except the yellow; but the red began to be a little faded, inclining very much to purple. After this succeeded the fourth circuit of green and red. The green was very copious and lively, inclining on the one side to blue, and on the other side to yellow. But in this fourth circuit there was neither violet, blue, nor yellow, and the red was very imperfect and dirty. Also the succeeding colours became more and more imperfect and dilute, till after three or four revolutions they ended in perfect whiteness. Their form, when the glasses were most compressed so as to make the black spot appear in the centre, is delineated in figure 151, where *a*, *b*, *c*, *d*, *e*; *f*, *g*, *h*, *i*, *k*; *l*, *m*, *n*, *o*, *p*; *q*, *r*; *s*, *t*; *v*, *x*; *y*, *z*, denote the colours reckoned in order from the centre, BLACK, blue, white, yellow, red; VIOLET, blue, green, yellow, red; PURPLE, blue, green, yellow, red; GREEN, red; GREENISH BLUE, red; GREENISH BLUE, pale red; GREENISH BLUE, reddish white."*

239. Having made these observations, Newton proceeded to measure the diameters of the rings, still making use of the same glasses; but he took care now simply to put them one upon the other, without any pressure being exerted which might bend them or alter their natural curvature; then placing his eye above the rings, as nearly perpendicularly as he could, he measured the diameters of the six first in the brightest part of their breath,

---

* The colours whose names are in capital letters, are those which form the commencement of each ring, reckoning from the central spot; and the dimmest rings are those designated in figure 151 by the shaded portions.

which, for the first and nearest the centre, was found to be nearly in the confines of the white and yellow; for the second, between the orange and the yellow; for the third, at the yellow; for the fourth, between the yellowish green and red; and for the fifth and sixth, between the greenish blue and red, always nearly in the middle of the space occupied by each ring. Now it may be readily shown by geometry, that the interval between two spheres, or between a plane and a sphere, which touch each other, increases in the ratio of the square of the distances from Top.141 the point of contact. Newton took, therefore, the squares of all the diameters which he had measured, and by comparing these squares with each other, beginning with the smallest, he found that they proceeded according to the progression of odd numbers, 1, 3, 5, 7, 9, ...; whence he inferred that the thicknesses of the air at the places where the rings appeared to be limited by the diameters, were also in the same progression.

In order to make these observations with accuracy, it is necessary to place the eye as nearly as possible in the direction of the vertical, which passes through the centre of all the rings, that we may avoid the dilatations which take place, when the rays are oblique to the surface of the glasses. For the same reason, it is necessary to measure the diameters, not in the plane of incidence at which the light arrives, for this would give unequal obliquities to the rays which come to the eye from the extremities of the same diameter; but transversely to the direction of incidence and perpendicularly to its plane, as represented in figure 152. Finally, as the diameters are measured on the superior surface of the glass, while the rings are actually formed at the inferior surface, it is necessary to take account of the refractions which they experience in traversing it, and to apply a correction for this effect. Newton neglected none of these precautions; and we shall perceive that they were indispensable, when we consider how extremely small the thicknesses were which he was endeavouring to estimate.

240. Newton measured also the diameters of the rings in the dimmest parts; which, for the first, he found to correspond to the middle of the central spot; for the second, to the dark violet; for the third, to the deep blue; for the fourth, fifth, and sixth, to the commencement of the greenish blue. He likewise took the squares of these diameters, in order to ascertain the corresponding thicknesses of the plate of air; and he found that they answered to the progression of even numbers 0, 2, 4, 6 . . . . , 0 standing for the central spot. Consequently, the thicknesses of the plate of air in the dimmest parts of the first six rings, are in the same progression. "And," Newton adds, "it being very nice and difficult to take these measures exactly; I repeated them divers times at divers parts of the glasses, that by their agreement I might be confirmed in them. And the same method I used in determining some others of the following observations." Thus we see that the mind of this extraordinary man was not easily satisfied.

241. The preceding law affords already an explanation of a phenomenon which is readily observed in forming the rings, even with an object-glass whose radius is very small. It is that these rings approach each other according as they are further from the central spot, and are so much the more crowded as they are more distant. Indeed, since the squares of the diameters answer to the progression of odd numbers 1, 3, 5, 7, ... the diameters themselves will be proportional to the square roots of these numbers; they will therefore form the following series, in which the diameter of the first ring in its brightest part, is represented by unity.

Squares of the diameters.	Diameters.	Differences of the consecutive diameters.
1	1,00000	0,73205
3	1,73205	0,50402
5	2,23607	0,40968
7	2,64575	0,35425
9	3,00000	0,31663
11	3,31663	0,28892
13	3,60555	0,26743
15	3,87298	
101	10,04938	0,09902
103	10,14890	

Thus we see that the differences of the diameters of the consecutive rings go on diminishing continually from the central spot, which is conformable to observation.

242. Hitherto we have spoken only of the law, according to which the diameters of the successive rings vary; to make the

results complete, it is necessary to know the absolute magnitude of some one among them. By measuring this magnitude with extreme care, on the glasses employed in the preceding experiments, Newton found that, for the most lucid part of the sixth ring, it was equal to $\frac{55}{100}$ of an inch. But some time after, fearing that he had not determined the diameter of the convex glass, with sufficient accuracy for so delicate an observation, he renewed the experiment with an object-glass doubly convex, whose two sides had been wrought upon the same sphere. The mean focal distance of this glass was 83,4 inches, and its ratio of refraction $\frac{17}{11}$; whence, by the formula of article 48, we have for the diameter of each of its surfaces 181,964, or, in whole numbers, 182 inches. Newton placed this convex glass on a plane one, in such a manner that the dark spot appeared in the middle of the coloured rings, without any other pressure being exerted than that of the weight of the glass. After this, he measured the diameter of the fifth dark ring, in the most exact manner possible, and found it precisely equal to the fifth part of an inch. He took this measure with a pair of compasses on the superior surface of the convex glass, having his eye elevated about 8 or 9 inches above this surface, and nearly perpendicularly over the centre of the glass, which was $\frac{1}{6}$ of an inch thick. According to this arrangement, the true diameter of the ring at the second surface of the glass, must be a little larger; and it is known by calculation that in order to obtain it, it is necessary to increase the apparent diameter by $\frac{1}{79}$ of its value, or, which amounts to the same thing, to multiply it by $\frac{80}{79}$; and as it was found to be $\frac{1}{5}$ of an inch, this augmentation makes it $\frac{1}{5} \cdot \frac{80}{79}$ or $\frac{16}{79}$ of an inch. Whence it follows that its true radius was equal to half this quantity, or $\frac{8}{79}$ of an inch.

243. Now when a convex glass is placed upon a plane glass, the thickness of the air, or the distance between the glasses, at the perimeter of any ring, is equal to the square of the semidiameter of the ring, divided by the diameter of the sphere upon which the glass was wrought. Applying this rule Geom. 228. to the case in question, the semidiameter of the fifth dark ring being $\frac{8}{79}$ of an inch, its square will be $\frac{64}{6241}$, which being divided by 182, the value of the diameter of the sphere, gives $\frac{64}{6241 \cdot 182}$ or $\frac{64}{1135862}$; that is, $\frac{1}{17747,84}$ of an inch. But, from the preceding observation, the thicknesses of the air at the perimeters of

the different dark rings, follow the progression 0, 2, 4, 6, 8, 10, ..., where the fifth is represented by 10, and the central spot by 0. It is evident then, that if we take the fifth part of the thickness of the air at the place of the fifth ring, where it is as 10, we shall have its thickness at the place of the first ring, where it is as 2. Accordingly, the fifth part of the fraction $\frac{1}{17747,84}$, or $\frac{1}{88739}$, is the thickness of the plate of air at the perimeter of the first of the dark rings. Newton repeated the same experiments on several other convex glasses of different curvatures, and even on portions of convex glasses in order to avoid the errors that might arise from inequalities in the formation of these glasses, and he always obtained results conformable to the preceding.

244. Now it may be presumed that these glasses were not all strictly similar in composition; there must then be differences in their refracting power. And if, notwithstanding these differences, they all agreed in giving the same thickness for the same rings, we must conclude that the colours thus reflected by the several thicknesses of the plate of air, belong to these thicknesses, and do not depend upon the nature of the surrounding medium. We shall see hereafter that this consequence is general. Whatever be the reflecting substance, the tint which it reflects, under a perpendicular incidence, at a determinate thickness, is independent of the nature of the surrounding medium and the intensity of this colour is all that is variable.

The measurements which have been stated, were taken in the manner indicated in figure 152, perpendicularly to the plane of reflection drawn through the central spot; so that the angles of incidence and reflection, made by the rays falling upon the plate of air, were very nearly the same for the different rings. It only remains, then, to ascertain what these were, in order to determine the incidence under which the observation was made; for we have already said that the rings increase in magnitude, according as the luminous rays become more oblique to the plate of air. Now Newton informs us that when he made his observations, his eye was elevated from 8 to 9 inches above the plane of the rings, and that it was 1,25 distant from the incident rays; hence we have by calculation for the incidence of the luminous rays upon the plate of air, an angle of about 4°. But from the laws of augmentation of the rings which have just been established, we find that, on account of the smallness of this angle, they may be referred to the case of perpendicular incidence, by diminishing them in the ratio of radius to the secant of 4°, that is, in the ratio of 10000 to 10024; in other words, we multiply them by the fraction $\frac{10000}{10024}$. This being done, the thickness of the plate of air at the place of the first dark ring, and under a perpendicular incidence, is reduced to $\frac{1}{88850}$. Another experiment, of which we shall not here give the details, makes it $\frac{1}{89063}$. The arithmetical mean is about $\frac{1}{89000}$, and this is the result upon which Newton relied. Now since the first dark ring is represented by 2, when the first ring in its brightest part is expressed by 1, it follows that by taking half of $\frac{1}{89000}$, we shall have, for the thickness of the air at the brightest part of the first ring, $\frac{1}{178000}$ of an inch.

245. In general, as the thicknesses of the plate of air, at the brightest parts of the rings, answer to the arithmetical progression 1, 3, 5, 7, 9, while the thicknesses of this same plate, at the darkest parts, are as the intermediate progression, 2, 4, 6, 8, 10, it is evident that, the absolute thickness of the first, or of any one whatever of these rings being known, we can immediately find that of all the rest; so that the different values of these thicknesses, expressed in fractions of an inch, will be for the brightest parts of the rings,

$$\frac{1}{178000},\ \frac{3}{178000},\ \frac{5}{178000},\ \frac{7}{178000};$$

and for the darkest parts,

$$\frac{2}{178000},\ \frac{4}{178000},\ \frac{6}{178000},\ \frac{8}{178000}.$$

These values relate only to a perpendicular incidence. To render them general, we must determine the dilatations which the rings undergo, by a change of inclination of the rays to the plate of air which reflects them.

246. Nothing is more easy than to observe these variations. We have only to view the rings first, with the eye placed as nearly as possible perpendicularly to the surface, and then move it gradually from this position in order to view them obliquely; for then we see them dilate circularly on all sides. To discover the law of dilatation, Newton measured the diameter of the same ring at different degrees of obliquity, taking any one of the colours which compose it; and hence he deduced the thickness of

the air, which reflected this colour under each obliquity. But to extend these measures to the greatest incidences, where the visual rays fall very obliquely upon the plate of air, he was obliged to dismiss the object-glasses and substitute prisms. For, on account of the small curvature of the object-glasses, the luminous rays which pass from the second surface into the plate of air, are always nearly parallel to the incident rays. In order, therefore, that they may fall very obliquely upon the plate of air, it is necessary that the visual rays should have very nearly the same obliquity to the surfaces of incidence. Then the curvature of the upper glass deranges the rings; and on this account, as well as the unfavourable position of the eye, the measurement of the diameter becomes difficult and uncertain. We can avoid these sources of error, by forming the rings between prisms, by means of refracted light, which, entering at one surface and emerging at the other, may thus be brought into the thin plate under the greatest incidences. But to avoid the effects of dispersion, it is necessary that this light should be simple and homogeneous. I have given a detailed account of this process in the *Traité de Physique.* I will only remark here, that by combining the results of this method, with those obtained by means of object-glasses, Newton composed the following table, which extends to all possible angles of emergence from the plate of air, from 0 to 90°.

Angle of incidence at the second surface of the glass.	Angle of emergence into the air.	Diameter of the ring.	Thickness of the air.
0°00′	0°00″	10	10
6 26	10 00	$10\frac{1}{13}$	$10\frac{2}{13}$
12 45	20 00	$10\frac{1}{3}$	$10\frac{2}{3}$
18 49	30 00	$10\frac{3}{4}$	$11\frac{1}{2}$
24 30	40 00	$11\frac{2}{5}$	13
29 37	50 00	$12\frac{1}{2}$	$15\frac{1}{2}$
33 58	60 00	14	20
35 47	65 00	$15\frac{1}{4}$	$23\frac{1}{4}$
37 19	70 00	$16\frac{4}{5}$	$28\frac{1}{4}$
38 33	75 00	$19\frac{1}{4}$	37
39 27	80 00	$22\frac{6}{7}$	$52\frac{1}{4}$
40 00	85 00	29	$84\frac{1}{13}$
40 11	90 00	35	$122\frac{1}{2}$

247. The two first columns need no explanation; only it may be remarked that, from the manner in which the observations are made, the angles of incidence and emergence may be reckoned indifferently, on the incident or refracted ray. For, the measures of the rings being taken from *M* to *M*, on the transverse diameter perpendicular to the central plane of incidence *OCH*, the visual rays which limit them at these extremities, traverse the two surfaces of the plate of air, at points where their tangents are sensibly parallel. Hence it follows, that the observations of Newton, although made upon curved plates, belong really to plates of equal thickness. But still we may apply the results to curved plates, when the curvatures of the surfaces, at the points where the luminous ray traverses them, differ so little, that the angles of incidence and emergence, measured upon one or the other, shall not give, according to the table, sensible variations in the diameters of the rings. This would be the case, for example, in the thin plates of air, comprehended between object-glasses of very slight curvature, even when the measures of the rings formed by them, were taken upon the diameter situated in the plane of incidence; only it would be necessary to apply different corrections to its two extremities, in order to make allowance for the unequal incidence of the rays at these points. Fig. 153.

248. The third column of the table expresses the successive diameters which the same ring takes when seen under different

obliquities, its diameter when the incidence is perpendicular, being represented by 10. The last column, formed of the squares of the third divided by 10, expresses the successive thicknesses of the plate of air, where this same ring is reflected under the different obliquities, 10 representing the thickness at which it is reflected under a perpendicular incidence. This table is general for all the rings, whatever be their distance from the central spot; for, by a very remarkable property, the proportion according to which the diameters of the several rings increase, is independent of the rank which they occupy, and the particular tints they reflect.

249. This table shows that the same colour is successively reflected at a greater thickness, as the incident rays become more oblique; consequently, if we observe the same point of the plate of air successively under the different obliquities, the colours which pass through this point will be those which before were reflected by less thicknesses. Thus it may be said, that by rendering the incident rays more oblique to the plate of air, we produce the same effect as if this plate were rendered thinner.

In order to make a correct and advantageous use of these results they must be reduced to an algebraic formula, which shall express them, if not rigorously, at least with sufficient exactness for observation. Newton has done this, and the formula may be seen in the *Traité de Physique.*

The rings being thus concentric, it is evident that the absolute dilatation of the diameter of each ring by reason of the obliquity, is less, in proportion as this ring is nearer the central spot; so that the contiguous rings, by being thus dilated, must be rendered more conspicuous and more distinct, as observation in fact shows.

250. According to the same law, the dilatation ought to be strictly nothing, only in the middle of the central spot, where the glasses touch each other; in fact, we observe a slight yet sensible dilatation, in the perimeter of the central spot, when we view it very obliquely. Now, when it is thus extended, it encroaches upon the parts of the plate of air, which, under a perpendicular incidence would reflect the tint approaching nearest to the black, that is, white. Consequently, the reflection from the plate becomes nothing at these points, without having its thickness completely annihilated. Accordingly, when we view the glasses at any other inclination whatever, and even

under a perpendicular incidence, we must perceive that the total transmission takes place, not only at the precise point where the glasses touch each other, but also at some distance about this point. This furnishes a reason why, when the contiguous glasses are very slightly curved, whatever be the perfection of their sphericity in other respects, the extent of the dark spot ordinarily exceeds considerably, that which we should reasonably ascribe to the point of contact. This result, apparently so trifling, is in reality very important; for it seems to contradict directly the theory of the propagation of light by undulations, since according to this theory, the reflection could never become nothing at the centre of the rings, except when the thickness of the plate at this point becomes nothing also. Euler, one of the warmest advocates for the theory of undulations, was well aware of this objection, and his only expedient for removing it was to say that it was not constant, and that Newton might possibly have been deceived in his observation.

251. Hitherto we have considered only the rings produced by reflection; it now becomes necessary to attend to those produced by transmission. In this case the central spot is white, surrounded by a blackish circle, to which succeeds another white circle, and then various other circles differently coloured. But these colours are much fainter than those produced by reflection, and it is difficult to distinguish them accurately, unless the incident rays are considerably inclined to the plate of air, for the purpose of augmenting their brightness. Newton observed them through his large object-glasses, and found them to be in the following order, beginning at the white central spot; WHITE, yellowish red, black, violet, blue; WHITE, yellow, red, violet, blue; GREEN, yellow, red; BLUISH GREEN, red; after which the degree of colour becomes insensible. The first yellowish red, intermediate between the white central spot and the blackish ring, was so pale and so little different from white, that it could hardly be discerned. The same is the case with respect to the first blue, which surrounds the dark central spot, in the reflected rings, as was noticed in the first observations upon these rings.

252. By comparing the order of these tints with those given by the reflected rings, we find that they are complementary. For example, the white central spot is complementary to the black central spot of the reflected rings, and thus it is formed by

the light which traverses the air in this place, without undergoing any reflection at its second surface. In like manner, the blackish circle which surrounds the yellowish red, is the necessary consequence of the white circle which surrounds the dark spot in the reflected rings; and one of these circles must be the opposite of the other, because it is formed of the portion of white light which escapes reflection at this place. Accordingly, the dusky circle formed of transmitted light will become darker, according as the reflection from the thin plate is greater; and it would be completely black if the reflection could be total. But this extreme case never exists; for in all transparent bodies, with whatever medium we surround them, the reflection which takes place under a perpendicular incidence is always very faint. The opposition of the successive rings is also indicated by the order of the colours, which are found to be complementary to each other in the two series. Moreover, Newton has directly established this opposition, by measuring the diameters of the transmitted rings in their brightest and dimmest parts. He has represented the effect of it in figure 154, where *AB* and *A'B'* are the surfaces of the glasses, one plane and the other spherical, touching each other at *C*. The dark lines, drawn between the two, are their distances from each other in arithmetical progression; the colours written above are seen by reflected light, and those below by transmitted light.

253. We have as yet considered only the coloured rings produced upon the plates of air comprehended between two glasses. We have determined by exact measurement, the thicknesses at which they are formed, as well with a perpendicular incidence, as with oblique incidences. But in order to attain to more general notions respecting this phenomenon, let us consider it as exhibited in other substances, for instance, in the plates of water. To form these plates in the most simple manner, Newton first adjusted together two object-glasses, one plane and the other convex, as if he had been going to form coloured rings in a thin plate of air; and when the rings appeared, he slightly wetted the borders of the glasses without deranging them. The water immediately insinuated itself between them by the effect of capillary attraction, and gradually displacing the air, took the form of the space interposed between the glasses. Thus a thin plate of water was produced between two surfaces of glass, and

coloured rings appeared in this, as they had in the plate of air the order was the same as well as the arrangement of the colours, but their tints were darker and of less extent. Newton measured their diameters in the brightest parts; and the squares of these diameters answered to the arithmetical progression of odd numbers 1, 3, 5, 7 . . . . . He measured them also in the darkest parts, and in this case their squares are in the progression of even numbers 0, 2, 4, 6 . . . . , precisely as in the case of air. But the diameters of the rings of the same rank were smaller than those of the plate of air, in the proportion of 7 to 8; whence it follows that the corresponding thicknesses which are proportional to the squares of the diameters, were to each other as 49 to 64, or nearly as 3 to 4, that is, in the ratio of the sine of incidence to the sine of refraction, when light passes from water into air; and perhaps, adds Newton, we may lay it down as a general law, that if other substances of whatever nature, are interposed between the two glasses, the thicknesses at which the same rings are formed, will be proportional to the sines of the refractions which the rays undergo in these substances, when they enter them under the same incidence; so that each ring will require a thickness so much the smaller, as the interposed substance exerts a greater refracting power.

254. In all the preceding experiments, the thin plates of water and air, comprehended between two surfaces of glass, have been surrounded by a medium of a greater refracting power than themselves. To render these observations complete, it remains to examine the colours produced under opposite circumstances; that is, when the plate has a greater refracting power than the surrounding medium. This is the course pursued by Newton; and with this view, he examined particularly the colours produced in soap bubbles, thus rendering subservient to important discoveries, what had before been considered only as a childish amusement.

Newton condescended not only to direct his attention to these bubbles, but he employed his ingenuity in forming them, in such a manner that they might become the subject of exact observation. This requires certain precautions. In the first place, we dissolve in distilled or rain water, a piece of solid soap of the best quality, and in such quantity that the solution may not be completely saturated. This being done, we dip into it the ex-

tremity of the bowl of a pipe or a glass tube, which is capable of detaching a small column of the liquid by virtue of capillary attraction; we then withdraw the tube, and after having wiped the outside, we blow into it gently from the other end. The liquid column, yielding to this pressure, expands, and as the viscidity of its parts prevents them from separating, it swells into a ball which adheres to the lower extremity of the tube by one of its points. Then, if we cease blowing, and leave the tube open, the capillary attraction which it exerts upon the water of the ball, together with the pressure which the ball itself exerts upon its own surface, will cause it gradually to contract, and at length to return entirely into the tube. But we can avoid these variations, either by closing up the tube with soft wax, to prevent the air from escaping, after the ball is blown, or by not suspending it from the tube, and leaving it to float freely on the surface of the solution. When we employ this last method, it is necessary that the vessel containing the solution, should be sufficiently large to prevent the capillary action of its sides, from giving to the liquid surface a sensible curvature. It is also necessary that the vessel should be filled completely to the edges, for otherwise the bubble would be drawn insensibly towards them and would be broken against them. But all these precautions will not avail, if we leave the bubbles exposed to the free air; for this fluid, being always agitated, disturbs by its motions, the regularity of their equilibrium, and accelerates, by drying, the evaporation of the film of water of which they are composed, and they would very soon burst. In order to give them greater durability and observe them in a constant and tranquil state, it is necessary, if
Fig. 155. we wish to let them hang at the end of the tube, to introduce this, charged with the liquid, into a receiver of thin glass, the upper orifice of which is closed by a perforated stopper fixed about the tube and removed and replaced with it. Then we blow up the bubble within the vessel, and if we close up the tube with wax, it may be kept sometimes for whole hours, without having its volume sensibly varied. In this disposition, the liquid tending always to flow towards the bottom of the bubble, the greatest thickness will be toward this part; and departing from this, the film gradually diminishes in thickness as we approach the top until within a small distance from its point of suspension, where the capillary action of the tube occasions a new increase of thick-

ness. This last cause of inequality does not belong to the first method, in which we let the bubble swim freely on the surface of the solution; in this case it is perfectly hemispherical, and the diminution of thickness, being determined by its weight alone, continues from the bottom to the top with the most perfect regularity. Newton adopted this method of preserving the bubbles, always taking the precaution to cover them with a receiver of thin transparent glass, to preserve them from the action of the air. In order to observe them conveniently, it is necessary to place the apparatus before an open window, where we can see a great extent of horizon, and receive by reflection, the white light of the sky. Moreover, that no foreign light may mix with this reflection, the interior of the vessel containing the solution must be of some dark colour; and lastly, a black cloth must be hung up beyond the bubble, on the side opposite to the eye, to intercept the luminous rays coming in this direction from external objects, which might otherwise traverse the bubble and arrive at the eye along with the reflected rays which we wish to observe. When the whole is thus disposed, we observe on the bubble several concentric horizontal rings, the colours of which are very lively and disposed with perfect regularity. They first appear on the summit of the bubble where the thickness is least; but according as the water subsides towards the bottom, the film grows thinner, the rings are seen to dilate gradually and extend over the whole surface. After the different sets of colours have thus appeared successively at the centre of the rings, a black spot is formed there, at first very small, but which gradually dilates in its turn, till at length the bubble bursts, and in this manner it always terminates. As these colours are more vivid and more extended than those which are produced by thin plates of air, we can better distinguish their order, succession, and different species. This observation, one of the finest with which the physical sciences have made us acquainted, is of indispensable importance, in order that we may become familiar with the peculiar characteristics of each tint, that we may understand perfectly their succession, and that we may be able to follow out the laws unfolded by Newton in the manner in which they naturally present themselves.

255. Setting out from the lowest, and consequently the thickest part of the bubble, and ascending to the highest, which is

also the thinnest, we observe, simultaneously or in succession, seven distinct series of colours, in the following order; namely, RED, blue; RED, blue; RED, blue; RED, green; RED, yellow, green, blue, purple; RED, yellow, green, blue, violet; RED, yellow, white, blue, black. The number of these series, the manner in which their colours succeed each other, the very space occupied by them, every thing is similar to what we have already observed in thin plates of air and water comprehended between two glass surfaces.

256. "The three first successions of red and blue were very dilute and dirty, especially the first, where the red seemed in a manner to be white. Among these there was scarce any other colour sensible besides red and blue, only the blues (and principally the second blue) inclined a little to green.

"The fourth red was also dilute and dirty, but not so much as the former three; after that succeeded little or no yellow, but a copious green, which at first inclined a little to yellow, and then became a pretty brisk and good willow green, and afterterwards changed to a blueish colour; but there succeeded neither blue nor violet.

"The fifth red at first inclined very much to purple, and afterwards became more bright and brisk, but yet not very pure. This was succeeded with a very bright and intense yellow, which was but little in quantity, and soon changed to green; but that green was copious and something more pure, deep, and lively, than the former green. After that followed an excellent blue of a bright sky colour, and then a purple, which was less in quantity than the blue, and much inclined to red.

"The sixth red was at first of a very fair and lively scarlet, and soon after of a brighter colour, being very pure and brisk, and the best of all the reds. Then after a lively orange followed an intense bright and copious yellow, which was also the best of all the yellows, and this changed first to a greenish yellow, and then to a greenish blue; but the green between the yellow and the blue, was very little and dilute, seeming rather a greenish white than a green. The blue which succeeded became very good, and of a very fair bright sky colour, but yet something inferior to the former blue; and the violet was intense and deep with little or no redness in it. And less in quantity than the blue.

"In the last red appeared a tincture of scarlet next to violet, which soon changed to a brighter colour, inclining to an orange; and the yellow which followed was at first pretty good and lively, but afterwards it grew more dilute, until by degrees it ended in perfect whiteness. And this whiteness, if the water was very tenacious and well tempered, would slowly spread and dilate itself over the greater part of the bubble; continually growing paler at the top, where at length it would crack in many places, and those cracks, as they dilated, would appear of a pretty good, but yet obscure and dark sky colour; the white between the blue spots diminishing, until it resembled the threads of an irregular net-work, and soon after vanished and left all the upper part of the bubble of the said dark blue colour. And this colour, after the aforesaid manner, dilated itself downwards, until sometimes it hath overspread the whole bubble. In the mean while at the top, which was of a darker blue than the bottom, and appeared also full of many round blue spots, something darker than the rest, there would emerge one or more very black spots, and within those other spots of an intenser blackness; and these continually dilated themselves until the bubble broke.

"If the water was not very tenacious the black spots would break forth in the white, without any sensible intervention of the blue. And sometimes they would break forth within the precedent yellow, or red, or perhaps within the blue of the second order, before the intermediate colours had time to display themselves.

257. "By this description you may perceive how great an affinity these colours have with those of air . . . , although set down in a contrary order, by reason that they begin to appear when the bubble is thickest, and are most conveniently reckoned from the lowest and thickest part of the bubble upwards.

"Viewing in several oblique positions of my eye the rings of colours emerging on the top of the bubble, I found that they were sensibly dilated by increasing the obliquity, but yet not so much by far as those made by thinned air . . . . . For there they were dilated so much as when viewed most obliquely, to arrive at a part of the plate more than twelve times thicker than that where they appeared when viewed perpendicularly; whereas in this case the thickness of the water, at which they arrived when viewed most obliquely, was to that thickness which exhibited them by perpendicular rays, something less than as 8 to 5. By the best of my observation it was between 15 and 15½ to 10; an increase about 24 times less than in the other case.

"Sometimes the bubble would become of an uniform thickness all over, except at the top of it near the black spot, as I knew, because it would exhibit the same appearance of colours in all positions of the eye. And then the colours which were seen at its apparent circumference by the obliquest rays would be different from those that were seen in other places, by rays less oblique to it. And divers spectators might see the same part of it of differing colours, by viewing it at very differing obliquities."

258. By observing, in this manner, the tints at different places on the bubble, under a perpendicular incidence, and by comparing them with those of the same rank and the same nature, observed on a thin plate of water or air comprehended between two glasses, Newton was enabled to deduce the relative thicknesses of the bubble in its different parts. Then, by considering the different parts to which the same tint withdrew, when followed under different obliquities, he was enabled to deduce the thickness of the water, which belonged to it under different obliquities. By these observations, together with various others, conducted on the same principles, he constructed the following table, analogous to that which he had formed for the thin plate of air.

Incidence of the rays upon the water.	Refraction of the rays in passing into the water.	Thickness of the water.
0°0′	0° 0′	10
15	11 11	10¼
30	22 1	10⅘
45	32 2	11⅘
60	40 30	13
75	46 25	14½
90	48 35	15⅕

The two first columns express the obliquities of the rays to the surface of the water; that is, their angles of incidence and refraction; the calculation is made upon the supposition that the sines of these angles are to each other as 4 to 3, in the same manner as if the water were pure, though probably the soap which is dissolved in it, slightly modifies its refracting power. In the third column, the thickness of the bubble, by which any colour whatever is produced, is expressed in parts of which 10 compose the thickness proper to produce this colour, when the rays are perpendicular. We see by this table, that the same colour is successively reflected at a greater thickness, according as the incident rays become more oblique; and consequently, if we were thus to observe the colours on a lamina whose thickness is constant, according as the obliquity increases, the limits would contract towards the centre of the rings, as if the lamina became thinner.

Not only are these results analogous to those obtained, article 246, for thin plates of air, but they may be connected together by the same law; we have only to change the constant ratio of the sine of incidence to the sine of refraction, which was $\frac{11}{17}$ when the light passed from the glass into the air, and which is found equal to $\frac{4}{3}$, according to Newton, when the light passes from the air into water, as we here suppose. With this single change we find all the numbers of the table. This agreement is a new and very striking confirmation of the identity which exists between the phenomena of colours produced on thin laminæ of water surrounded by air, and those produced on thin plates of air or water comprehended between two object-glasses.

259. Analogous variations are observed in the colours which are reflected from thin sheets of glass blown by means of a lamp to the point of bursting, and from the leaves of mica reduced to an extreme degree of tenuity. For if we place such sheets horizontally over a black ground, and observe the reflection of white light from the sky under different incidences, by raising or lowering the eye, we find that their colours also change, and change in the same manner as those of the plates of air or water, when we observe the orders of the rings, which arrive successively at the same distance from the central spot, and consequently at the same thickness. But as the range and extent of these variations are much less in plates of water than in those of air, for equal changes of incidence, on account of the great difference of refraction which the rays undergo; so they are also a little less in glass and mica than in plates of water, because these substances refract a little more strongly than water. Similar variations also, but so feeble as to be almost insensible, are

observed in the colours which appear on *oxydable* metals, particularly polished steel and copper, when they have been heated in the open air. And this is what we should expect, for the metals are then covered with a thin lamina of oxyd, which, as it has less action upon light than the pure metal, on account of the small refracting power which oxygen has been found to possess, must produce a reflection, although it is applied to the surface of the metal, and consequently exhibit colours, if it is sufficiently thin. Moreover, these colours must vary very little with the incidence, because they are produced by a substance whose action upon light is very powerful. We are even able to calculate the limits of these variations, when the ratio of refraction of the lamina is known; and observation is found to verify the calculation. If the ratio of refraction is not known, we can deduce it from this comparison, and hence obtain likewise the thickness of the laminæ which reflects this or that colour. We shall hereafter attend to these applications.

260. So perfect an identity of results, for substances and refractions so different, evidently proves that the order of the rings, their arrangement, and the nature of their tints, are subject to the same laws in all kinds of laminæ. The only difference is in the absolute value of the thickness at which they are formed, and in the more or less rapid variations they experience from the obliquity of incidence of the luminous rays. Their general laws are thus completely known to us; but there are still compound laws, because the phenomena themselves are compound. Indeed all the experiments hitherto related, having been made with white light from the sky, which contains all sorts of rays unequally refrangible, the variety of tints which result from it at different thicknesses, shows that reflection does not take place equally at each thickness, upon all the kinds of rays. Hence, in order to have a complete analysis of the phenomena, we must enter into a particular examination of the rings formed by each kind of rays. When the law of these elementary results is ascertained, we can deduce from it the effects produced by the mixture of all the rays which compose white light, and thus recompose the phenomenon after having decomposed it.

This is precisely what Newton did. He recommenced in a dark room, his experiments upon the thin plate of air contained between two glasses. Having decomposed a ray of white light by the prism, in order to obtain the different simple rays, he caused them to fall one after another, upon a sheet of white paper, which reflected them by radiation in all directions, and he placed his eye in such a manner as to be able to view the coloured paper by reflection in the glasses, and in the intermediate plate of air. In this manner he discovered the following phenomena, which he established in all their particulars, with the greatest care, and which we shall describe in very nearly the same terms he employed.

261. (1.) Each simple ray produced rings of its own colour, either by reflection or by transmission. The rings were entirely red in the red light, yellow in the yellow light, and so on.

(2.) In each species of light, the reflected rings were separated by dark intervals, which rendered them much more distinct than if they had been in open day, and enabled him to discern a much greater number of them. They approach each other more and more, according as they are more distant from their centre, which forms the first and innermost of the dark rings.

It is to be observed, that Newton does not say that these intervals between the bright rings, seen by reflection, were *black*; but *dark*. Indeed, in order that he might be able to see them absolutely *black*, the first and second surface of the glass, situated on the side of the eye, must have reflected no light at all. This complication of reflection is inevitable; it would take place, even if we should employ a single thin insulated lamina, since the first surface of such a lamina would still produce a partial reflection, the effect of which would reach the eye at the same time with the light of the rings. It is proper to separate here what the experiment necessarily unites; but this remark may enable us to appreciate the scrupulous fidelity which Newton always observed in his enunciations.

(3.) The dark intervals which separate the luminous reflected rings, form in their turn luminous rings when seen by transmission; and there were among them dimmer intervals corresponding to the places at which the light was most abundantly reflected. Still these dim spaces were not perfectly black, because the reflection from the plate of air was far from being total, even in the most brilliant part of the reflected rings; and the same is true of all thin transparent laminæ of whatever substance, as has been before remarked.

(4.) By examining the luminous reflected rings, Newton found that they were not formed of simple mathematical lines; each of them occupied a certain circular space, in which the intensity of the light went on diminishing indefinitely from the middle in both directions.

(5.) By measuring the diameters of the reflected rings in the brightest parts of their breadth, he found that, for each particular species of rays, the squares of these diameters answered to the arithmetical progression of odd numbers 1, 3, 5, 7, ... ; consequently the thicknesses of the laminæ, at the perimeters of the successive rings, formed also a similar progression; for the thicknesses are proportional to the squares of the diameters. When the glasses were illuminated by the most brilliant part of the spectrum, which corresponds to the limit between the yellow and the orange, the absolute diameter of the sixth ring was found to be very nearly the same, as he had before found it in the experiments made in open day, when he measured it at the brightest point of the compound ring.

(6.) By measuring also the diameters of the dark rings, comprehended between the bright ones just mentioned, he found that for each particular species of rays, the squares of the diameters answered to the arithmetical progression of even numbers 2, 4, 6, 8, ... ; and consequently the thicknesses of the air at the perimeters of these rings were in the same progression.

(7.) By other measurements taken upon the transmitted rings it was found that their brightest parts corresponded to the darkest intervals between the reflected rings, and reciprocally that their darkest parts corresponded to the brightest parts of the reflected rings. Hence we see, that in the transmitted rings, the thicknesses of the air, in the bright parts, answer to the progression of even numbers 2, 4, 6, 8, ... , and in the dark parts, to the progression of odd numbers 1, 3, 5, 7 . . . . .

(8.) The absolute dimensions of the same ring, or rather of rings of the same order, were different in the different colours. Thus the exterior diameter of the fifth ring, for example, when it was formed by the first degrees of extreme red, was greater than that of the same ring, when formed of the rays which compose the middle red; and this last was greater than when the ring was formed of the first rays of the orange, and so on in the order of refrangibility of colours to the violet, which formed the smallest rings. The width of the rings at their perimeter, that is, the extent occupied by all the degrees of their light, differed, in a similar manner, for the different colours; it was greatest in the first red rays, less in those of the middle red, less still in the orange, and so on to the violet, where it was least of all.

(9.) The simple rings formed of each colour, were the smallest possible, when the rays traversed the plate of air perpendicularly, and they increased according as the incidence became more oblique, conformably to the law of article 246.

262. The preceding observations completely explain the compound phenomenon presented by rings formed of natural light. For this light, being only a mixture of rays of different colours in determinate proportions, it follows that when a beam of such a mixture falls upon a thin plate of air, interposed between two glasses, each simple ray must form rings according to its own peculiar laws; and as the absolute magnitude of these rings is different for the rays of different colours, there must be a separation between them large enough to admit of their being distinguished. This separation will not be so distinct as in the observations made with simple rays, because the rings of different colours must partly cover each other, and lap the one upon the other, by a quantity which may be unequal in their different successions, in such a manner as to produce that infinity of different tints, exhibited in the experiment. But although this successive superposition of the simple rings may, in fact, be the key to the phenomena, yet we cannot be assured of the fact, till we have measured with accuracy the absolute magnitude of the diameters and breadths of the different rings formed of these rays; for, these being once known, it becomes a simple problem of arithmetic, to find the species and quantity of each simple colour which may be reflected or transmitted at each determinate thickness; and, accordingly, if we calculate the effects of the composition of all the colours by the rules given in the former part of this work, article 169, we shall obtain, with the utmost accuracy, the numerical expressions of the intensities and tints which ought to exist in each point of the compound rings, and these results may be easily compared with those of experiment. In a word, having a glimpse of what may possibly be the cause of the phenomena, we now want precise measurements in order to establish the fact, and convert our conjectures into certainty.

263. These measurements Newton made. He took the diameter of the simple rings *of the same order*, at the interior and exterior part of their perimeters, considering them successively at the limits of the different colours of the spectrum, beginning with extreme violet. According to his constant practice, he took care to connect these results by a mathematical law, which should represent them with sufficient accuracy. He thus found that the diameters, whether interior or exterior, were to each other sensibly as the cube roots of the numbers $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1, which represent the lengths a musical cord must have, in order to produce all the notes in an octave; that is, if we represent by 1 the interior diameter of a certain ring, when it is formed by the red rays which compose the extreme part of the spectrum, $\sqrt[3]{\frac{8}{9}}$ will express the interior diameter of the same ring, when it is formed by the rays which are the limit between the red and the orange; and so on to $\sqrt[3]{\frac{1}{2}}$, which will express the exterior diameter of the same ring, when it is formed of the last violet rays taken at the other extremity of the spectrum.* Knowing the ratios of the diameters, Newton took the squares of these ratios, and they gave him the proportions of the thickness which the plate of air must have at the beginning and end of the observed rings. Similar measurements, made upon the different orders of rings formed by the same simple colour, showed that the intervals of the thicknesses where the reflection took place, were sensibly equal to those where transmission took place; so that if we designate generally by e_1, the thickness of the air at the commencement of the first bright ring formed by any of the simple rays, this ring ends with the thickness $3\,e_1$, and thus occupies an interval of thickness equal to $2\,e_1$; after which comes the first dark ring, occupying also the same interval of thickness $2\,e_1$; next to this in course,

---

* The numbers here employed by Newton, have to each other a very singular relation, which was first noticed by M. Blanc. It is this; if we multiply together two terms taken at equal distances from the extremes, for example, $\frac{9}{16}$ and $\frac{8}{9}$, $\frac{3}{5}$ and $\frac{5}{6}$, or $\frac{3}{5}$ and $\frac{3}{4}$, we shall always have a constant product $\frac{1}{2}$, which is equal to that of the two extreme terms. I shall hereafter make known the remarkable consequences which result from this relation.

comes the second bright ring, commencing at the thickness $3\,e_1$, and ending at the thickness $5\,e_1$, and so of the rest. Combining this law of succession for the different orders, with that of the distribution of the different tints in the same order, we perceive that one absolute thickness, measured at the commencement, middle, or end, of any ring whatever, formed by one simple colour, is sufficient to enable us to calculate the value of the first thickness e_1, in relation to this colour as well as to all the others; and thence to deduce the limiting thicknesses $3\,e_1$, $5\,e_1$, $7\,e_1$, ... belonging to all the orders of rings.

264. We can, therefore, employ for this purpose the very exact value obtained by Newton, for the thickness of the air, at the middle of the first bright ring, formed by the rays, which make the limit between the orange and yellow; a thickness which we have seen to be equal to $\frac{1}{178000}$ of an inch, or $\frac{1000}{178}$, taking the millionth of an inch for unity. This will be the value of $2\,e_1$ for this colour; and consequently e_1 will be equal to $\frac{500}{178}$ or 2,80899. Upon this fundamental datum, together with the ratios found to exist between the different colours, is founded the following table, in which e^n, E^n, represent the thicknesses at which the ring of the *n*th order begins and ends.

TABLE

Representing the thicknesses of the air at which the different rings begin and end, expressed in ten millionths of an inch.

General expression of the thicknesses e_n, E_n, at which each ring, of the order n, begins and ends.	Extreme violet.	Limit of the violet and indigo.	Limit of the indigo and blue.	Limit of the blue and green.	Limit of the green and yellow.	Limit of the yellow and orange.	Limit of the orange and red.	Extreme red.
1st ring e_1	1,99849	2,16154	2,25671	2,42071	2,61866	2,80899	2,93207	3,172206
$E_1 = 3\,e_1$	5,99547	6,48462	6,77013	7,26213	7,85598	8,42697	8,79621	9,516618
2d ring $e_2 = 5\,e_1$	9,99245	10,80770	11,28355	12,10355	13,09330	14,04495	14,66035	15,861030
$E_2 = 7\,e_1$	13,98943	15,13078	15,79697	16,94497	18,33062	19,66293	20,52449	22,205442
3d ring $e_3 = 9\,e_1$	17,98641	19,45386	20,31039	21,78639	23,56794	25,28091	26,38863	28,549854
$E_3 = 11\,e_1$	21,98339	23,77694	24,82381	26,62781	28,80526	30,89889	32,25277	34,894266
4th ring $e_4 = 13\,e_1$	25,98037	28,10002	29,33723	31,46923	34.04258	36,51687	38,11691	41,238678
$E_4 = 15\,e_1$	29,97735	32,42310	33,85065	36,31065	39,27990	42,13485	43,98105	47,583090
5th ring $e_5 = 17\,e_1$	32,97433	36,74618	38,36407	41,15207	44,51722	47,75283	49,84519	53,927502
$E_5 = 19\,e_1$	37,97131	41,06926	42,87749	45,99349	49,75454	53,37081	55,70923	60,271914
6th ring $e_6 = 21\,e_1$	41,96829	45,39234	47,39091	50,83491	54.99186	58,98879	61,57347	66,616326
$E_6 = 23\,e_1$	45,96527	49,71542	51,90433	55,67633	60,22918	64,60677	67,43761	72,960738
7th ring $e_7 = 25\,e_1$	49,96225	54,03850	56,41775	60,51775	65,46650	70,22475	73,30175	79,305150
$E_7 = 27\,e_1$	53,95923	58,36158	60,93117	65,35917	70,70382	75,84273	79,16589	85,649562

The results of this table may be represented by a geometrical construction which will enable us to comprehend the whole at one view. We shall explain it after the manner of Newton.

265. We remark, in the first place, that the series of quantities $e_1, E_1, e_2, E_2, e_3, E_3, \ldots$, for the several colours, forms an arithmetical progression e_1, $3\,e_1$, $5\,e_1$, $7\,e_1$, ... of which the common difference is $2\,e_1$, and which answers to the order of odd numbers. If we wish to represent this result geometrically, we have only to divide an indefinite straight line ZZ', into an indefinite number of parts, equal to each other and to e_1, and mark the successive points of division by the numbers 1, 2, 3, 4, 5, 6, ... Then from the thickness 0 to the thickness $Z\,1 = e_1$, the rays of this colour will be transmitted; from the thickness $Z\,1$ to the thickness $Z\,3$, they will be reflected; from $Z\,3$ to $Z\,5$ they will be transmitted; and thus they will be alternately transmitted and reflected through the whole extent of the straight line ZZ'. The maximum of reflection will take place in the thicknesses $Z\,2$, $Z\,6$, $Z\,10$, ... which answer to the arithmetical progression of odd numbers 1, 3, 5, 7, ...; and the maximum of transmission will take place at the thicknesses $Z\,4$, $Z\,8$, $Z\,12$ ..., which answer to the progression of even numbers 0, 2, 4, 6, ...; so that if we wish to know what effect will be produced by any given thickness represented by ZX, we have only to apply this length along the straight line ZZ', beginning at Z; and the point X where it falls in the direction ZZ', will show whether it will produce reflection or transmission, and to what order of rings it belongs. Fig. 156.

But this construction upon a single straight line is applicable only to rays of a single colour, and indeed only to those of this colour which belong to a determinate part of the spectrum. To render it general, it is sufficient to remark that the values of e_n and E_n for the different colours, when n is the same, are proportional to the values of e_1 which belong to these colours; and consequently we can represent them by the ordinates to a straight line, of which the $e_1 s$, are abscissas. This is the object of the following construction given by Newton.

Upon an indefinite axis CZR, take from any point C the abscissas CZ, CV, CI, CB, CG, CY, CO, CR, proportional to the numbers 0,6300; 0,6814; 0,7114; 0,7631; 0,8255; 0,8855; 0,9243; 1; which, according to experiment, express the ratios of Fig. 157.

the different values of e_1 for the limits of the seven principal colours of the spectrum. Then, through the points *Z*, *V*, *I*, ... *R*, the extremities of these abscissas, and perpendicularly to the axis *CZR*, draw the indefinite ordinates *ZZ'*, *VV'*, ... *RR'*; and, taking on the first a length *Z* 1, equal to the value of e_1 for the last violet rays of the spectrum, which border on the black, apply this space along this ordinate successively to the points 2, 3, 4, 5, ...; lastly, draw from the point *C*, the dotted lines *C* 1. *C* 3, *C* 5, ..., to all the odd divisions. The intersections of these straight lines, with the ordinates belonging to each colour, will determine the values of e_n and E^n, at which the reflection of this colour begins and ends in the several orders of rings. Thus the spaces 11'33', 55'77', ... comprehended between *C* 1 and *C* 3, *C* 5 and *C* 7, ... represent the intervals of thickness in which some reflection takes place, and the intermediate spaces 00'11', 33'55', ... represent the intervals in which the incident light is entirely transmitted. Lastly, the trapezoids contained between these lines and the ordinates which limit the seven divisions of the spectrum, indicate the particular intervals of thickness which belong to the reflection or transmission of all the degrees of simple light, which produce the sensation of the same colour. For instance, the trapezoids contained between the ordinates *ZZ'* and *VV'* comprehend all the degrees of violet; those which are contained between *VV'* and *II'* comprehend all the degrees of indigo, and so of the rest. But it is necessary to add to these geometrical limitations, an important modification, namely, that the transmission is total in the places which are assigned to it, whereas the reflection which is at first hardly sensible at the limits where it commences, gradually increases till it attains a certain maximum, for which the thickness is indicated by the mean lines 22', 66', 10 10' . . . .; after which it decreases again by the same degrees, and even in these thicknesses where it is greatest, it is still only partial. It should also be remarked that the intervals of thickness which correspond to reflection, are always found in nature to be a little larger than those which answer to transmission, especially in the first orders of rings. But Newton, who was well aware of this difference, considered it too small and too little susceptible of exact estimation to be worth regarding.

266. Although in the construction of this figure as well as the table which it illustrates, we have only had in view the reflection of colours by thin plates of air, they are both applicable to plates of any nature whatever, since we have seen already, that the coloured rings are formed according to the same laws in all substances. The only variation is in the absolute values of the thicknesses e_1, at which they are formed, which are less in proportion as the substance has a greater refracting power. We may, therefore, with this single modification, regard the consequences of our construction as general.

By means of this construction, we can find at once whether a certain colour is reflected or transmitted at a given thickness; for if we represent this thickness by *ZX*, we have only to apply it from *Z* along the ordinate *ZZ′*, and through the point where *X* falls, draw a line *XX′* parallel to the axis *CZR*; and then see whether this parallel passes through one of the spaces assigned to the reflection or to the transmission of the proposed colour. Also, if we wish to know what sorts of colours will be transmitted or reflected at this thickness *ZX*, we have only to observe whether any portions of this line pass through any part of the spaces 00′11′, 33′55′, ... where transmission takes place; for these portions will indicate the transmission of the corresponding colours; and on the contrary, those portions which pass through the intermediate spaces 11′33′, 55′77′, ... will indicate a reflection, which will be more abundant for each section, in proportion as the line *XX′* passes nearer the middle of these spaces, where they are traversed by the mean lines 22′, 66′, 10 10′ . . . . &c. Suppose, for example, that we wish to know what sort of green, will appear by reflection in the third ring, at the place where the reflection of this colour is brightest, we mark on the transverse line 10 10′ the middle of the trapezoid belonging to green; and, drawing through this point a line *v m v′* parallel to *CZR*, we find that it passes through the lower extremity of the space appropriated to yellow, and the upper extremity of the space appropriated to blue, in the same ring. But in all the rest of its course, the line *v m v′* passes through spaces belonging to transmission, whence we may conclude that the species of green reflected at this thickness will be principally composed of simple green, mixed however with a little blue and yellow, but still constituting a very good green.

267. We are thus enabled, by means of this figure, to explain in all its details, the phenomenon of coloured rings formed by natural light, when it is reflected by a thin lamina of air or any other substance; for all the simple rays which compose this light, entering the thin lamina together with an equal incidence, it follows that each must form rings there according to its own proper laws; and the second surface of the lamina must reflect or transmit them at the same thicknesses at which it would have reflected or transmitted them, if the simple rays had traversed it separately or successively. Since then, the same thickness may reflect separately the rays of different species, as we have seen, it may also reflect them simultaneously, while it allows all the others to pass. Hence it follows that if the lamina is throughout of the same thickness, it will reflect in all its points the same mixture of rays, under each given obliquity; and consequently, if we are at a sufficient distance to cause the rays which it sends to the eye from its different points to be sensibly parallel, it will appear of a uniform colour. But if the thickness varies, the colours will also vary in the different points of the lamina, according to the thickness. Thus, when the lamina is comprehended between two spherical object-glasses, between which the space constantly increases in thickness in every direction as we depart from the point of contact, there must be formed about this point an infinite number of circular rings of different shades, as we in fact observe in the coloured rings formed by daylight.

268. We can even determine the order in which the colours of these rings must succeed each other as we depart from their common centre. For this purpose, it is sufficient to conceive a straight line, which, being at first coincident with *CZR*, gradually removes from it, remaining always parallel to it, and passing in this manner through all the alternate spaces where reflection and transmission take place. When this line first leaves *CZR*, it traverses a space in which little or no reflection is produced, on account of the extreme thinness of the lamina; after which it arrives at 1, that is, at the feeblest beginnings of extreme violet. But as soon as it has traversed, ever so little, the space which belongs to this colour, it meets also those belonging to the blue and green, which, with the violet will compose a blue; then it will enter the yellow and red, which, with this blue, will compose a white. From the slight inclination of the line 11′ to the

axis *CZR*, it is evident that this passage from violet to white must be very rapid. In a lamina of air, for example, the beginning of the violet in the first ring answers to the thickness 1,99849, as shown by the table, article 264; and the commencement of the red in this same ring, or the limit of the red and the orange, answers to the thickness 2,93207; whence it follows that the separation of colours does not take place except at an interval of thickness equal to 2,93207 — 1,99849 or 0,99358; and consequently, it must be very difficult to distinguish it unless the thicknesses vary with extreme slowness. The white being once formed in the first ring, it will continue to be reflected more or less perfectly, while the moving line passes from 1 to 3; after which, the colours which compose it successively disappearing, it will change first into a compound yellow, then into a red, and this red will finally disappear at 3′. Here the colours of the second ring begin; and between these and those of the first ring there is a small dark interval, at least if we adopt as rigorously exact the limits of reflection fixed by Newton. For the extreme thickness where the red of the first ring *ends*, is 9,516618, according to our table; and the thickness where the violet of the second ring *begins* is 9,99245; whence it follows that between these two limits, there is an interval equal to 9,99245 — 9,516618, or 0,475832, in which no colour is reflected; consequently there must be formed by transmission in this place, a very narrow white ring. But the existence of these two rings may be modified by the more or less considerable extension of the sensible limits of reflection; for if the thin plate of air, upon which the preceding limits were taken, could suddenly exert a greater reflecting power, without its other properties being changed, we might observe sensible quantities of light, at the places where before we did not perceive any; and the intervals of thickness occupied by the bright reflected rings becoming more enlarged, the two first rings might extend so far, as to cause the narrow dark ring which separated them, in the determinations of Newton, to disappear. Indeed, we shall find substances hereafter, which present this superposition in a manner sufficiently distinct to be observed, not in this self-same phenomenon of the rings, but in a series of facts which are governed by precisely the same periodical laws, on a scale of thicknesses much more extensive. Returning, however, to the experiments of Newton, we see that beyond the dark narrow ring in question, the colours of the second ring commence and succeed each other in order, while the moving line passes from 5 to 7; these are more vivid than those in the first ring, because they are more dilated and separated from each other, as the figure itself indicates, by the greater inclination of the line 77′ to the axis *CZR*. In consequence of this separation, there no longer intervenes a white between the blue and yellow of this ring, as in the first, but a mixture of orange, yellow, green, blue, and indigo, all which colours, joined together, must compose a faint imperfect green. In like manner, the colours of the third ring succeed each other in order; first comes the violet, which mixes a little with the red of the second ring; for it begins at the thickness 17,98641, and ends at 21,98339, whereas the red of the second ring ends only at the thickness 22,205442; whence it follows that these two colours are reflected together through the whole of the violet of the preceding ring. We hence see why this violet is not separately perceived, but instead of it a reddish purple. Next come the blue and green, which are less mixed with other colours, and for this reason are more lively than in the former case, especially the green. Then follows the yellow, of which the part next the green is distinct and good; but the other part, next the red, which comes immediately after, consists of a yellow which, as well as the red, is mixed with the violet and blue of the fourth ring; whence result different degrees of red inclining strongly to purple. This violet and blue, which should succeed this red, are mixed and confounded with it, so that in place of them there succeeds a green. This green at first inclines to blue, but it very soon becomes a fine green; and it is the only unmixed and lively colour which appears in this fourth ring; for in proportion as it inclines to yellow, it begins to mix with the colours of the fifth ring, by which mixture the yellow and red immediately succeeding, become very faint and dirty, especially the yellow, which, being the faintest of the colours, can hardly be perceived. After this, the different rings and colours are intermixed and confounded more and more, until after three or four successions in which the red and blue especially predominate, all the species of colours become mixed together in nearly equal degrees, and constitute an uniform white.

269. As we have observed that the rays of one colour are transmitted at the same place at which the rays of another colour

are reflected, it is evident that if we take the complement of the reflected colours, the succession of which has been stated, we shall have the order and succession of the transmitted colours. Thus, as we moved the line parallel to *ZR*, from *Z* to 1, we found that, in this space, the reflection was insensible; consequently the transmission there will be total, and this will produce the white central spot which was observed in the transmitted rings. When the line has arrived at 1, the violet and blue rays will begin to be reflected, and of course their complement in the transmitted rings, will form first a yellowish white and then a yellowish red; but these colours will be extremely faint and difficult to be distinguished, because the first violet rays are not reflected at first in great abundance, as is indicated by the distance of the moving line from 22′, the place of maximum reflection; and when it begins to approach nearer, it comes at once upon 1′, the commencement of the red; so that the white light transmitted, losing thus a part of all its colours, and in a proportion very little different from that which produces white, still continues white as before, or at least does not take any other determinate colour. But at the same time its intensity is weakened, and it is the least possible in the thicknesses where the reflection is strongest, which answers to the middle of the white in the first of the reflected rings. This accounts for the black, or rather the dark grey ring, which succeeds the first yellowish red in the transmitted rings.

270. The moveable line continuing to depart from the axis *ZR*, arrives next at 3. Then the violet rays begin to escape reflection altogether; next the blue, the green, the yellow, and finally the red escape in their turn. But as the line 33′ is still very little inclined to the axis *ZR*, the same thing happens here as in the passage from 1 to 1′; that is, we cannot distinguish the succession of these different colours, on account of the slight difference in the thicknesses by which they are produced, and also on account of their faintness; so that at the most we perceive only a little violet and blue, which immediately change to white when the rule arrives at 3′; for we have seen that it does not yet reach 5, since *Z*5 exceeds *R*3′, by a quantity equal to 0,475832 millionths of an inch. This produces the second white transmitted ring, and this ring continues while the line passes from 3′ to 5. Then, the colours of the second ring beginning to be reflected,

the different parts composing this white, successively separate; first the violet, then the violet and blue, then the violet, blue, and green, and so on; which changes it successively into the colours complementary to the preceding, that is, into yellow, red, violet, and blue. These are the colours of the second transmitted ring, and they are more distinct than those of the first, because the line which limits them is more inclined to *ZR* than 11′ was; which separates them more from each other. Continuing thus to depart from *ZR*, and taking always the colour complementary to that of the reflected ring, we obtain successively the colours of the transmitted rings, in the order stated in article 251, from observation.

271. After this examination, we can easily understand why the transmitted rings are always paler than the reflected rings. In transmission, the rings formed by each species of simple light are not separated by dark intervals, because the reflection is never total at any part of the thin lamina; consequently their succession does not offer alternations of intensity periodically increasing and decreasing; whereas the simple rings seen by reflection, are separated from each other by intervals absolutely black, which prevents them from encroaching so much upon each other, when they are all reflected at once. On this account the transmitted rings would be more distinct if we were to increase the reflecting power of the thin lamina; since the variations of intensity would be more considerable in the different parts of the simple rings, and of course their mixture would be less uniform. We effect this purpose when we give the rays a greater inclination to the lamina, by looking very obliquely; for then the reflection becomes fifteen or twenty times greater, than under a perpendicular incidence. We then find that the colours of the transmitted rings become much more sensible; and, on the contrary, those of the reflected rings are much less sensible, because the simple rings which compose them spread out and encroach more upon each other.

272. It is very easy to verify all these theoretical indications, by observing the colours which are developed in soap bubbles, by reflection and transmission; but in order that we may be able to attend to all the details, it is necessary to form these bubbles, according to the directions given in article 254, by blowing them in a close vessel, and letting them hang at the end of the tube with which they are blown. As this disposition enables

us to preserve them for whole hours, we have time to pass progressively through all the varieties of thickness and colour. But the process is rendered still better, if we blow, one above the other, two bubbles, separated from each other by a very thin plate of water, as represented in figure 158. For this purpose, when we have formed the first bubble, we bring, with the finger, to the upper orifice of the tube, a small drop of water, which soon descends to the other orifice. We blow this water gently, and there results another bubble adhering to the first. The aqueous partition which separates them is ordinarily nearly plane, and more or less inclined to the horizon. This inclination causing the water which forms the partition to flow down, it gradually grows thinner, and becomes thinnest at its highest part. Hence there results a succession of stripes coloured with all the tints of the rings, which appearing first at the highest part of the lamina, descend by degrees to the bottom, developing more and more the richness of their hues; and in their successive formation, as well as in their different changes, they present all the varieties of colour before indicated by theory.

273. Hitherto we have considered only the order and species of simple colours, which are reflected or transmitted at each thickness. In order to recompose the phenomena completely, we must determine the nature of the tints which result from their mixtures. This requires that we should know the law according to which the intensity of reflection varies in the extent of each simple ring; for, these intensities being known, the composition of the colours may be deduced by the process explained in article 169. We might then assign with precision the thicknesses at which the most distinct shades of each ring must appear. Newton has done this, and he has constructed a table of results for the seven orders of rings whose colours are sensible. I shall present it as he has given it in his Optics. I cannot give all the numerical details of the construction, and Newton himself did not. I think, notwithstanding, that I have succeeded in obtaining them, and I propose to make them, at another time, the subject of a particular exposition. I will simply remark here, that having first calculated this table only for plates of air, Newton extended it to those of water and glass, by means of the law indicated above, that in different substances, the same tint is reflected at thicknesses reciprocally proportional to their ratios of refraction. For instance, if this ratio is $\frac{3}{4}$ for the rays which

pass from air into water, the plates of water which reflect a certain tint, must have $\frac{3}{4}$ of the thickness which the plates of air had. We may observe this ratio in the table of Newton, and experiments which we shall soon relate, show that the rule is general.

Colours reflected.	Thickness in millionths of an inch.			Names of the Colours, or substances having them.
	In air	in water	In glass	
1st *Order.*				
Very black	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$	
Black	1	$\frac{3}{4}$	$\frac{20}{31}$	
Beginning of black	2	$1\frac{1}{2}$	$1\frac{2}{7}$	
Blue	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{20}$	Whitish sky blue.
White	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$	Tarnished silver.
Yellow	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$	Straw colour.
Orange	8	6	$5\frac{1}{6}$	Dried orange-peel.
Red	9	$6\frac{3}{4}$	$5\frac{4}{5}$	Geranium Sanguineum.
2nd *Order.*				
Violet	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$	Iodine.
Indigo	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$	Indigo.
Blue	14	$10\frac{1}{2}$	9	Cobalt blue.
Green	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$	Water, aquamarine.
Yellow	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$	Lemon.
Orange	$17\frac{2}{9}$	13	$11\frac{1}{9}$	Orange.
Bright red	$18\frac{1}{5}$	$13\frac{3}{4}$	$11\frac{5}{6}$	May-pink.
Scarlet	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$	
3rd *Order.*				
Purple	21	$15\frac{3}{4}$	$13\frac{11}{20}$	Flax-blossom.
Indigo	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$	Indigo.
Blue	$23\frac{2}{5}$	$17\frac{11}{20}$	$15\frac{1}{10}$	Prussian blue.
Green	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$	Meadow green.
Yellow	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$	White wood.
Red	29	$21\frac{3}{4}$	$18\frac{5}{7}$	Rose red.
Bluish red	32	24	$20\frac{2}{3}$	Purplish red.
4th *Order.*				
Bluish green	34	$25\frac{1}{2}$	22	
Green	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$	Emerald green.
Yellowish green	36	27	$23\frac{2}{9}$	
Red	$40\frac{1}{3}$	$30\frac{1}{4}$	26	Pale rose.
5th *Order.*				
Greenish blue	46	$34\frac{1}{2}$	$29\frac{2}{3}$	Sea green.
Red	$52\frac{1}{2}$	$39\frac{3}{8}$	34	Pale rose.
6th *Order.*				
Greenish blue	$58\frac{3}{4}$	44	38	Light sea-green.
Red	65	$48\frac{3}{4}$	42	Paler rose.
7th *Order.*				
Green blue	71	$53\frac{1}{4}$	$45\frac{4}{5}$	Very faint.
Ruddy white	77	$57\frac{3}{4}$	$49\frac{2}{3}$	Ditto.

274. If we compare this table with figure 157, we shall perceive the manner in which each of the colours it contains, is composed of functions of its simple elements. It has also many other uses which will be explained hereafter; but there are some which may be explained now.

The first and most simple is that of finding the thickness of a plate of any substance from the colour which it reflects under a perpendicular incidence, when its ratio of refraction is known. This colour being known, the table determines the thickness of air which corresponds to it, and if we divide this by the ratio of refraction of the substance in question, we shall have the thickness of this substance. For example, experiment has shown that the ratio of refraction of common Siberian mica with two axes, may be estimated at 1,53, that of air being 1. Suppose that we detach a leaf of this substance sufficiently thin to reflect blue of the third order, under a perpendicular incidence. According to the table, the thickness of air which would reflect this blue is 23,4; that of the mica, therefore, must be $\frac{23,4}{1,53}$ or 15,3. This thickness differs very little from that of a sheet of glass which would reflect the same colour; because there is very little difference between the ratios of refraction of the mica and glass.

275. Another use of the table is to determine at once the ratio of refraction of plates and their thickness, when we have observed the tints which they reflect under two known incidences, for instance, when the incident rays are successively parallel or perpendicular to their surface. We have only to combine the indications of the table with the general law established by Newton, for the variations of tints under different incidences. In the *Traité de Physique*, may be seen the formulas which express this combination, and their application to a sheet of mica thus observed. It is from this that I have obtained the ratio of refraction indicated above for this substance. It remains yet to show how we are to know that a tint observed belongs to a particular order. This will be explained hereafter.

276. The theory of coloured rings above given from Newton, does not make known the physical cause which produces the decomposition of light in very thin laminæ of bodies; but this decomposition being admitted as a fact, it reduces this fact to its most simple elements; it shows why the rings of different col-

ours are superposed; it assigns the laws of their superposition, and it accounts for all the diversity of tints which results from their mixture. These properties combined with the other laws of optics, that is, with those of refraction and reflection, must necessarily be sufficient to explain completely all the modifications which the tints of the rings can experience, when they are made to undergo the refracting or reflecting action of bodies. If some newly discovered phenomena have appeared to elude these laws, it is because we have not sufficiently distinguished the causes which modify the intensity of light, from the causes which effect its decomposition in rings. This distinction is a very important one; for we have already observed that we may weaken the intensity of the rings, without changing the nature of their tints; and we shall see hereafter that this diminution of intensity may be carried so far as to make them completely and strictly disappear. But the laws we have established respecting the formation of the rings, are not invalidated by these phenomena, for they relate only to the mode of separation and distribution of the colours, and not to the absolute quantity of reflected light, which may be varied by causes totally foreign from these laws.

Newton, wishing to establish his discoveries respecting the rings in every possible way, has related a great number of facts apparently very singular, but which, when analyzed by his theory, are very easily accounted for. We shall not follow him into all these details; it is sufficient to have shown that all the phenomena which relate solely to the combination of tints, are necessarily comprehended in his theory. We shall, however, relate one of his experiments which shows that the alternations of reflection and transmission still subsist when we have gone much beyond the thickness, at which we perceive rings with compound light.

When we view thin laminæ of air, water, or glass, through a prism, we perceive many more rings than when we view them directly with the naked eye; insomuch that instead of 8 or 9, which we ordinarily observe, we can count 30, 40, and even more, which are very narrow and close to each other; and judging by their proximity and the space which they occupy, we may conclude that they follow each other in this manner to a much more considerable number, even to some hundreds. In order to understand the reason of this phenomenon, we must first attend to the most simple case, namely, that in which the rings are formed of a single kind of rays, the red, for example. Then, if we observe them formed upon a lamina, the thickness of which gradually increases from the centre to the circumference, such as those of water and air comprehended between two spherical surfaces, we shall see that the first rings are the farthest separated from one another; the differences of their diameters grow less and less, as they become more distant from the central spot; and lastly, at a certain distance from this spot they are so close to each other, that the eye can no longer discern any intervals, and they appear to compose one continuous colour. If we view these rings through a prism, they will not experience any sensible separation, because the rays which compose them being equally refracted by the prism, all their images appear equally displaced, at least if we neglect the small differences which arise from the unequal incidence of these rays upon the faces of the prism. If we now cause the assemblage of all the colours which compose white light to fall upon these same laminæ, we shall constantly perceive, if we still view them directly with the naked eye, much fewer distinct rings than before, owing to the superposition of these rings and to their encroaching upon each other, as we explained above when speaking of the experiments of Newton on this subject, made in the dark room. But if we view the laminæ through a prism, refraction will transfer all the images in the same direction, but to unequal distances, the violet rays which are most refrangible, being removed farthest, and the red rays least. Then if we suppose the prism placed as represented in figure 161, the consequence will be that in all the parts of the rings situated beyond the centre C, the violet rings, which before were by their nature smaller in each order than the red, will be found to approach nearer to them; and if the difference of dispersion is sufficiently great to compensate entirely for their primitive inferiority they may finally join them; and as the same will take place very nearly with respect to all the intermediate colours between the red and the violet rings, the result will be that all these rings, being thus concentrated together, will appear white and separated by dark intervals. The case will then be the same as if they had been formed by rays of a single colour, and consequently we shall be able to perceive a much greater number than before the prism was interposed. But in the position of the prism which we have supposed, this increase of distinctness will only take place with respect to the parts of the rings situated beyond their centre, relatively to the prism; and, on the contrary, with respect to the parts situated on this side the centre, the prism will only render them more confused; for its effect, in this part, will be to bring still nearer to the centre, the violet rings which before were the nearest, and thus remove them still further from the red rings; and this extension of the coloured fringes, by causing them to encroach more upon one another, will contribute to mix them more together, and will sooner compose an uniform white tint, in which no colour will be sufficiently distinct to be perceptible. It is evident, therefore, that if we wish to separate this part of the rings, we must change the direction of the prism. This is done in figure 162. But reciprocally, the parts situated beyond the centre will now become more confused and indistinct, as we find by observation.

277. Now since we are, in this manner, enabled to see rings in the part of the plate of air where we could not see them with the naked eye, it follows that such a plate may appear to the eye to be of a continuous and uniform white, when, in reality, the rays of light form rings there which the prism would separate and render sensible. This may be observed, not only in laminæ of air comprehended between two object-glasses, but also in soap bubbles; for before they have attained the degree of thinness necessary to reflect sensible colours, the prism enables us to discover in them concentric rings. In the same manner, laminæ of mica, water, or glass blown by means of a lamp, although they are not thin enough to appear coloured to the naked eye, exhibit, when viewed by the prism, a vast number of small irregular rings, which undulate in a thousand different ways upon their surface, according to the insensible inequalities of their thickness. And, as observed by Newton, we shall easily comprehend the reason of these phenomena, if we consider that all these rings, infinite in number, already exist in the laminæ when we view them with the naked eye, although on account of the extent of their circumference and the high order to which they correspond, they are so mixed and confounded together, that they appear to compose an uniform white; whereas the prism recovers them from this confusion by separating them. To perform this experiment in a satisfactory manner, we must place the thin laminæ over some

black body, and view them through the prism disposed as in figure 163.

In fine, all the phenomena described in Newton's theory of coloured rings, confirm us in the conclusions to which we were before led, respecting the nature of the luminous rays themselves; namely, that the colorific properties of these rays do not depend upon any alteration or modification produced in them by the media which they traverse; but that they belong to the nature of the rays themselves, and exist in them at the moment they emanate from luminous bodies; that they are transported with the rays to all distances indefinitely, and preserved without alteration in all media.

Fits of easy Reflection and easy Transmission.

278. After having established by experiment the fundamental laws which regulate the distribution and succession of the coloured rings formed upon thin laminæ, Newton recognised, by means of them, a new physical property in the particles of light, a property which not only accounts for all the particular phenomena observed by this great man, but also explains a multitude of other facts apparently of a totally different nature, and which were entirely unknown to him. In order to make it evident that this property results necessarily from the phenomena without the intervention of any hypothesis, I shall state in their order the propositions of Newton, adding only such explanations and remarks as are necessary to prove that they are general; and I shall afterwards show how each of them is a faithful expression of some phenomenon observed in the rings.

279. In the first place it must be remembered that the transmission of light is progressive. This fact was first made known by the eclipses of Jupiter's satellites, and has since been confirmed by the aberration of the fixed stars. These phenomena agree in proving that light employs 8′ 13″ in traversing the mean distance of the earth from the sun. We find, moreover, that the motion of light is uniform through the whole of this space, and indeed throughout the extent of Jupiter's orbit. Observations of this kind do not indicate a sensible difference in the velocities of the luminous particles of different refrangibilities; for if there

were a sensible difference in this respect, when a satellite entered the shadow or emerged from it, its disc would appear successively tinged with the different prismatic colours, which is not the case. The common velocity thus determined is that which light has in a vacuum; for the celestial spaces may be considered as destitute of all ponderable or refracting matter. When the luminous particles traverse media, the parts of which act upon them by attractions at a small distance, their velocity in these media is to their velocity in a vacuum, as the sine of incidence in the vacuum is to the sine of refraction in the material medium. Whence it follows that the velocity of light in bodies, is always greater than in a vacuum, and increases with their refracting power.

280. We are now prepared to attend to the new properties of light which Newton established as consequences of his observations upon thin laminæ.

First proposition. Every luminous particle, which traverses a refracting surface of whatever kind, acquires by this very act a certain transient disposition, which, from that time, during the whole course of the particle in the same medium, is periodically reproduced at equal intervals; and the consequence of each return of this disposition, is, that the luminous particle is *easily* transmitted through a second refracting surface, if one then presents itself; while, on the contrary, at each intermission of this state, it is *easily* though *not necessarily* reflected by such a surface. These successive states or dispositions, Newton calls *fits of easy transmission and easy reflection;* and the distance traversed by the particle between the returns of two fits of the same nature, he calls the *interval of the fits;* so that *the length of each fit* is half of one of these intervals. If these definitions be expressed analytically, we can predict the kind and stage of the fit, which a luminous particle will be found in at any instant whatever, in a given medium, when we know these elements for the instant of its entrance into the medium, and also the length of the fits for this particle. In the *Traité de Physique*, I have given formulas which express the dependance in question.

This is only a general enunciation of the fact and the law of the alternate transmissions and reflections, which occur at different thicknesses, in the same place, under each given incidence. Only Newton represents these alternations as indefinite, and attributes them to a physical property of the luminous particles, which renders them susceptible of being thus modified by the refracting surfaces of bodies; these are the two points which we are to examine.

281. When we observe the compound rings with the naked eye, we are unable to perceive more than 7 or 8 distinct successions or alternations; and we can predict beforehand, that in every other series of phenomena which follows similar laws, we can never perceive more. But the analysis, and if we may so express it, dissection, which we havemade of the phenomenon, has taught us that this limitation is owing simply to the encroachment and superposition of rings of all colours, formed by the different simple rays of which white light is composed. Accordingly, we have discovered a much greater number of rings, when we have formed them with a beam of simple light, and we have arrived at the same object in a still easier manner, by taking rings themselves compound, and separating them by the prism, in virtue of their unequal refrangibility. These observations prove that the alternations of reflection and transmission extend to thicknesses much greater than we at first suspected; and from the manner in which the rings crowd upon each other according as the thickness increases, we may conclude that the alternations are still produced, at thicknesses much greater than those at which we cease, even with the prism, to distinguish them. Indeed, other experiments of Newton, which may be seen in the *Traité de Physique*, show that these alternations exist in glass, at thicknesses which amount even to a quarter of an inch; and the same process might be employed to make known their existence at still greater thicknesses. Now, as such thicknesses exceed by many thousand times, the distance at which the variability of the attractive and repulsive forces of media can be sensible, we are obliged to conclude that the alternations of reflection and transmission, if continued so far, must be continued indefinitely.

282. Hence it becomes evident that these alternations depend upon some physical modification impressed upon the luminous particles, in their passage through the first refracting surface; and which they afterwards preserve throughout the whole extent of the medium which they traverse; for otherwise, when the luminous particles came to the second surface of this medium, their reflection or transmission would no longer depend upon

their distance from the first surface, especially at thicknesses where we know that they could no longer be subject to its action directly. The dependance of the particles upon this first surface, with respect to the power of being reflected and transmitted, when they are no longer directly influenced by it, proves necessarily the existence of a durable modification, which they receive conformably to the enunciation of Newton.

It is important to observe that these modifications of the luminous particles are not attended with an absolute necessity of being transmitted or reflected at certain intervals, but only a disposition or *facility* for one or the other of these effects; for very thin laminæ of mica or glass, which, when in the air, reflect very vivid colours, exhibit, when wet at their second surface, the same colours, but much fainter; so that a certain number of luminous particles, which before were reflected by this second surface, are transmitted, when its repulsive force is weakened or counteracted by the presence of the exterior medium. This phenomenon takes place at the second surface not only of thin bodies, but also of thick ones; since the intensity of reflection is equally diminished in these, when we bring them in contact with a medium, the refracting power of which, whether greater or less, differs less from their own than that of air or a vacuum. In this case also, as in the preceding, the reflection, whatever be its intensity, always takes place in the system of colours more or less compound, which the body is capable of reflecting at its second surface, according as it is thin or thick. In the same manner, when a ray of white light, after having traversed a certain space of air, encounters the surface of another medium, as water for instance, a part of the luminous particles which compose this ray, is disposed to be reflected at the common surface of the water and air; and, if the thickness of air is very great compared with the thicknesses which give coloured rings, there is, as we shall prove hereafter, nearly an equal quantity of particles in each of these two dispositions. Accordingly, the reflection is abundant, if the second surface of the air is limited by a thick plate of water; but it becomes very feeble and almost insensible, when the thickness of this plate does not exceed the millionth part of an inch; for, according to what we have observed in soap bubbles, such a plate does not reflect any light at the second surface, and reflects almost none at the first. In this case, therefore, the disposition which the incident particles may have for reflection, when they arrive at this first surface, is without effect; and consequently it is attended with only the facility, and not the necessity of being reflected. In order that the reflection may take place, it is not enough that the luminous particle be favourably disposed for reflection, but the energy of the reflecting force must be such, that in this favourable state of the particle, it shall destroy completely its velocity and turn it into a contrary direction. In like manner, the tendency to transmission ceases to be effectual, when the reflecting force is sufficiently energetic to repel the particles which have this tendency; and this is the case in very oblique reflection from most polished bodies, particularly metals, since, in this case, they reflect, more than half of the incident light, as we shall hereafter prove. From these examples, we perceive that Newton rigorously followed the phenomena when he gave to the fits denominations which might indicate, not an *absolute* necessity of reflection or transmission, but a *conditional disposition*, as indicated by the phrases *fits of easy reflection and easy transmission.*

283. It is to be observed moreover, that when a ray of simple light as it comes from a luminous body, falls directly, or by refraction, upon a refracting medium, into which it penetrates, the luminous particles transmitted, which are all actually brought into the state of easy transmission by traversing the refracting surface, do not all possess this disposition in a manner equally perfect. Indeed, if this equality were to take place, as the fits of all the particles would afterwards be of equal lengths and equal velocities, the ray being supposed homogeneous, it is evident that throughout the whole distance they must all be modified exactly in the same manner; and, consequently, when they came to the second surface of the refracting body in which they were moving, they must either be reflected or transmitted all at once. But this never happens; for even if we choose the most perfectly homogeneous light, there will never be reflected more than a certain proportion, depending on the nature of the refracting body and that of the surrounding medium.*

---

* This division not only takes place in the case of thick bodies, but also in that of thin laminæ; for, in observing the coloured rings formed by reflection at their second surface, we remarked that the reflection was far from being total, even in the middle of each simple ring.

Since then two ambient media of unequal refracting powers, being successively applied to the second surface of the same body, determine the more or less abundant reflection of the same homogeneous ray, which arrives at this surface, it is necessary that some cause or other should determine the preference of the luminous particles, which yield at once to the weakest of the two reflections; and since this preference takes place among particles perfectly homogeneous, it must be owing to some inequality in the physical dispositions which the particles bring with them, for reflection or transmission at the same surface. Therefore, as these particles have equal velocities and fits of equal lengths, it is absolutely necessary that their inequality should be traced back to the first refracting surface, and thence in its principle, to their first emission from the luminous body.

In order to discover in what this inequality consists, we must consider that when the luminous particles pass from a fit of easy transmission to a fit of easy reflection, whatever be the nature of these fits in other respects, it is extremely probable that they do not take place in an abrupt and sudden, but in a gradual and progressive manner, losing by little and little their disposition to be transmitted, and when it is entirely gone, immediately acquiring a contrary disposition, that is, a tendency to be reflected; this is at first very feeble, but gradually increases to a certain maximum, after which it diminishes by the same gradations. Now let us suppose that an infinite number of luminous particles, homogeneous and emanating simultaneously or nearly so, from a luminous body, are found, in departing from its surface, to be in all the different stages of the two kinds of fits, either in virtue of the very act of emission, or as being parts of the different points of the infinitely thin, incandescent plate, through which the radiation may always take place; this will be sufficient to produce, throughout the whole range of these particles, the differences of disposition which experiment makes known to us. Indeed, when they arrive together at the first surface of the reflecting body, which reflects a part and refracts the rest, the reflection will undoubtedly take place, from preference, with respect to those incident particles which are found most disposed to it. If, for example, the reflecting force were infinitely great, all these particles would be reflected, whatever fit they might be in; if the reflecting force were less, the particles which at the moment of their incidence were in a fit of easy transmission, would be the first to transmit themselves; and we can conceive of such a degree of energy that these would all be transmitted, while those which were in a fit of easy reflection, would all be reflected, whether they were at the commencement, in the middle, or toward the end of this fit. Lastly, with a reflecting force still more feeble, those which were nearly at the commencement or the end of such a fit would be transmitted, and those only which were in the most energetic part of the same fit would be reflected. The number of these last would thus go on decreasing with the intensity of the reflecting force, to the last degrees of this force, when there would be reflected only an infinitely small number consisting simply of those particles which were precisely in the middle and strength of their fit. In all these cases, from the moment when a certain number of luminous particles were transmitted through the first refracting surface of a body, they would all find themselves brought by this very act, to the state of easy transmission; but they would take this state in a manner more or less complete, according to the more or less favourable dispositions they were in at the moment of their incidence, and this inequality continuing through the whole extent of the refracting body to its second surface, would determine the choice of the particles which should there be reflected, from preference, by a given repulsive force.

284. In order to render this completely evident, let us represent by *AB* the length of a fit of a certain species of luminous particles in a given medium, and let us designate the variable energy of each fit, at its different stages, by the ordinates of a curve *AMB*, the nature of which is entirely arbitrary. This curve must pass through the points *A* and *B*, the extremities of the fit; its ordinates at these points must be nothing, and from this they must continually increase by equal degrees to the middle *O* of the fit, where they will attain their maximum *OM*. If we wish to extend this construction to all the alternate fits which the particle experiences in the whole course of its passage through the given medium, we have only to take upon its direction, beginning at the point *B*, a series of lengths *BC*, *CD*, ... equal to each other and to the length *AB* of the first fit. Then, if we construct upon each of these the curve of intensity *AMB*, alternately on one side and the other of the axis *ABCD*, the sinuous Fig. 164.

line *AMBM′C* . . . . which results, will express the kind and intensity of the fit with which the particle is affected, in any point whatever of its course. Now let us suppose that a ray formed entirely of luminous particles like these, falls upon the first surface of the medium, and is in part transmitted; then all the particles transmitted will be brought by the refracting surface into a state of easy transmission; but according to what we have discovered of their primitive inequalities, they will not all come into this state in the same manner; and as their number is supposed infinite, they may exhibit all the imaginable stages of it from the commencement to the end. Let us construct then for each of them the series of successive fits, as they are continued, Fig. 165. in departing from the refracting surface *SS*, and we shall see what must take place when a second surface is presented successively at different distances from the first.

Note

285. To begin with an extreme case, we will, in the first place, suppose that the medium contiguous to this second surface is such that the reflection takes place with respect to *all* the luminous particles which are presented to it, in any stage whatever of the fit of easy reflection. The possibility of this, as we have seen above, depends wholly upon the ratio which exists between the refracting forces of the body and of the contiguous medium. If then, in this construction, we cause a straight line *S′S′* to move parallel to *SS*, for the purpose of representing the limit of the second surface, it is evident that the reflection will never become entirely nothing, however small the thickness given to the refracting body, unless this thickness be strictly nothing; for, from the moment the line *S′S′* moves from *SS*, there will be a certain number of luminous particles in a state to be reflected. This number, at first very small, will progressively increase as *S′S′* withdraws from *SS*, till at length, this line having arrived at I_1, a distance from the surface equal to i, the length of a fit, all the light transmitted will be in a state of easy reflection, and consequently the reflection will be total. But this will take place only at the precise distance in question; for, from the instant that *S′S′* moves farther from *SS*, there will be a certain number of particles which will pass to the state of easy transmission, and will consequently escape the reflecting forces, since we have limited the possibility of reflection to those which are found in some stage or other of the contrary state. Thus, the proportion

of light reflected will gradually diminish according as $S'S'$ removes from I_1; and will finally become nothing at I_2, a distance from the first surface equal to $2i$. From this point, the reflection will begin again to increase according to the same periods; it will be total at I_3, nothing at I_4, and so on through the whole extent of the refracting body; hence all the phenomena, both of transmission and reflection, will be limited by the successive terms of the two following series;

Total transmission $0 \quad 2i \quad 4i \quad 6i \ldots.$
Total reflection $i \quad 3i \quad 5i \quad 7i \ldots.$

The first indicates the precise thicknesses at which the reflection is strictly nothing, and the second those at which it becomes total.

286. If, instead of supposing the two surfaces of entrance and emergence exactly parallel, we suppose them inclined to each other at an angle so small that the intensity of the reflecting forces may not be sensibly changed, the simple variation of thickness resulting from this inclination, will produce between the first and second surface, all the varieties of distance which we have supposed, as well as all the alternations of reflection and transmission which result from it; that is, the light transmitted perpendicularly through the first surface, will be reflected or transmitted at the second, according to the thickness through which it passes to meet it. Accordingly, this second surface, seen by reflection or transmission, will present, in its different points, luminous alternations, exactly analogous to the different orders of rings formed of homogeneous light, upon the laminæ of water or air, contained between two object-glasses. Only, from the energy which we have here attributed to the reflecting forces, it will be found that the luminous zones, transmitted and reflected, will have equal breadths, and will be separated in each series by black lines infinitely narrow, two circumstances which do not take place in the coloured rings produced by the refracting bodies which nature presents.

The reason is, that these bodies have not, with respect to the media which surround them, a repulsive force so powerful as we have supposed; for, whatever proportion we give to their thickness, the reflection at their second surface is never total, at least under a perpendicular incidence, the only case at present under consideration. Consequently, when this second surface is at one of the distances suited to produce the most abundant reflection, as I_1, I_3, I_5, ... in which case all the incident light is in a state of easy reflection, there is still a certain number of particles which elude the reflecting forces, in spite of the favourable disposition they are in; whence it will be seen that this disposition is effectual only beyond a certain degree of energy. If, for example, we suppose the whole of the incident light to be distributed equally among all the stages of the fit, and there be one half which escapes reflection, this will be a proof that the reflection takes place only in the particles which have passed through more than one fourth and less than three fourths of their fit; if there be one fourth which escapes, the limits of the reflection will be $\frac{i}{8}$ and $\frac{7i}{8}$, and so of the rest. In general, if we designate the first of these limits by e, the second will be $i - e$, since they must always be equally distant from the middle of the fit. This being settled, if we apply to these new conditions, the geometrical construction above used, it is evident that the intensity of the reflection will be nothing, not only when the second surface $S'S'$ coincides with the first SS, but also through all the interval of thickness, where it is distant by a quantity less than e. At this precise thickness, only one of the refracted particles will be capable of being reflected; this will be that, which, at its entrance, had terminated a fit of easy transmission; for, having then commenced a fit of easy reflection, when it afterwards arrives at the thickness e, it will have made an advance equal to e, in the period of this fit, and will therefore be capable of being actually reflected; and it will continue so from this point to the thickness $i - e$, where it will be at the same distance from the end of its fit. The other particles, which were originally less advanced in their fit of initial transmission, must arrive successively at thicknesses greater than e, in order to begin to be reflected; so that from e the intensity of the reflection will always increase to $i - e$, where the first particle begins to pass from the efficacious part of its fit. Beyond $i - e$ this particle will be transmitted; but since we suppose all the particles, at their entrance, uniformly distributed among the different degrees of transmission, there will be at this very instant another particle, originally less advanced in its state of initial transmission, which will have attained the proper degree of reflectibility, and will be reflected in place of the first; the intensity of reflected light will then become constant at this thickness, and will continue so to $i + e$, where the reflection begins for that luminous particle, which, at its entrance into the body, had commenced a fit of easy transmission. But beyond this limit no new particles will present themselves to replace those which have successively escaped reflection; so that the intensity of the reflected light will begin to decrease, and continue decreasing to $2i - e$, where it will become nothing, there being then no refracted particles in the efficacious part of a fit of easy reflection. Consequently, the transmission will then be total, and will remain so to the thickness $2i + e$, where a new period of reflections will commence, and so on throughout the whole extent of the medium.

287. If we would realize these alternations upon a single lamina of a refracting body, it is sufficient to give to the two surfaces a very slight inclination to each other; and by causing a portion of simple light to fall perpendicularly upon the first, we may observe by reflection upon the second, an alternate succession of luminous and black stripes, limited to the different thicknesses which our calculation has assigned. This is precisely the phenomenon of the reflected rings formed by homogeneous light. We may also observe, by transmission, alternations of luminous and dark stripes analogous to the transmitted rings; but the dark intervals will be nowhere entirely black, since there will still be a certain portion of light transmitted even in the mean thicknesses i, $3i$, $5i$ . . . . at which the reflection is most abundant.

We can calculate the proportion of light transmitted in these places, when we know the initial state of each of the refracted particles, and the distance at which reflection begins and ends toward the extremities of each fit. We thus find, that in proportion as the reflecting forces become weaker, the intervals of thickness occupied by the lucid rings diminish, and become less different from the intervals of transmission, to which they are exactly equal when the reflecting forces become infinitely feeble. This case is the limit of those which our experiments can present; but the feebleness of the reflecting power of most transparent bodies, under a perpendicular incidence, presents it almost exactly. This is the cause of the equality remarked by Newton, and which appeared to him to be sufficiently exact to satisfy him without proceeding farther.

288. In what we have last said, we have considered the refracted light, as being, at its entrance into the body, uniformly distributed among all the possible degrees of a fit of easy transmission. It is possible that this uniformity may not rigorously exist, and I may add, it is very probable; for refraction never takes place except in sequence of an exterior reflection, which must suffer to pass, especially those incident particles, which are in the most energetic stages of a fit of easy refraction, and which, consequently, if they were refracted, would be the less disposed to take a contrary state; whence it may be presumed that the transmitted light, at the moment of its entrance into the refracting body, contains a predominant proportion of particles in the most energetic stages of a fit of easy transmission, and perhaps very few or none, which have only reached the feeblest stages of the beginning or end of such a fit. This very probable circumstance tends also to equalize the intervals of thickness e, $2i - e$, at which the reflection and transmission take place. It is attended with no other modification to any of the conclusions we have established, than that which results from another law of intensity, in the proportions of reflection, at the different periods of each lucid ring.

289. Hitherto we have supposed the light, whether reflected or refracted, to be composed of a single species of luminous particles. If we suppose it to contain different species, each one will proceed by laws strictly similar, only the fits will not be of the same length for all. It remains then to determine them. Such is the object of the following propositions established by Newton.

When the luminous particles which form the eight limits of the colours of the spectrum, after having traversed the same refracting surface, enter into the same medium under a perpendicular incidence, or in general under a common incidence, the intervals of the successive fits of easy transmission and easy reflection, belonging to these different species of particles, are to each other exactly, or very nearly, as the cube roots of the squares of the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, which roots have respectively for their values 1; 0,9243; 0,8855; 0,8255; 0,7631; 0,7114; 0,6814; 0,6300; the first representing the length of the fit belonging to extreme red; and the luminous particles whose nature is such as to be comprehended between these several limits, have likewise their intervals of fit comprehended between the several numbers thus calculated. This is a mathematical consequence of the ratios before found by experiment, between the thicknesses of the plates which reflected or transmitted the different colours of the same ring, under a common obliquity.*

---

* We take this opportunity to point out the remarkable relation that exists between these numbers, and which was first noticed by M. Blanc. If we multiply together the two extreme terms 1 and $\frac{1}{2}$, or any two others taken at equal distances from these extremes, as $\frac{8}{9}$ and $\frac{9}{16}$, $\frac{5}{6}$ and $\frac{3}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$, the product will be $\frac{1}{2}$. Consequently, if we multiply in the same manner the values of the fits corresponding to these terms, we shall ikewise have a constant product, equal to the cube root of the square of one half, or 0,6300. This may be verified by the numbers given above, as expressing the lengths of the fits for the different limits of the simple colours. Accordingly, if we conceive all the colours of the spectrum distributed in their order round the circumference of the same circle, in a manner analogous to that by which Newton calculated their mixtures, and which is explained in article 169, the length of fit corresponding to the number of degrees n, being multiplied by the length of fit corresponding to $360^\circ - n$, must give for a constant product $\frac{1}{2^{\frac{2}{3}}}$ or 0,6300, in conformity with the preceding remark. The simplest way of satisfying this condition, is to represent the value of any fit by $\frac{1}{2^{\frac{2n}{3c}}}$; c representing the entire circumference, and n the number of parts, to which the particular species of simple rays we wish to consider, corresponds. If we make this expression successively equal to the values of the fits which correspond to the different limits of the simple colours, we shall obtain the values of n, which belong to these several limits. If, for example, we represent the entire circumference c, by the fraction $\frac{79}{120}$, as Newton did in the place above cited, we find that the respective portions of this circumference which the several colours occupy, beginning with the red, are expressed by the fractions $\frac{1}{8{,}939}$; $\frac{1}{16{,}318}$; $\frac{1}{9{,}993}$; $\frac{1}{8{,}989}$; $\frac{1}{9{,}993}$; $\frac{1}{16{,}318}$; $\frac{1}{8{,}939}$. These differ very little from the fractions $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{10}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{16}$, $\frac{1}{9}$, which Newton

290. As yet we have considered only a single medium. In order to extend the definitions of the fits from one medium to another, Newton established the following proposition.

If the luminous particles, of whatever species, pass *perpendicularly* into different media, the intervals of the fits of easy transmission and reflection, in any two of these media, are to each other, as the sine of incidence to the sine of refraction, when the particles under consideration pass from one to the other. This is the mathematical generalization of the ratios before observed between the thicknesses of water and air, which reflect or transmit the same tint under a perpendicular incidence. Hence it follows, that for each kind of luminous particles, the length of the fits, *under a perpendicular incidence*, is always the same in the same medium, whatever be the bodies traversed before coming to it.

According to this rule if we take for unity the thickness of air, which, when viewed perpendicularly, reflects a certain tint, the thickness of a vacuum which reflects this same tint would be greater in the proportion of the ratio of refraction of air, that is, as 3388 is to 3389; so that the nicest precision would hardly detect a difference. Accordingly, when we have formed coloured rings by compressing a plate of air between two object-glasses, the size and colour of these rings do not appear to un-

---

employed; and if we reduce them to degrees, as in article 169, the lengths of the arcs thus obtained will be;

$RO = 61^\circ\ 10'\ 23''$
$OY = 33\ \ 31\ \ 10$
$YG = 54\ \ 43\ \ 15$
$GB = 61\ \ 10\ \ 23$
$BI = 54\ \ 43\ \ 15$
$IV = 33\ \ 31\ \ 10$
$VR = 61\ \ 10\ \ 23$

These arcs differ so little from those of Newton, that we might substitute them in their stead, without making any appreciable alteration in the estimates of the compound colours. Thus we see that this mode of distribution, so singular in appearance, is closely connected with the values of the fits. Whether Newton was aware of this connexion and followed it, or whether it is derived solely from the nature of the numerical results, which the experiment furnished him, is not known.

dergo any change, if we put the glasses into a vacuum, or heat them strongly in order to expel the air from between them. Mazéas, who first made these trials, was very much astonished to find no difference, and this circumstance did not fail to be urged as an objection to Newton's theory; whereas it is, as we perceive, a consequence of it.

291. By means of the two last propositions, the length of the fits, under a perpendicular incidence, will be determined generally for every species of refracting medium and of luminous particles, if their value for a single case be ascertained. This may be easily done from the observations of Newton upon the thicknesses of air, which reflect or transmit any colour whatever under a perpendicular incidence. Let us take for an example, the luminous particles which form, on the spectrum, the limit between the yellow and orange. We have found that the alternations of their transmission, and reflection expressed in parts of an inch, succeed each other at the mean thicknesses which follow;

Transmission . . . . 0 $\frac{2}{178000}$ $\frac{4}{178000}$
Reflection . . . . . . $\frac{1}{178000}$ $\frac{3}{178000}$ $\frac{5}{178000}$.

Accordingly, *the length of a fit* for this species of light will be $\frac{1}{178000}$ of an inch; and double this quantity or $\frac{2}{178000}$, which becomes $\frac{1}{89000}$, will be *the interval* of two fits of the same nature, whether of transmission or reflection. If we combine this result with one of the preceding propositions, we shall obtain the intervals of the fits for the different species of particles, which form the eight limits of the colours of the spectrum.

The following table contains their values in millionths of an inch.

Colours.		*Length of the fit for different colours.*			
		In a vacuum.	In air.	In water.	In glass.
Extreme violet		3,99816	3,99698	2,99773	2,57870
Limit of	violet and indigo	4,32436	4,32308	3,24231	2,78908
	indigo and blue . .	4,51475	4,51342	3,38507	2,91188
	blue and green . .	4,84284	4,84142	3,63107	3,12350
	green and yellow	5,23886	5,23732	3,92799	3,37891
	yellow and orange	5,61963	5,61798	4,21349	3,62450
	orange and red . .	5,86586	5,86414	4,39811	3,78331
Extreme red		6,34628	6,34441	4,75831	4,09317

292. The numbers which express the fits in air, are deduced from the table of article 264, by doubling all the values of e, belonging to the limits of the different colours. The numbers of the other columns are obtained by multiplying these first results by $\frac{3380}{3381}$ for a vacuum, $\frac{3}{4}$ for water, and $\frac{20}{31}$ for the glass which Newton used.

We are now to connect together the reflections and transmissions which take place for the same ring under different obliquities. For this purpose Newton modified the intervals of the fits, according to the table of article 246, or rather according to the law which he deduced from it and which he found to be applicable to water and to all other substances, as explained in article 258.

Although these ratios were established by observations upon curved plates, the application which Newton here makes of them is not the less legitimate, because the thicknesses compared were all deduced from measurements taken upon the transverse diameter of the rings; so that the luminous rays which limited each diameter, traversed the thin plates at points where the tangents of its two surfaces were sensibly parallel; this rendered constant the thickness which separated them.

But the values of the oblique fits, obtained from these observations, could not be employed, if the two surfaces traversed by the ray, were so much inclined to each other that the lengths of the fits might be sensibly different at the entrance and emergence. The experiments of Newton do not decide how the fits would in that case change near the second surface, and it is a point yet to be inquired into.

293. The preceding definitions, obtained from actual experiment, characterise all the modifications which the fits experience in the act of refraction. Those which they receive from reflection yet remain to be determined; but the observations hitherto related, will not serve to solve this problem, because the thinness of the plates employed, prevents us from observing separately the influence to which the rays are subjected in them before and after being reflected at their second surface; or at least, the indications to be furnished by such experiments cannot be perceived till after we have separated, by some other method, the different actions thus produced. This Newton has effected by a new series of observations. He has rendered

the rings sensible upon thick plates, in which the two paths of the rays, before and after the interior reflection, could thus be distinguished. From these new phenomena he derived the following proposition.

When the luminous particles of any species whatever, having arrived at the second surface of the body in which they move, experience either the specular or radiant reflection, they take after reflection new fits, in departing from the reflecting surface; and the lengths of these fits are the same as they would have been, if the particles, coming from the exterior medium to the body where they are found, had entered this body with the obliquity derived from reflection. This proposition completes the characteristics of the fits.

Application of the preceding Theory to the Reflection of Rays of Light which have traversed thick Media.

294. The fits of luminous particles being completely defined by the preceding considerations, we come now to develope the consequences which result from them with respect to the reflection and refraction of light at the second surface of thick bodies, in order to see whether these consequences are conformable to observation.

To begin with the sources of these phenomena, let us first consider a luminous body placed in an indefinite medium, like the air; and following in imagination the different luminous particles which emanate from it, let us see what must be their tendency to reflection or refraction, at every distance. For the solution of this problem, we must have given the nature of the medium, that of the luminous particles emitted, the direction of their introduction, and the initial state of each of them at the instant it escapes the action of the radiating body. With the two first data, we can calculate the length of the fits of each particle, and by adding this length continually to itself, beginning with the primitive position and state, we shall have all the successive returns of the same state or the opposite one. Then, if we place at any point whatever a surface whose reflecting force is given with respect to the medium which surrounds it, we can determine from the state of each luminous particle, whether it will yield or not to reflection. These will be the modifications belonging to each particle. If afterwards, we wish to predict the phenomena as to colour that will arise from their mixture, we can do it by combining their colorific properties according to the method of Newton already employed for a similar purpose.

295. But this combination will not be necessary, except when the medium traversed by the light is extremely thin; for, if it has sufficient extent to allow the least refrangible of the particles to experience in it only 12 or 15 fits, the effect of reflection will become sensibly constant, at least with respect to our senses, and the reflected ray will always appear of the same colour as the incident light. This is the case with respect to reflection at the second surface of thick bodies.

To understand the cause, we must remember that in the general division of the spectrum a certain extent is occupied by the violet, another by the indigo, another by the blue, and so on for the seven principal colours; that is, the sensation of each of these colours does not strictly belong to one single class of rays whose refrangibility is mathematically fixed, but may be excited by rays whose refrangibility is in a slight degree different, yet so nearly alike as to make us confound them. Accordingly, in the phenomena of colour, we may consider together, the effects of any one of these different groups. Beginning, for example, with the violet, let us suppose that the light emitted contains only the varieties of particles which are capable of producing the sensation of this colour, and let us suppose that all these escape from the luminous body at the same time, and in the same stage of a fit of the same nature. Then their unequal refrangibility will give to their fits unequal lengths; and, according to the table of article 291, if the length of the most refrangible is 3,99698, that of the least refrangible will be 4,32308, which gives for each fit a difference of extent equal to 0,3261. Consequently, at the same distance from the luminous body the most refrangible of the violet particles will have experienced more alternations than the others; and it is easy to perceive that after 13 fits and about a quarter, the difference 0,3261, continually repeated, will have become equal to 4,32308, that is, to an entire alternation; so that when the rays of one description are in a fit of easy transmission, the other will be in a fit of easy reflection.

Thus, at this distance from the radiating body, there will be violet particles intermediate between the preceding, which will be in all possible stages of the two kinds of fits. If then a reflecting surface be met in their passage at this distance, provided its repulsive force is not strictly nothing, there will be a certain number of violet particles, which will undergo reflection, the rest undergoing transmission.

296. This result may be rendered sensible by the same construction that Newton employed to represent the relations between the thicknesses of bodies and the colours which they reflect. We have only to consider the divisions 1–3, 3–5, of each vertical line, as representing the lengths of the fits of the luminous particles to which this vertical belongs. In the particular case represented by the figure, the common point of departure of the luminous particles is fixed in *ZR*, at the the middle of a fit of transmission; and each of the equal divisions contained between two transverse lines 00′11′, 11′22′, 22′33′ . . . . . represents the length of half a fit. Then the value assigned above for the number of fits which produces complete inversion, falls between the 26th and 27th division of the first column *ZZ′*. Now if we draw through this point a line *XX′*, parallel to the axis *ZR*, we perceive that it in fact cuts the vertical *VV′* almost exactly in the point where it is cut by the transverse line answering to 24 24′; that is, at this distance from the point of departure, the most refrangible of the violet particles are brought nearly to the middle of a fit of easy reflection, while the least refrangible particles having experienced one alternation less, are nearly in the middle of the fit of easy transmission which immediately precedes. Fig. 157.

If we repeat the construction for any other more considerable thickness, we shall always find in this manner violet particles in the same stages of two consecutive fits, and consequently in states exactly opposite; but, in consequence of the lengthening of the squares occupied by the several colours, the opposition will relate to particles comprehended between the extremes of refrangibility.

297. It will be found, moreover, that beyond these limits the same parallel may pass over squares of different orders belonging to the same colour; so that, for the same thickness there will be found in these different orders, some particles of the same

colour which may be reflected, and others which may be transmitted. Lastly, if we continue to increase the thickness, the number of orders of the several colours, which will mix together, and the lengthening of their squares will both become so great that they will always offer to the reflecting forces a sensibly constant quantity of light, the particles of which will be in all possible stages of the two kinds of fits. Hence, for each given value of these forces, the intensity of reflection will become constant at all greater thicknesses, and the particles which present themselves being distributed among all the possible stages of the fits of different orders, the total quantity of light reflected will be equal to that which the same surface of the same media would reflect in the whole extent of one order, that is, in the entire space of one simple ring.

What we have now said with respect to the violet particles will apply equally to the other divisions of the spectrum. With respect to each of these, then, there are limits of thickness beyond which the number of reflected particles will become constant, for each given value of the reflecting forces, and equal to the total number which these same forces would reflect in the whole space of one simple ring.

Now if we collect together all the quantities of the seven kinds of light thus reflected in each simple ring, they will form white; white then is the colour that will be reflected by all reflecting surfaces, whatever be their nature when they are placed at distances from the luminous body sufficiently great to allow the whole diversity of fits time to display itself. Moreover, the intensity of this white will then become constant for each kind of reflecting body at whatever distance we place it; but it will be different for different bodies, according to the greater or less energy which their own nature and that of the surrounding medium give to their reflecting power.

298. These results are completely verified by observation; when a ray of light has traversed a wide space of air, if we receive it upon a polished body of any colour, whether white, black, grey, green, yellow, or red, the portion which is reflected specularly, making the angle of reflection equal to the angle of incidence, is always white. It is true, that if the reflecting body is sufficiently dense, it sends off likewise a sensible portion of coloured light, which it scatters in every direction, as if by

a proper radiation. But this dispersion indicates a mode of reflection different from the first; and it may be shown that the portion of light thus reflected has acquired particular characteristics, which prove that it has penetrated into the reflecting body, and that it is reflected from a certain depth in its substance. We cannot, therefore, apply to it simply the considerations drawn from the succession of fits in the first medium, these being adapted only to those particles whose reflection takes place in this same medium, merely by the influence at a distance of the reflecting body. Now it is a fact that this first reflection always gives a ray of the same colour as the incident light, conformably to the theory, and without any regard being had to the nature of the reflecting body.

In order to be completely assured as to the distinct characteristics of these two kinds of reflection, we take two opaque vessels, the insides of which are rendered black, that they may send no coloured light to the eye, we pour the most limpid water into one, and the blackest ink into the other, and then view external objects by reflection from these two liquids. We shall see these objects in their ordinary colours, and there will be no appreciable difference of colour as they appear from the two liquids. Here then are two bodies, of which one appears transparent and the other dark, yet each reflects all the colours. The act, therefore, by which this first species of reflection is produced, is entirely independent of the property which the body may have of appearing coloured to us. The same will take place, if, instead of ink, we pour into one of the vessels solutions of carmine, indigo, or any other colouring substance. These liquids will all reflect similar images. Yet if we were to consider their mass, they would have very different colours; the solution of carmine would appear red, and that of indigo, blue. It is true that if these solutions were very highly charged, they might at length tinge the images of objects with their own colours, and more strongly as they were more concentrated; but this addition would at most only weaken a little the intensity of the white produced by the first kind of reflection, without desstoying it; and, in general, it would not sensibly alter it. Thus, a cylindrical stick of sealing-wax of the brightest red, when exposed to the white light of the sky, reflects through its whole length a line of white light which always appears on those parts of its surface where the rays can be sent to the eye, making the angle of reflection equal to the angle of incidence; and the brightness of this line is such, that at the places where it is observed, we can hardly distinguish the proper colour of the wax, which at other times affects us so vividly.

In these different examples, the portions of incident light which give the seven principal colours, after traversing a sufficient thickness of the same medium, which is air, are equally disposed to reflection; this produces the identity of colours reflected *at a distance* by all kinds of bodies. But each body, when applied to the same medium, sends a greater or less number of particles of each colour, according to the stages of their fits at which it is capable of reflecting them. Hence the intensity of reflection varies with different bodies, while the colour reflected remains the same. This variation of intensity may also be obtained with the same body applied to the second surface of the same thick medium. For this purpose, it is sufficient to reduce the thickness of this body until it becomes less than the distance at which the reflecting forces are sensible; for then, as the plates taken away, diminish its reflecting power, a portion of the particles which it would have reflected before must now escape, being too far from the middle of their fit of easy reflection, to yield to the degree of force which remains. The intensity of reflection which it is capable of producing, will therefore, continue to diminish, as we reduce its thickness. When these are so thin as not to reflect light sensibly at their second surface, they do not appear absolutely black; we observe still at their first surface, a feeble reflection, which produces a white ray, if the incident light is white, and which, in general, does not change the natural colours of objects.

299. These constant rules by which reflection is determined in all possible cases, from the state of the luminous particles when they arrive at the reflecting surfaces, furnish unanswerable arguments to prove that reflection is not produced by the collision of the luminous particles, impinging against the matter of bodies. For if this immediate action were the cause of the phenomenon, it would become more easy and more frequent according as the thickness of bodies increased, till they reached the term of opacity. And if there happened to be a certain thickness at which all the particles of light were transmitted, it

would be impossible that, at less thicknesses, any should be reflected. Now we have seen that the contrary takes place when we illuminate a thin plate of any substance with a single kind of simple light; for, if the thickness of this plate is variable, luminous rings are formed upon it by reflection which are separated from one another by dark intervals. Moreover, on the hypothesis of a collision, it is impossible to conceive how the same thickness of water, oil, air, or glass, when exposed to light under a certain incidence, should present a sufficient number of material particles to reflect certain kinds of rays, while under a different incidence, they transmit these and reflect other kinds. Lastly, when two plates of glass touch each other, or very nearly touch, we cannot, on the supposition of a collision account for the fact, that there should not be at the point of contact and for some distance about this point, a sufficient number of vitreous particles to reflect a sensible quantity of light, while enough are found immediately beyond, when the plates are only a slight degree more distant from each other. All these phenomena, which harmonize perfectly with the hypothesis of reflection at a distance, and are indeed necessary consequences of it, become so many physical impossibilities, on the hypothesis of an immediate contact between the particles of light and those of the reflecting bodies.

Explanation of the proper and permanent Colours of Bodies.

300. After having analyzed all the phenomena of specular reflection, whether in thin plates or in thick media, Newton endeavoured to comprehend radiant reflection under the same laws. It is in virtue of this property, that bodies always appropriate to themselves a certain portion of incident light, which they afterwards send out into space as if by actual radiation, and which, having always a tint peculiar to each substance, determines the proper colour which it appears to us to possess. In order to understand how this separation can take place, it is to be observed, in the first place, that a variety of chemical and philosophical experiments agree in proving that the material particles of bodies have dimensions exceedingly small, compared with the distances which separate them. It is even possible that

in those bodies which appear to us most dense, the extent of the interstices may exceed, by many thousand times, the bulk of the elementary and impenetrable particles, which constitute the material elements. Indeed, if we suppose that the ultimate particles of bodies are united in groups of two and two, three and three, four and four, or a greater number, in such a manner that in each group, there shall be certain intervals between the particles, and that the different groups shall have much larger intervals between them; these groups themselves may be considered as united in their turn, two and two, three and three, four and four, so as to form groups still larger, and separated from one another by still greater distances. Now if we conceive the elementary particles to be very dense, we may, by multiplying thus the successive orders of groups, compose systems which shall present all the degrees of density and rarity we wish. If we suppose, for example, in each order, the sum of the spaces comprehended between the groups to be only equal to their total bulk, a body which has only one order of such groups, will contain only $\frac{1}{2}$ of its bulk of matter; with two orders, it will contain only $\frac{1}{4}$; with three, $\frac{1}{8}$; with four $\frac{1}{16}$; with five, $\frac{1}{32}$; and so on; so that if we continue to multiply the orders of different groups, we shall form systems of an indefinite rarity. It is thus, that in the celestial spaces, the particles of a planet, although separated from each other, form a group of a certain density, which constitutes the body of the planet. Several planets, infinitely distant from each other, compared with the intervals between their particles, but infinitely near, compared with the distances of other bodies in the universe, form a rarer system, a group of a more compound order. An assemblage of such systems, separated from one another by other intervals, almost infinite when compared with the orbits of each planet, forms another system still more rare, of which nebulæ furnish an example. Finally, we may still conceive of assemblages of nebulæ, and so on, without end.

301. It being once granted that the particles of bodies are placed at a distance from one another, this mode of constitution is evidently the most general that we can imagine. Now in order to explain how such systems can have peculiar colours which continue the same under all aspects, it is sufficient to suppose that the groups of particles the most compounded are very

small, and that either by their density or their nature, they refract light much more powerfully than the medium or media interposed between them. The first condition may be inferred from the impossibility of distinguishing these elementary groups with the best microscopes; the second, as we shall soon see, is necessary in order that their colours may be permanent under all inclinations.

When a beam of light penetrates such a system, a certain number of rays may pass between the groups, without traversing them, and emerge again into space. This will be the portion of light which the body is capable of transmitting; but there is also a certain portion of rays which will encounter the groups, and be forced to traverse them. These, when they arrive at the first surface of each group, will experience there a partial reflection; but the effect will be insensible, on account of the perfect homogeneity of the groups placed before and after it. Then the luminous particles penetrating the group, and experiencing its action, which is supposed very energetic, will acquire fits much shorter, and a succession much more rapid than they had in the surrounding medium. For this reason, when they arrive at the second surface of a group, there will be some among them disposed to be reflected, and others to be transmitted. Those which actually undergo reflection, will form the proper colour of the group, which, in many cases, will have only a very feeble intensity, on account of the attraction of the surrounding groups, which may be very sensible. Moreover, this colour will be the same under all incidences, if, as we have supposed, the refracting power of the group be very great,* and if, in addition to this, it can be traversed only near its centre, the lateral transmissions being prevented and turned aside, by the inflection which light always experiences in passing near the extremities of bodies. The portion of light which has thus traversed a first group, and escaped reflection at its second surface, will continue to be transmitted until it encounters another which produces upon it similar effects. Then, if the first group has not reflected all the rays, which, in the incident light, were proper to compose its colour, the second group will reflect a part of the rest, and so on from group to group, until the collection of rays which form this colour in the incident light is completely exhausted. The sum of these reflections then will compose the total colour of the entire body, which will thus go on increasing in intensity with the thickness, as long as the groups which last receive the incident light have any to reflect, and the portion of this light, which shall have escaped the successive reflections, will form the colour transmitted by the body.

---

* This does not prevent that of the entire body from being very feeble.

302. If the phenomenon took place exactly as we have described it, the proper colour of bodies would always rigorously present one of the tints of the reflected coloured rings, and the colour transmitted would be exactly complementary to this tint. But it is seldom that the identity of the proper colour with one of the rings is complete, and still more seldom that the colour transmitted is the complement of the one reflected. It must be observed that light does not traverse bodies, even the most transparent, with the perfect freedom we have supposed; a part is always absorbed by the body, either because it combines with the material particles, or because it is modified by their action in such a manner, as no longer to produce on our organs the sensation of vision. Now we may easily convince ourselves that this absorbent action is exerted upon the different simple rays with unequal energies, the intensity and order of which are different for each substance. For if we pour liquids of different colours into conical glass vessels, and view them by transmission, the colour of each almost always appears to vary, according to the degree of thickness which the rays have traversed, and the progress of absorption is generally found to vary for different liquids, as well as the order in which the rays are successively absorbed. This phenomenon, therefore, as it modifies the nature of the transmitted light, must necessarily destroy, almost always, its relation to the reflected tint; and the reflected tint itself may also be altered to a certain degree, though undoubtedly not to the same extent, by a similar cause, that is, by the absorption which the rays composing it experience, in returning from the second surface of the groups to the exterior surface of the body. Still, in consequence of the small extent of this path, the reflected tint must generally be less altered than the other and indeed its analogy to the reflected rings in very

many instances is most evident, as may be proved either by the nature of the observed tint, or by the nature of those into which it degenerates when the composition or texture of the substance undergoes slow and progressive variations. In the *Traité de Physique*, I have given a large number of examples of these resemblances, taken both from the changes of colour which may be produced in bodies by chemical operations, and from those which nature produces spontaneously by analogous operations in vegetation.

303. All these examples evidently indicate that the proper colour of a body is a mere accident, resulting from the dimensions and arrangement of its particles; as is rendered in the highest degree manifest by the following experiment of M. Thénard. This skilful chemist having carefully distilled some phosphorus seven or eight times, with a view to obtaining it extremely pure, found that it had acquired after these operations a new and unexpected property. When melted in hot water, it became transparent, and of a yellowish white, as usual. Being left to cool slowly, it became solid, remaining of the same colour and semitransparent; but if, in its melted state, it was thrown into cold water, and stirred with a glass tube to make it cool suddenly, it immediately became opaque and absolutely black. Yet it had not changed its nature, for upon being melted anew, it recovered its colour and transparency, and preserved them in its solid state when left to cool slowly; so that the same solid piece of phosphorus was capable of being rendered at pleasure, successively yellow or black, and transparent or opaque. This remarkable experiment shows most fully that transparency and opacity, colour and the want of colour, are merely modifications resulting from the arrangement and dimensions of the material groups of which bodies are composed. M. Clément and myself repeated the experiment upon a quantity of phosphorus prepared by M. Thénard, and we had occasion to observe a phenomenon which renders this transition still more striking. Having thrown our melted phosphorus into cold water, a certain number of small globules, perhaps 10 or 12, remained diffused in different parts without losing their liquidity or transparency. It appears that either from the slight coldness of the water or from some other cause, the particles had gradually arranged themselves, as they do by slow cooling; but if we only touched one of

them with the extremity of a glass tube, this slight motion, or perhaps the mere attraction of the solid matter of the tube, instantly determined the solidification of the globule, and it became at once absolutely black. This trial, repeated successively upon all the parts, was always followed by the same result. The slightest agitation, then, was sufficient to cause the particles to arrange themselves in one way or the other. In like manner, when water has had its temperature reduced several degrees below the point of melting ice, without ceasing to be liquid, if we throw in the smallest crystal of ice, or, as I believe, any small solid body, capable of being wet by the water, still liquid, either of these substances instantaneously determines the congelation.

304. I shall close this subject with an account of an experiment by Dr Brewster which appears to me to be one of the most proper for confirming the influence which the arrangement of the material particles is capable of exerting, under a variety of circumstances, upon colour. Almost every one is aware of the vivid and brilliant colours presented by the mother of pearl. It appears that they are peculiar to this substance, as much as those of any other natural body are peculiar to that body. Yet they result simply from the constitution of its surface, and from certain small and imperceptible wrinkles which furrow it, without any connection whatever with the nature of its particles. For if we take an impression of the pearl, like that of a seal, upon fine black wax, or any other substance capable of receiving an exact impression, the surface of the wax or other substance, acquires the property of the pearl and exhibits the same colours.†

Return of Rays reflected interiorly at the second Surface of thick, transparent Media.

305. When a ray of light, after having penetrated obliquely a plate, with parallel faces, of any thickness, is reflected specularly at its second surface, and returns to emerge through the first, the equality of its paths through the plate, before and after reflection, gives rise to certain conditions, which determine the

---

† See note upon the colours of striated bodies.

possibility or impossibility of its emergence. These conditions may be established in an abstract manner, by means of the properties which Newton has ascribed to the fits; and their results may be compared with those of experiment. In this manner, the effects of reflection upon the fits are established conformably to the definitions of Newton; and, by applying them to thin plates, nearly parallel, and which exhibit the phenomena of coloured rings, we are enabled to ascertain precisely what takes place before and after interior reflection, by which the rays are produced. This delicate subject can only be suggested here; those who would pursue it further, are referred to the *Traité de Physique.*

Explanation of Coloured Rings and Fits upon the Hypothesis of the Undulations of Light. Principle of Interferences.

306. If light really consist of matter emitted from luminous bodies, the fits are a necessary property, since they are a literal enunciation of the alternations of reflection and transmission, which the coloured rings present. But if we suppose light otherwise constituted, these alternations may be represented differently.

Descartes, and after him Huygens and many other philosophers, have supposed the sensation of light to be produced in us, by undulations excited in a very elastic medium, and propagated to the eye where the sensation takes place, in the same manner as the undulations excited in the air and propagated to the ear, produce the sensation of sound. This medium, if it exist, must fill the celestial spaces, since it is through these spaces that the light of the heavenly bodies comes to the eye. It must also be very elastic, since the transmission of light takes place with so great a velocity. At the same time it must have an infinitely small density, since the most careful examination of ancient and modern astronomical observations, does not indicate in the planetary motions any sensible resistance. This medium must evidently penetrate all terrestial bodies, since all transmit light when they are sufficiently thin, and since they modify, by their contact, its interior reflection. Moreover, its density in different substances must vary according to their nature, since the unequal refractions which they exert upon the same rays, prove that the propagation of these rays takes place with unequal velocities. But what must be the proportions of these densities, for different substances, crystallized and uncrystallized? How is the luminous ether introduced into each of them in this state, and retained there? How is it confined in such a manner as not to admit of its spreading without? Moreover, how is this medium, so rare, unresisting and intangible, agitated by the particles of bodies which appear to us luminous? These are so many points that must be well understood, or at least well defined, before we can have an exact idea of the conditions by which the undulations are to be produced and propagated; but they have not as yet been distinctly settled.

307. Nevertheless, if we conceive a body endued with the power of exciting an instantaneous agitation in any point of such a medium, supposed for the present to be equally dense throughout its whole extent, this agitation will be propagated spherically by the same laws as that of the air, except that the velocity will be much greater. Each particle of the undulating medium will then be agitated in its turn, and afterwards come to a state of rest.

If these agitations are repeated in the same point, there will result, as in air, a series of undulations analogous to those of sound; and, as in the latter we observe periodical alternations of condensation and rarefaction, corresponding to the alternations of direction which constitute the vibrations of the sonorous body, so we may suppose that the successive and periodical vibrations of luminous bodies produce similar alternations in the luminous undulations. Lastly, as the succession of sonorous undulations, when it is sufficiently rapid, produces on the ear the sensation of continued sound, the quality of which depends upon the rapidity of the opposite vibrations, and the laws of condensation and velocity which the nature of these vibrations excites in each sonorous wave; so, with analogous conditions, the ethereal undulations may produce sensations of light in the eye, which shall vary with the diversity of the conditions. Hence the difference of colour. In this system the lengths of the luminous undulations correspond to the fits of Newton, and the length of the former is, as we shall see hereafter, precisely quadruple that of the latter; the velocity of propagation, in each case, depends upon the ratio of the elastic force of the fluid to its density.

308. When a sonorous undulation excited in the air, arrives at the surface of a solid body, its impulse produces in the parts of this body an actual though insensible motion, which sends it back. If the body, instead of being solid, be a gas, there is still an impulse, which sends the sonorous undulation back, but a sensible undulation is produced in the gas, depending upon the impression which its surface has received.* The luminous undulations must therefore produce a similar effect, when the medium, traversed by them, is terminated by a body, in which the density of the ethereal fluid is different; that is, there must be produced a reflected wave and a transmitted one; thus giving rise to reflection and refraction. In this system the intensities of the rays must be measured by the moving force of the fluid in motion, that is, by the product of the density of the fluid into the square of the velocity of its particles.

In order to establish these analogies which appear so remarkable, we must follow out the consequences by calculation. We must determine by the laws of motion in fluids, how the direct waves must be refracted when they pass from one medium to another; what is the ratio of the angle of refraction to the angle of incidence in substances crystallized and uncrystallized, and what is the value of this ratio for each simple ray, according to the nature of the vibrations which belong to it. M. Poisson has succeeded in solving completely the two first problems, and his analysis has indicated the same laws of refraction which are established by experiment. He has likewise deduced from it the phenomenon of interior reflection at the point of contact of two media of different refracting powers, a phenomenon which might have seemed, at first view, incompatible with the hypothesis of undulations, by appearing to require that the undulations, excited in the first fluid, should be reflected there without communicating motion to any other contiguous fluid. The analysis of M. Poisson removes this difficulty, by showing that the second medium actually receives condensations and velocities, but so small and decreasing so rapidly with the distance, that they cease to be sensible at any distance physically appreciable. But this analysis, which assigns so exactly and so completely the smallest particulars of the luminous undulations, offers nothing by which we can account for the phenomenon of dispersion, or rather, nothing which is not opposed to it. This circumstance constitutes an objection to the system of undulations as strong as the analysis which leaves it unexplained, is refined and complete. M. Poisson, in the course of his calculations on reflection, remarked also a peculiarity which the system of undulations indicates, and which experiment does not present; but he observes that this disagreement might vanish, if regard were paid to the particular state of the two media in the strata bordering upon their common surface; so that we are not obliged to consider it as a positive objection. He has also applied this analysis to the calculation of the coloured rings formed by homogeneous light, considering each of them as formed, not by the alternations of reflection and transmission, which light experiences at the second surface of the thin lamina, as is done in the system of emission, but by the simultaneous return, in the organ, of the waves reflected at the two surfaces of the lamina, in opposite stages of undulatory motion. We cannot here give a more extended view of this valuable paper; but those who may wish to pursue the subject further, will find an abstract of it, by the author himself, in the 22d volume of the *Annales de Chimie et de Physique.* They will also find in the same volume the results of the discussion between M. Poisson and M. Fresnel, respecting the manner in which the undulatory system is to be regarded.

---

* This phenomenon is observed in the sounds produced with organ pipes, when we have introduced successively strata of gas of unequal densities, for instance, of atmospheric air and hydrogen gas. The sounds which such a compound is capable of producing, have been calculated by M. Poisson, and the law of the succession is found to be perfectly conformable to what direct experiments have shown. Now this same theory indicates the mode in which the agitations, excited in one fluid, are communicated and divided, when they arrive at another contiguous fluid.

309. When the ear hears at the same time two regular and continued sounds, it distinguishes, besides the two sounds, the epochs at which the undulations of the same nature arrive together or separate. If the periods of these returns are very rapid, we hear a third sound, the tone of which may be calculated beforehand from the epochs of the coincidences; but if the returns of these coincidences are more rare, so that the ear can distinguish them separately, we hear a series of beats

which succeed each other with greater or less rapidity. The mixture of two rays which arrive together at the eye, under circumstances properly chosen, produces a similar effect which was first noticed by Grimaldi, but of which Dr Young was the first to show the numerous applications. The best manner of producing this phenomenon is the following, proposed by M. Fresnel.

We introduce into a dark room a solar beam, reflected in a fixed direction by means of a heliostat, and transmit it through a magnifying glass of a very short focal distance, which causes it to converge at its focus almost in a mathematical point. From this the rays of which the pencil is composed, diverge in all directions, and form a cone which increases in magnitude in proportion to the distance. In this cone, at the distance of 7 or 8 feet, we place two metallic mirrors, inclined to each other at a very small angle, so as to receive the rays under nearly the same incidence; and placing the eye at a certain distance, we view at the same time on each mirror the image of the luminous point. In this manner we perceive two images, separated by an angular interval depending upon the inclination of the mirrors, their distance from the luminous point, and the distance at which the observer is placed. But what constitutes the essential circumstance of the phenomenon is, that if we make use of a magnifying glass of a short focus, we perceive between the places of the two images, a series of luminous stripes, parallel to each other, of different colours, and directed perpendicularly to the line which joins the two images. When the incident light is simple, the stripes are of the colour of this light, and separated by dark intervals. Their direction depends simply on the direction of the planes of the mirrors, and not on any influence produced by the physical borders which terminate them; for we can turn each of the mirrors upon itself, which will change the position of the borders without changing the common section of the two planes, and the stripes will suffer no alteration.

310. For the sake of simplicity, let us suppose the incident light homogeneous. This case is easily realized for any of the colours, by decomposing with a prism a beam of light rendered stationary by a heliostat; we may likewise realize the same condition, but only for the red, by looking at the stripes through certain coloured glasses, which suffer none but this colour to

pass. Then, if we consider any one of the bright stripes formed between the two images, we can calculate, by means of the position of the magnifying glass, that of the mirrors and the eye, the direction and range of the luminous rays which form this stripe and which come from each mirror. The following are the results of this calculation. 1. The middle of the space comprehended between the two luminous points is occupied by a bright stripe, formed of rays whose ranges from the luminous point to the eye are of equal lengths, so that their difference is zero. 2. The first bright stripe on the right and left of this is formed of rays whose difference of range is constant, and equal to a certain quantity which we shall denote by l. 3. The second bright stripe is formed of rays whose difference of range is $2l$. 4. In general, the difference of range for each bright stripe is one of the terms of the series $0, l, 2l, 3l, 4l \ldots\ldots$ 5. The dark stripes, intermediate between the preceding, correspond to differences of range expressed by $\frac{1}{2}l$, $\frac{3}{2}l$, $\frac{5}{2}l \ldots\ldots$ 6. The numerical value of l is exactly quadruple the length which Newton has assigned to the fits, for the species of simple light upon which the observation is made.

311. The analogy of these laws to those of the rings is evident. The explanation, upon the hypothesis of undulations is the following; the interval l is precisely equal to the length of a luminous wave, that is, to the distance between the points of ether, which, in the succession of waves are found to have at the same instant similar motions and states. When the ranges of the two rays which interfere, differ by exactly half this quantity, at the place where they cross each other, they occasion at that place opposite motions and states whose stages are precisely similar. Moreover, the motions produced by these partial undulations take place in nearly the same direction, on account of the slight inclination of the rays to each other. Accordingly the motions which they tend to produce are to be subtracted, the one from the other, and as they are exactly equal they mutually destroy each other; so that the point of ether where this phenomenon takes place, is not agitated at all. The eye, therefore, when placed at this point must experience no sensation of light. The same must take place in all the points where the difference of range of the rays which *interfere*, is $\frac{3}{2}l$, $\frac{5}{2}l$, $\frac{7}{2}l \ldots\ldots$, because they arrive there also in precisely opposite states. On the con-

trary, for the bright stripes in which the difference of range is l, $2l$, $3l$, $4l$, ...the rays which compose them arrive in precisely similar states and stages of motion. Accordingly, the partial agitations which they excite are to be added together, and the phenomenon of light is produced. This mode of considering the combination of luminous undulations and the alternations of shade and light which result from them, has been called by Dr Young the *principle of interferences.*

312. The phenomenon of the alternations of shade and light is *certain ;* if it were *possible* only upon the hypothesis of undulations, it would establish this hypothesis and overthrow that of emission. But it does not seem to me absolutely to possess this character, so much to be desired whichever side it may favour, because it would be decisive. Without violating any rule of logic, we can equally conceive of the principle of interferences upon the hypothesis of emission, by making the result which it expresses a condition of vision.

Indeed, the phenomenon of stripes does not prove that the luminous rays exert a physical influence upon each other in certain circumstances; it only shows that the eye has or has not the sensation of light, when placed at the point where the rays answer to these circumstances. It proves also that an unpolished surface placed there and viewed from a distance, appears bright or obscure. Now, as to the first case, it is possible that vision may cease when the retina receives simultaneously rays which are in opposite stages of their fits; and as to the second, when such rays arrive at an unpolished surface, and are afterwards diffused by radiation in all directions, it is evident that having the same ranges to pass through, from each point of this surface to the eye, they will preserve, when they reach the retina, the same relation as to stages which they had when they arrived at the reflecting surface; so that if they were then in opposite states of fits, their opposition will be the same when they reach the retina, and of course there will be no sensation. I do not pretend that this explanation is certain or necessary in itself; it is both if light be material, because it is merely an enunciation of the phenomenon; but, that the phenomenon from which it is derived may not be decisive against the hypothesis of emission, it is sufficient that it implies no physical contradiction.

313. Dr Young has also applied the principle of interferences to the explanation of coloured rings, whether reflected or transmitted by thin laminæ. According to him, when we look at a thin plate, the light which is reflected at its first surface interferes in the eye with the light which is reflected from the second; this interference, therefore, does or does not produce the sensation of light, according as the difference of range of the two rays puts them into stages of similar or opposite undulations. But then at the point where the thickness of the plate is nothing, the difference of range is also nothing; it ought consequently to produce a bright spot, whereas the spot is actually black. Accordingly, Dr Young introduced, in this case, a new principle, namely, that reflection in the thin plate causes the rays to lose an interval $\frac{1}{2} l$, exactly equal to half the length of an undulation. By means of this modification the two rays reflected at the two surfaces of the plate, at the point where the thickness is nothing, acquire opposite dispositions, and consequently do not produce the sensation of light when they arrive together at the eye; after which, for the other neighbouring thicknesses, the law of periodical undulations accounts for the alternations of bright and obscure rings. This law thus modified, satisfies the measures of the coloured rings, observed under a perpendicular incidence; but for oblique incidences, it does not agree at all with the laws established by Newton. The difference is in the most oblique incidences. It presents also, with respect to the intensity of the rings, some difficulties depending upon the relative proportions of the two descriptions of light which form them by their interference; and this delicate subject has given rise to a very interesting discussion between M. Poisson and M. Fresnel, which may be found in the 22d and 23d volumes of the *Annales de Chimie et de Physique.*

314. We have considered only the rings which are seen by reflection; those which are seen by transmission, are formed according to the system of undulations, by the interference of the rays whose transmission is direct, with the rays, which, after being first reflected at the second surface of the plate, are also reflected by the first when they return to it, and are thus sent towards the eye, at which they arrive without other modification. In this case, the point where the thickness is nothing, gives a white spot according to experiment; and consequently

there is no constant to be added as in the reflected rings; but the subtraction or addition of a constant is necessary in many other cases. Hence we see that, in reality, the principle to which we here ascribe the phenomena, produces and expresses nothing but their periodical appearance.

According to this system the thicknesses at which the rings are formed, indicate the length of the undulations in each substance. Now for the same mode of vibration of the luminous body, the length of the waves must be equal to the path which the light describes during the time in which this vibration is performed; since then, we find the undulations shorter in substances of a greater refracting power, the velocity of transmission must, in this case, be less according to the same law, that is, it must be reciprocally proportional to their ratio of refraction.

315. If we consider the alternations of shade and light, as produced by the superpositions of luminous undulations of the same or a different nature, we can give a physical sense to the explanation of the phenomenon; and this is the manner in which Dr Young first presented the principle of interferences; but we may also detach it, as he has done, from every foreign consideration, and present it as an experimental law. In that case it is expressed by the following propositions.

(1.) *When two equal portions of light, in circumstances exactly similar, after having been separated, coincide again in nearly the same direction, the sensations which they would separately produce, unite with, or counteract each other, according as the difference of time employed in their separate paths, is an even or odd multiple of a certain semi-interval, which is different for the different colours, but constant for each kind of simple light.*

(2.) *In the application of this law to different media, the velocities of light must be supposed to be reciprocally proportional to the ratios of refraction of the media; so that the rays will move more slowly in substances more dense, and more rapidly in substances more rare, according to these ratios.*

(3.) *In reflection from the surface of a rarer medium, from some metals, and under several other circumstances, a semi-interval is lost.*

(4.) *The lengths of these intervals are always quadruple the lengths of the fits assigned by Newton to the same colours.*

316. In order to give an example of these laws, suppose that when two simple and homogeneous rays interfere and form

stripes, in the experiment with the two mirrors, we interpose in the path of one of these rays a very thin plate of glass, which it is to traverse alone. According to the second condition, its course through the glass will be less rapid than in the air, in proportion to the ratio of refraction. Consequently, when, after emerging from the glass and continuing its course, it encounters the ray with which it interferes, its ratios of interval will be different; and in order to recover the same intervals we must conceive another ray so directed that its retardation in the glass shall be compensated by the shortening of the path belonging to its new direction; consequently the coloured stripes which were formed, before the interposition of the glass, will be displaced, and by a quantity which may be calculated from the thickness of the plate and its ratio of refraction. This result is confirmed in the strictest manner by the observation of M. Arago to whom we are indebted for the experiment.

Accordingly, if we observe the displacement of the stripes thus produced by a given plate, which may be done with great precision, we can evidently deduce from this observation the ratio of refraction for the plate in question; we can also compare the refractions of different substances together, by interposing equal or unequal laminæ of each in the direction of the two rays which interfere. M. Arago and M. Fresnel have tried this method, and it has been found so exact, as to enable them to measure differences of refraction which could not be measured in any other way.

Diffraction or Inflection of Light.

317. Diffraction is a modification which luminous rays undergo, when they pass near the extremities of bodies. They are in this case turned out of their direct course, and unequally, according to their different refrangibility. This phenomenon may be rendered evident by the following experiment. We introduce into a dark room a fixed solar beam reflected horizontally by a heliostat, and receive it perpendicularly upon a white vertical screen, situated about 18 or 20 feet from the window. In this case, if the aperture by which the ray is introduced is circular,

and at least $\frac{1}{25}$ of an inch in diameter, the circular image of the sun, projected upon the screen, will experience no very sensible alteration in its whiteness. But if we place, in the axis of the beam about 7 or 8 feet from the window, a circular plate of metal, pierced with a very fine needle, and intercepting all the light except what passes through this small hole, and if we receive this light upon the same screen, or what is still better, upon a glass plate situated at the same distance, the face which receives the ray being very slightly ground, and the eye being placed behind the other; we shall no longer see one single circular spot of white light; but this spot will be surrounded by several coloured rings concentric with it, the total extent of which will considerably exceed that which the beam would have occupied, if the rays which compose it, had followed their rectilinear direction; for, from the dispositions we have supposed, these rays would only form with each other extremely small angles. We are obliged therefore to conclude, that in passing through the small hole in the metal, they have experienced a modification which dilates them into a cone with a much greater opening, or rather into several cones according to their different refrangibility. As a proof that the inflection is produced in the aperture or at an infinitely small distance from it, we have only to bring the ground glass upon which the rings fall, gradually nearer, and we shall see them unite and contract more and more, as if they emanated from the aperture. At a very small distance we find that the rays have experienced all the inflection which they ought to acquire; for if we place the eye very near the aperture and look through it, we shall see the first opening bordered with similar coloured rings, which are nothing but the images traced upon the retina by the inflected rays which fall upon it. Moreover, if this aperture be formed in a manner ever so slightly irregular, leaving a serrated edge, we shall perceive the small parcels of the metal glittering with a variety of colours, resulting from the unequal diffractions which they produce in the different rays.

If we now replace the glass at a suitable distance for exhibiting the rings in a sensible manner, and examine attentively the order of their colours, we shall see, that in each ring the blue and violet are on the inside, and the orange and red on the outside, precisely as in the rings reflected between two spherical glasses;

so that the same luminous particles, the reflection of which in each order took place at the greatest thicknesses, experience here, at an equal distance from the centre of the image, the greatest deviations.

318. In order to examine this analogy more accurately, we must follow the process of Newton, and decomposing the solar beam by a prism, project successively upon the small aperture, the different simple colours, always taking care to have their incidence perpendicular to the plate in which the aperture is made. When this is done, we perceive only rings formed of these several colours, and separated by intervals absolutely black. Hence it follows, not only that the inflection causes each kind of simple light to deviate, but that this deviation has different degrees and not a continuous progression; in so much that each kind of simple light, deflected by the influence of the small aperture, is thrown in part upon the first ring of this colour, or upon the second or third, but never upon the intervals which separate them. This also is analogous to the periodical intermissions of reflection in thin or thick plates; and in the same manner also we find that the diffracted simple rings are greater in the blue than in the violet, greater in the green than in the blue, and so on to the red, where the rings are largest.

319. We cannot here make known the ratios of magnitude of these rings in the different orders and colours; but what has already been said is sufficient to show how the phenomenon may be varied. As in the case of capillary attraction, the action exerted by the interior contour of circular tubes, is also exerted at the interior surfaces of all other tubes, whatever be their form, on account of the small distance at which it is sensible; so the inflecting action exerted at the perimeter of a circular aperture upon the luminous particles which pass very near its borders, must also be exerted according to similar laws, though perhaps with unequal intensities at the different extremities of bodies, terminated by right and curved lines; and luminous fringes are in fact always formed about these lines, the colours of which are arranged in the same manner as in the rings diffracted through a circular aperture.

Let us take, for example, two plates of the same substance, brought to an edge, and place their rectilinear extremities parallel, at a small distance from each other; then, in order to produce a small variation in the distance between these extremities, suppose one of them fixed, and the other adapted to a frame moving with a screw, that it may be made to approach the first gradually, as the moveable wire of a micrometer gradually approaches the fixed wires. This apparatus, invented by 'sGravesande, is represented in figure 174. We substitute it in place of the single plate, pierced with a circular aperture, and cause the most central parts of a compound beam to fall perpendicularly between the edges of the plates. Then, if we first separate the plates to some distance from each other, ½ an inch, for example, the screen or ground glass, remaining at the same distance as before, we shall perceive no very sensible traces of colour in the transmitted light; there will only be formed upon the screen a white image, rectangular like the interval between the plates. But if we bring them slowly towards each other, keeping them parallel, the longitudinal sides of the image will be bordered on the inside with several very fine white lines, more luminous than the rest of the image; and as the plates approach still nearer, these luminous lines will change into coloured fringes, separate from each other, which will throw themselves to considerable distances on both sides of the shaded part, still leaving between them a white rectangular image, more dilated than the preceding. If we examine the colours of these fringes when they are well developed, we shall find exactly the same order as in the circular rings; that is, violet on the inside, red on the outside, and the intermediate colours in the intermediate zones of the several stripes. This being observed, if we still continue to bring the plates nearer, the successive fringes will be thrown farther into the shade, which proves that the inflections producing these fringes are thus increased; and this increase will continue as long as the plates continue to approach.

320. This description supposes that the opposite edges of the plates are kept constantly parallel. When the frame which carries the moveable plate does not satisfy this condition, the fringes will no longer be composed of parallel stripes; but, according to the direction of the edges which produce them, they will take a trapezoid form, being largest on the side where the plates Fig. 17
approach nearest, and more contracted where they most diverge. In order to be able to produce at pleasure these different degrees

of inclination, we give to the fixed plate a motion of rotation, Fig. 176. kept steady by friction, about one of its points *C*, by means of a metallic rod fixed to its support, and passing through it at this point. Then by moving the part *M*, either directly or with a screw, we can give the plate the obliquity required. This serves also to establish a perfect parallelism; for if we first bring the moveable plate into contact with the other, rendering the motion of the axis *C* so free that the plate L' can be easily turned, the two edges will necessarily touch through their whole extent; and if we fix L' in this position, and remove *L* by the motion of its screw, their parallelism will continue, as may be ascertained by the parallelism of the fringes which serves as a very sensible index for this purpose. We must, however, take special care in producing the contact, not to press the plates strongly against each other by the action of the screw; for if they are thin, as they ought to be, in order that their edges may be perfectly rectilinear, this pressure would alter their shape and consequently produce irregularities in the configuration of the fringes.

321. If we bring the edges to a state of parallelism, and place the ground glass at a great distance compared with their separation, so that the number of fringes would not be sensibly increased by removing it farther, we find the following remarkable result, which M. Pouillet and myself have confirmed by a variety of experiments. *The deviations experienced by the several species of luminous particles in any media whatever, under a perpendicular incidence, and for a given interval between the plates, are determined by the lengths of the fits of the luminous particles in these media, and are proportional to them.* Hence it follows, that they vary from one medium to another, as the fits themselves, that is, in the inverse proportion to the ratios of refraction. Figures 177 and 178 represent the proportions and the distribution of parallel fringes, formed according to the preceding law, for the different kinds of simple colour.

As we progressively remove from each other, the two edges between which the light passes, the distance of the fringes from each other will continue to diminish till it reaches a certain limit after which their deviation will remain constant, even when one of the plates is taken entirely away. This proves that one edge is sufficient to form fringes in the light that passes near it; and hence we may conclude that the same effect will be produced by all the extremities of bodies, that limit the medium through which light is transmitted. Accordingly, if we introduce a beam of simple light into a dark room, and receive it at a considerable distance upon ground glass, we observe that the extremities of all the bodies near which this light passes, are bordered with luminous fringes, similar to those produced by the two plates when very distant from each other, and exposed perpendicularly to the luminous ray. This last condition is necessary to the precise enunciation of the phenomenon; for we may observe very large fringes between two distant edges, if we incline them considerably to the incident light; and, for the same reason, similar fringes are also formed by reflection, when the incident ray grazes the borders of surfaces under a very great obliquity.

322. But if the single body which forms fringes by its interposition, has dimensions so small that the beam of light can completely envelope it, projecting over the opposite sides, the simultaneous influence of these two edges produces new phenomena, which become sensible at less distances, according as the intercepted portion of light is narrower. For the sake of distinctness, suppose that the body is a small opaque rectangular lamina, only a few hundredths of an inch in width. Then, besides the exterior fringes formed on the parts situated towards the light, as when the body is of an indefinite size, other fringes appear within the shade. When the incident light is simple, these fringes are of the same colour with that; and are separated from one another by intervals absolutely black. Their number is always odd, so that there is always one at the centre of the shade. If we receive them first at very small distances, and then gradually at greater, they are at first very fine and almost imperceptible; then they gradually extend and become more distinct, and their intervals enlarge at the same time. If we receive them upon ground glass where we can observe their formation, we recognise in them the preceding peculiarities; but the observation is rendered very much more precise and distinct, if we employ the different arrangements invented by M. Fresnel, and which apply with equal advantage to all the experiments on diffraction. After having fixed the direction of the luminous beam by receiving it from a heliostat, M. Fresnel concentrated it by a strong magnifier, in a radiating point, almost mathematical, as in the fundamental experiment on inter-

ferences; and he placed the body in the divergent cone formed by this beam, either permitting all the rays to pass, or when he wished to operate with homogeneous light, causing the beam to pass through a glass which transmitted only the red rays. Then, instead of receiving the diffracted stripes upon ground glass, he received them immediately upon the eye, thus preserving much more of the brilliancy. Lastly, in order to join to this mode of observation a means of measurement, he applied to the eye a magnifier fixed upon a firm foot, and adapted to move horizontally, by means of a screw, along a scale divided into equal parts. Thus he was able to bring the axis of this magnifier successively before each bright or dark stripe; and he fixed precisely the position of this stripe, by referring it to a very fine wire extended before the glass exactly at its focus. In this manner he was able to measure upon the scale of equal parts, the intervals of the stripes, by knowing the distance through which the wire passed to apply itself to each of them; and this method is so exact that none of the most minute circumstances of the phenomena can escape observation; and hence nothing was wanting in order to determine the physical laws of these phenomena.

According to a very ingenious remark of Dr Young, the nature and arrangement of the stripes may always be represented with a very near approximation, by supposing that the light which falls upon the borders of the bodies interposed, darts off radiating in all directions from these borders, and interfering either with itself or with the light which continues to be transmitted directly.

Let us take, for example, the case where the body interposed in the luminous cone, is a simple opaque lamina with rectilinear borders. We have seen that in this case two systems of fringes are formed, one exterior with respect to the shade of the lamina, and the other interior. According to Dr Young, the interior are formed by the mutual interference of the luminous beams which proceed from the opposite borders of the plate, by virtue of radiant reflection. Indeed these two portions of light are precisely in the same situation as the two luminous points reflected in the experiment with the mirrors; also the disposition of the interior stripes, whether luminous or obscure, as well as the ratios of their intervals, are absolutely similar. If we conceive the series of points in space where the same kind of interference is produced at different distances behind the lamina, which gives the successive places where the same stripe appears, we shall find that these points are sensibly in a straight line; and their intervals when measured, are exactly conformable to what the calculation of the interferences indicates.

323. With respect to the exterior stripes, we may consider them as formed by the interference of the light directly transmitted with the light radiated by each border; but here, as in the case of the reflected rings, we must suppose the loss of an interval equal to $\frac{1}{2}$ *l*. We thus find that the successive positions in which each stripe ought to appear, for the different distances, are not in the same straight line, but in hyperbolic curves of the second degree; and this is perfectly confirmed by experiment. We must not infer from this, that in diffraction the motion of light is not rectilinear, for it is not the same ray of light which forms the stripe of the same order at different distances; and the rays alone are subjected to rectilinear motion. The change of ray at different distances may be inferred from this simply, that we can view the stripes in space, either with the naked eye or with a magnifier; for it is necessary that the rays which form them should converge and afterwards separate, in order that the magnifier may be able to receive them, and give a sensible image of their point of meeting.

In the case where the diffracted stripes are formed by the passage of a luminous beam between two plates with rectilinear borders, we may with a very near approximation, attribute them to the interference of the two portions of light which fall upon the opposite borders of these plates.

Nevertheless, there are several physical peculiarities of the phenomenon, which are not easily reconciled with this hypothesis. M. Fresnel has even shown that the measures of the stripes are not completely explained when they are taken with extreme precision. He has found that the feeble portion of light, capable of being reflected by the borders of bodies, is not sufficient to produce the intensities of the observed stripes, and that we must suppose the concurrence of rays which pass exteriorly to the sphere of contact of these borders. Accordingly, he considers all the parts of the direct luminous undulation, as so many centres of distinct agitations, the effects of which must extend spherically to all points of space to which they can be propagated; after which, for each of these points the definitive effect must result from the interferences of all the partial agitations which reach it. If this supposition be applied to the free propagation of a spherical wave, in a homogeneous medium, it gives a diminution of light reciprocally proportional to the square of the distance, agreeably to observation. But when a part of the wave is intercepted, it indicates in the different points of space to which it is afterwards propagated, alternations of shade and light, which agree minutely in their disposition and intensity, with what we observe in the diffracted stripes. This principle has enabled M. Fresnel to comprehend, with extraordinary precision, all the cases of diffraction, and to unite them in formulas which express under all circumstances their mutual dependance. But the exposition of these nice results would lead us beyond the design of an elementary work. While we admit all that is established in this case by experiment, it must be allowed that the theoretical deductions from the principle of partial agitations, as explained by M. Fresnel, has been called in question by geometers of the highest authority, particularly by M. Poisson. See *Annales de Chimie et de Physique*, for 1823.

324. I will here add the enunciation of a singular phenomenon, which is easily explained on the principle of interferences. When a narrow and opaque lamina forms interior fringes with respect to its shadow, these fringes may be made to disappear, by placing an opaque screen in contact with one of the borders, or interposing it to a certain degree in the luminous rays, either before or behind the opaque lamina. This phenomenon was discovered by Dr Young. M. Arago has added the circumstance, that the disappearance may be effected in like manner by the approach of a transparent screen sufficiently thick, and that it is progressive according to the thickness; so that thin screens at first only transfer the fringes from the side where they are found. It is remarkable that the quantity of this transference may be calculated according to the laws of interferences, when we know the thickness and refracting power of the plate. If two screens are placed on the two sides of the opaque lamina, the effect is equal to the difference of the transferences which each of them would have produced separately. MM. Arago and Fresnel have made use of this process in measuring the refracting power of gaseous substances, and it has been attended with

a precision of which no other method is capable. See *Annales de Chimie et de Physique*, for 1816 and 1817.

325. The principle of interferences is as yet the only one by which we have been able to explain the peculiarities of diffraction; and in this particular the phenomenon favours the theory of undulations. Nevertheless, if we examine carefully the details of the explanation, we shall find it to be rather a representation of the phenomenon, than a rigorous mechanical theory of it. It were a great desideratum, therefore, to account for this phenomenon upon the principles of the materiality of light, which have hitherto given us such clear ideas and such precise measures as to so many of its other motions. It would be necessary to determine the kind of forces which the particles of bodies exert upon the particles of light, in order to inflect them as they are inflected in the diffracted stripes, and to do this independently of the chemical nature of these bodies, and simply according to the form of the borders which terminate them. But this appears to be very difficult; and it is probable that if such forces exist, they are of a different character from any which is presented by the ordinary phenomena of refraction and reflection.

NOTES.

I.

On the Spherometer.

Fig. 17

THIS instrument was invented by M. Cauchoix for the purpose of measuring the curvature of spherical glasses. It consists essentially of three horizontal branches of steel, making with each other angles of 120°. At the extremities of these three branches and perpendicular to their direction, are three steel rods, the ends of which being drawn out into small cylinders and turned with extreme precision, are terminated by three planes of a very small extent. At the centre of the three branches is a screw accurately formed, the head of which carries a graduated dial. When we wish to employ this instrument for measuring the equality of curvature of a spherical glass, we first place it upon the surface of the glass and turn the central screw until it comes exactly in contact with the glass, which will be indicated by the circumstance that then the instrument may be turned upon the glass with perfect freedom, and without any sensible jostling in its motion. This being done, we transfer the instrument to other parts of the surface, when the least change of curvature will become appreciable, because the centre screw and the three points will no longer touch simultaneously. When the screw no longer touches, the weight of the instrument rests entirely upon the three feet, and its rotation produces a hard friction, and a sound very different from what it produced before. If, on the contrary, the screw descends too far, the instrument will jostle upon the three feet, in such a manner as to become sensible to the ear and the touch. The precision of these two indices is truly incredible; no other known process of art can be compared with it. To be convinced of this, we have only to place the spherometer upon a piece of plane glass, turn the screw until it comes in contact with its surface, and then turn it a little to the right or left, until the want of contact becomes sensible by the roughness of the friction or the sound of jostling; if then, we read by means of the index how slight a movement of the screw is necessary for this change, it will appear truly astonishing.

From what has been said, we may determine whether a glass surface supposed to be plane is really so; for when the screw of the spherometer has been brought just into contact on one part of this surface, we have only to transfer the instrument to the other parts of the glass without touching the screw, and we shall see if the contact still continues with the same precision.

Let us suppose this condition satisfied. If then we slide between the glass plane and the point of the screw, a plate with parallel faces, however thin we suppose it, it is evident that the spherometer will jostle. The quantity by which it is necessary to turn the screw to restore the contact will determine the thickness of the plate interposed. But the pressure of the screw upon the plate in this operation, might break it if it were very thin, or at least might mar its surface if it were susceptible of indentation. To avoid this, the plate must not be inserted directly under the screw. We should first place the screw upon a fragment of plane glass with parallel faces, whose equality of thickness has been previously verified by the spherometer. This fragment being placed upon the principal glass plane, we bring the screw into exact contact with the upper surface, the three other points resting on the principal plane; then we introduce between this last and the fragment, the plate which we wish to measure. After this interposition the spherometer will jostle, and the contact is restored by turning the screw; the distance through which the screw moves, marked by the index, will indicate the thickness sought; and this will be done without the slightest risk, however fragile or thin the plate may be.

II.

Polarisation of Light.

The polarisation of light is a property discovered by Malus, which consists in certain affections that the rays of light assume on being reflected by polished surfaces, or refracted by these same surfaces, or transmitted through substances possessing double refraction.

Though it would be impossible here to give a complete exposition of the details of these phenomena, we will at least describe some of the experiments by which they may be exhibited.

The first and principal of these consists in giving to light a modification, such that the rays composing a pencil will all escape reflection when they fall on a reflecting surface under certain circumstances.

As an instance, suppose a beam of solar light *SI* to fall on the first surface *LL* of a plate of glass, smooth but not silvered, making with the surface an angle of 35° 25′; it will be reflected in the direction *II′*, making the angle of reflection equal to that of incidence. Let it then be received on another plate of glass, smooth but unsilvered, like the former; generally speaking it will be again reflected with a partial loss. But the reflection will cease altogether if the second glass be placed like the first, at an angle of 35° 25′ to the line *II′*, provided also it be so turned that the second reflection shall take place in a plane *II′L′* perpendicular to that of the first, *SIL*. Fig. 166.

In order to make this disposition of the glasses more clearly intelligible, we may imagine that *II′* is a vertical line, that *IS* is directed north and south, and *I′L* east and west.

Before we enter upon the inferences to be drawn from this remarkable experiment, I will make a few observations on the manner of performing it.

Many kinds of apparatus may be devised to attain this end. A very convenient one is represented in figure 167. It is very simple, and is sufficient for all experiments on polarisation. It consists of a tube *TT′*, to the ends of which are fixed two collars which turn with sufficient friction to keep them fast in any position. Each of them bears a circular division which marks degrees. From two opposite points of their circumference proceed two brass stems *TV*, *T′V′*, parallel to the axis of the tube, and between them is suspended a brass ring *AA*, which may be turned about an axis *XX* perpendicular to the common direction of the stems. The motion of the ring is likewise measured by a circular graduation, and it may be confined in any position by screws. When a plate of glass is to be exposed to the light, it must be fixed on the surface of the ring; then it may be placed in any situation whatever with respect to the rays of light which pass through the tube; for the collar, turning circularly round the tube, brings the reflecting plane into all possible directions, preserving a constant inclination to the axis, and this inclination may be varied by means of the proper motion of the ring round its axis *XX*. The graduated circle which regulates this motion should mark zero when the plane of the ring is perpendicular to the axis of the tube, and the divisions on the two collars should have their zeros on the same straight line parallel to the axis. In constructing the apparatus one should take care that these conditions are fulfilled; but it is of no great consequence that they be so exactly, as any error may be compensated by repeating each observation on both sides of the axis, and taking the mean of the numbers of degrees found in the two opposite positions.

If it be desired, for instance, to repeat Malus's experiment described above, a plate of glass must be placed on each ring, and they must be disposed so as to be inclined to the axis at angles of 35° 25′. Then the graduated circle of one of the collars must be brought to mark zero, and the other 90°, that the planes of reflection may be perpendicular to each other. The tube must then be secured, and a candle placed at some distance in such a position that its rays may be reflected by the glass along the axis *TT′*. This will happen when on looking through the tube the reflection of the candle is seen in the first glass. Every thing being thus arranged, the reflected rays will meet the second glass at the same angle of 35° 25′; then according to the different positions given to the collar *T′T′* which carries this glass, the light proceeding from the second reflection will be more or less intense, and there will be two particular positions in which there will be no reflection at all, of those at least which are specularly reflected by the first glass. Care must be taken to put a dark object behind the glass *L′L′* on the side opposite to the reflected light, in order to intercept the extraneous rays which might be sent on this side from exterior objects, and which, passing through the glass, and arriving at the eye, would mix with the reflected rays that are the subject of the observation. The same precaution should be taken for the glass *LL*; and indeed as this is never used except to reflect light at its first surface, the back of it may be blackened once for all with India ink, or smoked with a lamp; it would not do to silver it for a reason that will be given hereafter.

For the light of the candle mentioned above may be substituted that coming from the atmosphere, which may be received into the tube when reflected by the first glass *LL*; but in this case to preserve to the rays the precise inclination required for the phenomenon, the field of the tube should be limited by some diaphragms with very small apertures placed within it. The first glass should be blackened or smoked as before mentioned to intercept any rays that might come by refraction from objects situated under it. In this manner, on looking through the tube, when the glass *LL* is turned towards the sky a small brilliant white speck will be seen, on which all the experiments may be made. The perfect whiteness of this spot is a great advantage; it is an indispensable qualification in many cases, where different tints are to be observed and compared; it is impossible to succeed as well with the flame of a candle or any other inflamed substance, as none of these flames are perfectly white. Lastly, the brightness of the incident light must be modified, so that the portion irregularly reflected by the two glasses may not be sensible; for this portion, being after such reflection in the state of radiant light, cannot be polarised in one single direction; the other part which is regularly reflected, alone undergoes polarisation, and therefore alone escapes reflection at the second glass.

Whatever be the nature of the apparatus employed, the process will always be the same, and the same phenomena of reflection will be observed on the second glass. To exhibit them in a methodical manner, which will allow us easily to take them all in at one view, we will suppose, as above, that *SIL* the plane of incidence of the light on the first glass coincides with that of the meridian, and that the reflected ray *II'* is vertical. Then if the collar *T'T'* which bears the second glass be turned round, this glass will also turn the reflected ray, making continually the same angle with it, and the second reflection will be directed successively to all the different points of the horizon; this being premised, the phenomena that will be observed are as follows;

When the second or lower glass is so placed that the second reflection shall take place in the plane of the meridian like the first, the intensity of the light, finally reflected, is at its maximum. As this glass is turned round it reflects less and less of the light thrown on it.

Finally, when the lower glass faces the east or west point, all the light passes through it, without being reflected at either surface.

If the collar be turned still farther round, the same phenomena recur in an inverse order, that is, the intensity of the light reflected increases according to the same degrees by which it before diminished, and attains the same maximum state when directed towards the meridian, and so on through the whole circle.

It appears then, that during a whole revolution of the glass the intensity of the reflected light has two *maxima* answering to the *azimuths* 0 and 180°, and two *minima* answering to 90° and 270°. Moreover, the variations are altogether similar on different sides of these positions. These conditions will be completely satisfied by supposing, as Malus does, that the intensity varies as the square of the cosine of the angle between the first and second planes of reflection.

The results of this interesting observation being thus collected into one point of view, we may draw this general conclusion from them, that a ray reflected by the first surface is not reflected by the second (under a particular incidence), when it presents its *east* or *west side* to the surface, but that in all other positions it is more or less reflected. Now if light be an emission, a ray of light can be nothing else but the rapid succession of a series of particles, and the *sides* of it are only the different sides of these particles. We must therefore necessarily conclude that these have faces endued with different physical properties, and that, in the present case, the first reflection turns towards the same point of space, faces, if not similar, at least endued with similar properties. This arrangement of the particles Malus denominated the *polarisation of light*, assimulating the operation of the first glass to that of a magnet which turns the poles of a number of needles all in the same direction.

Hitherto we have supposed that the incident and reflected rays made angles of 35° 25′ with the glasses; it is indeed only under that angle that the phenomenon takes place completely. If while the first glass remains fixed, the inclination of the second to the ray be ever so little altered, it will be found that the second reflection will not be entirely destroyed in any position, though it will still be at a minimum in the east and west plane. If again, the inclination of the ray to the second glass being preserved, that on the first be changed, it will be seen that the ray will never pass entirely through the second glass, but the partial reflections which take place at its surfaces are at a minimum in the above mentioned position.

Similar phenomena may be produced by means of most transparent substances besides glass. The two planes of reflection must always be at right angles, but the angle of incidence varies with the substance. According as the refracting power of this is greater or less than that of the ambient medium, the angle of polarisation, measured from the surface, is greater or less than half a right angle. We have seen that for glass this angle is 35° 25′; for sulphate of barytes it is only 32°, and for diamond only 23°. If glass plates be placed in essential oil of turpentine which has a refracting power almost exactly equal to that of glass, the angle of polarisation will be found to differ very little indeed from 45°. The reflection at the second surface is supposed to take place on the ambient medium which bounds the glass. In general, according to an ingenious remark of Dr Brewster, the angle of polarisation is characterized by the reflected ray being perpendicular to the refracted. The angles calculated on this hypothesis agree singularly well with experiment, and also confirm the rule given above for the different magnitudes of these angles, as will easily appear from figures 168, 169, and 170, in which the refracting power is supposed to be respectively greater than unity, equal to unity, and less than unity.

This law applies equally well to substances which, like the diamond and sulphur, never produce more than an incomplete polarisation, for the quantity of light reflected is invariably a minimum for the angle so determined.

If the mode of observation which we have applied to smooth glass plates be universally employed, it may serve to show that polarisation, when complete, is always a modification exactly of the same kind, for all substances; for when a beam of light has been once polarised, it will equally pass through all substances, with the exception mentioned above, provided each be presented to it under its proper angle, and whatever be the nature of the first or second substance employed, the variation of intensity in the light, after the second reflection, is always subject to the same laws.

To represent these circumstances geometrically, let us consider a ray *II'* polarised by reflection on a glass plate *LL*, and through any one of the particles composing it, let there be drawn three rectangular axes $c\,z$, $c\,x$, $c\,y$, the first coinciding with the ray, the second in the plane of reflection *SIC*, the third perpendicular to both the others. Then when the ray *II'* meets a second glass *L'L'*, placed so as to produce no reflection, the reflecting forces which emanate perpendicularly from the glass, must be perpendicular to the axis $c\,x$; moreover they must act equally on particles lying towards $c\,x$, and $c\,x'$, for if the glass be turned a little from the position of no reflection, the effects are found to be symmetrical on each side of that position. The action, therefore, of these reflecting forces, under this incidence, cannot make the axis $x\,c\,x'$ turn either to the right or left, any more than the force of gravity can turn a horizontal lever with equal arms. They cannot bring the axis into their own plane, in which we see it was in the first reflection, by which the polarisation took place on the glass *LL*. This proves that it is on that axis that the properties of the luminous particles depend. We shall for this reason call it the *axis of polarisation*, and suppose its direction similar and invariably determined for each particle. Further, for the sake of conciseness, we will call $c\,z$ the *axis of translation;* but we do not suppose this invariable in each particle, and we will consider it only as relative to its actual direction, in order to leave the particle at liberty to turn about its axis of polarisation. According to these definitions all the results that we have hitherto obtained may be expressed very simply and clearly in the following manner. Fig. 16

When a ray of light is reflected by a polished surface, under the angle which produces complete polarisation, the axis of polarisation of every reflected particle is situated in the plane of reflection, and perpendicular to the actual axis of translation of that particle.

If the incident particles are turned so that this condition cannot possibly be fulfilled, they will not be reflected, *at least under the angle of complete polarisation.* That happens when the axis of polarisation of an incident particle is perpendicular to the plane of incidence, the angle of incidence being properly determined *à priori*.

Generally speaking, when a polished surface receives a polarised ray under the angle at which it would itself produce complete polarisation, if it be made to turn round the ray without changing that angle, the quantity of light, reflected in different positions, varies as the square of the cosine of the angle between the plane of incidence, and the axis of polarisation.

When a ray of light has undergone polarisation in a certain direction, by the process above described, it carries that property with it, and preserves it without sensible alteration, when made to pass perpendicularly through even considerable thicknesses of air, water, and, in general, any substance that exerts only single refraction; but doubly refracting media alter, in general, the polarisation of a ray, and in a manner, to all appearance, sudden, communicating to it a new polarisation of the same nature in a different direction. It is only when crystals are held in certain directions, that the ray can escape this disturbing influence. Let us endeavour to compare more closely these two kinds of action.

That of crystals of one axis has been studied by Malus, who has comprised its effects in the following law. When a pencil of light naturally emanating from a luminous body, passes through a crystal of one axis, and is divided into two pencils having different directions, each of these pencils is polarised in one single direction; the ordinary one in the plane passing through its direction and a line parallel to the axis of the crystal, the extraordinary one perpendicularly to a plane similarly situated with respect to its direction. Each of these rays, when received on a plate of glass after its emergence, shows all the characters of polarisation that we have described.

This law obtains equally, when the ray has been polarised by reflection before its passage through the crystal. The two refracted pencils are always polarised, as if they had been composed of direct rays, but their relative intensities differ according to the direction of the primitive polarisation given to them; this direction must therefore have predisposed the particles to undergo from preference one or the other of the refractions.

These two laws were discovered by Malus. The analogy already remarked, between crystals of one axis and those of two, indicates sufficiently how it is to be extended to the latter; to find the direction of polarisation for the ordinary pencil, draw a plane through its direction, and through each of the axes of the crystal. If either of these axes existed alone, the ordinary pencil would be polarised in the plane belonging to it. Now it is actually found polarised in a plane intermediate between these two, and the extraordinary pencil perpendicularly to the analogous plane drawn through its direction between the two planes containing the axes. If the angle between these be equal to nothing, the crystal has but one axis, and the direction of polarisation is conformable to Malus's indications. This law has been directly verified on the two pencils refracted by topaz; as for other crystals in which it has been possible to verify it directly, we may, by the consideration of some other phenomena that will shortly be mentioned, infer that it applies to them also.

These laws of polarisation are applicable in all cases where the two pencils transmitted by a crystal are observed separately, but when they are received simultaneously, and in nearly the same direction, that of their apparent polarisation is found to be modified, and at the same time their coincidence produces certain colours, which M. Arago first observed, and of which M. Biot determined the experimental laws. The most simple arrangement for exhibiting these colours, is to place a thin lamina of some crystallized substance, in the direction of a white ray, previously polarised by reflection, and to analyze the transmitted light by means of a doubly-refracting prism. The light is thus separated into two portions, the colours of which are complementary to each other, and identical with those of the rings between two glasses. One of these portions appears to have preserved its primitive polarisation, whilst the other exhibits a new polarisation, of which the direction depends on that given to the axes of the crystal by turning the lamina round in its own plane.

Following gradually in this manner the direction of the polarisation given to a particle of light, transmitted through different thicknesses of a crystalline medium, it will be found to undergo periodical alternations, which, if light be an emission, indicate an oscillatory motion of the axes of the particles accompanying their progressive motion. M. Biot has designated this fact by the name of *moveable polarisation*, which is merely the expression of the observed results.

If the system of undulations be adopted, the colours of the two images may be attributed to the interference of the two pencils into which the incident polarised light separates, in passing through the lamina. This is what Dr Young does, and it is remarkable that calculations founded on this principle, gave him the nature of the tints, and the periods after which they recur, precisely as M. Biot had determined them by experiment. As to the alternations of polarisation, they become, in the undulatory system, a compound result produced by the mutual influence of the interfering rays, and it is easy to deduce from observation the conditions to which the mixture of the waves must be subjected to produce the new direction of apparent polarisation. M. Fresnel has done this, and the indications of his formula have been found conformable in all respects to the laws deduced by M. Biot from observation.

These interferences of the rays may be produced without the assistance of crystalline laminæ; we may equally well employ thick plates, provided the rays pass through them at very small inclinations to their crystalline axes. If the experiment be made with a conical pencil of light, large enough to give the various rays composing it inclinations sensibly different to the axes, so that they shall experience double refractions sensibly unequal,* these rays, analyzed after they emerge, offer different colours united in the same system of polarisation; and the union of these colours forms round the axes coloured zones, the configuration of which indicates the system of polarising action exerted by the substance under consideration. This kind of experiment is therefore very proper for exhibiting the axes, and indicating the mode of polarisation which any given substance impresses on the rays.

Upon the whole, the interferences of polarised rays offer very remarkable properties, many of which have been discovered and analyzed by MM. Arago and Fresnel with great ingenuity and considerable success, but as the limits of this work do not allow of a full exposition of them, we shall only cite one, which is, that rays polarised at right angles do not affect each other when they are made to interfere, whereas they preserve that power when they are polarised in the same direction. It is not only crystalline bodies that modify polarisation impressed on the rays of light; MM. Malus and Biot found by different experiments, made about the same time, that if a ray be refracted successively by several glass plates placed parallel to each other, it will at length be polarised in one single direction perpendicular to the plane of refraction. Malus by a very ingenious analysis of this phenomenon, has moreover shown that it is progressive, the first glass polarising a small portion of the incident light, the second a part of that which had escaped the action of the first, and so on. M. Arago, measuring the successive intensities by a method of his own invention, has shown that they are exactly equal to the quantity of light polarised in contrary directions at each reflection. A phenomenon analogous to this is produced naturally in prisms of tourmaline, which appear to be composed of a multitude of smaller prisms, united together, but without any immediate contact. All light passing through one of these prisms perpendicularly, is found to be polarised in a direction perpendicular to the edges, so that if two such prisms be placed at right angles, on looking through them a dark spot is seen where they cross. This property of the tourmaline affords a very convenient method for impressing on a pencil of rays a polarisation in any required direction, and for discovering such polarisation when it exists.

Moreover, M. Biot has discoverd that certain solid bodies, and even certain fluids, possess the faculty of changing progressively polarisation previously impressed on rays passing through them ; and by an analysis of the phenomena produced by these substances, he has shown that the same faculty resides in their smallest particles, so that they preserve it in all states solid, liquid, and aeriform, and even in all combinations into which they may happen to enter. M. Fresnel has found certain analogies between these phenomena and those of double refraction, which seem to connect the two together most intimately through the intermediation of total reflection.

Since reflection and refraction, even of the ordinary kind, modify the polarisation of light, we may expect to find this effect produced when rays of light are made to pass through media of regularly varying density. It is accordingly found that all transparent bodies which are sufficiently elastic to admit of different positions of their particles round a given state of equilibrium, as glass, crystals, animal jellies, horn, &c., produce the phenomena of polarisation, when they are compressed or expanded, or made unequally dense by being considerably heated and then cooled suddenly and unequally. These phenomena, discovered originally by Dr Seebeck, have been since studied and considerably extended by Dr Brewster, who has moreover remarked, that successive reflections of light from metallic plates produced phenomena of colour in which both M. Biot and he recognised all the characters of alternate polarisation.

Knowing, by what precedes, the experimental laws, according to which light is decomposed in crystals endued with double refraction, we may consider these effects as proper means for determining the mode of intimate aggregation of the particles of such bodies, and for affording some insight into the nature of their crystalline structure. Light becomes thus, as it were, a delicate instrument with which we examine the interior of substances, and which, insinuating itself between their minutest parts, permits us to study their arrangement, at which mineralogists previously guessed only by inspection of their external forms. M. Biot has shown the use of this method, applying it to a numerous class of minerals designated by the general name of *mica*, and he thinks he has decisive reasons for believing that several substances of natures extremely different as to their composition and structure have been improperly comprised under that name. He has also made use of the phenomena of alternate polarisation, to construct an instrument which he calls a *colorigrade*, which, producing in all cases the same series of colours, in exactly the same order, merely by the nature of its construction, affords a mode of designation just as convenient for comparison as that furnished by the thermometer for temperatures.

Many other experiments have been made, and are daily making, and many other properties have been discovered in polarised light; but we have been obliged to confine ourselves to the results, which are, perhaps not the most important part of the subject, although the easiest to explain; our aim in this rapid sketch being rather to stimulate than to satisfy the desire of knowledge on this branch of science which presents so vast a field for research both in theory and experiment, and which, though so lately discovered, has already furnished some useful applications to physics and mineralogy.

III.

The Relations of Light and Heat or Caloric.

In the body of this treatise we have considered only those properties of light which are made known to us by vision. In this single point of view, we have been made acquainted with a variety of physical characteristics belonging to this element. We shall now, for a moment consider the power which light has of heating bodies exposed to its action; and shall endeavour to ascertain whether this phenomenon proceeds from the identity of light and heat, or from the coexistence of both principles in the luminous rays.

We shall first state an important fact discovered by the great astronomer Sir William Herschel. Having proposed to measure the calorific energy of the different rays of the solar spectrum, he placed a very sensible thermometer in each of the seven divisions of colour marked by Newton; he then observed to what degree these thermometers rose in each of them above the point at which they stood in the surrounding air. He thus found that this degree was higher in the blue than in the violet, higher in the green than in the blue, and so on to the red, which produced a temperature higher than any of the others. But the most important fact was, that the maximum of temperature seemed to be a little beyond the extreme red, and wholly without the visible part of the spectrum. Hence Sir William Herschel inferred that the calorific property is not inherent in those rays alone, which produce in us the sensation of light, and that there exist among them other rays less refrangible than these, which possess only the property of heating.

These delicate experiments were repeated by other philosophers with various success. Some, like Sir William Herschel, discovered traces of sensible heat beyond the visible part of the spectrum; and others either perceived no indications of heat at all, or only such as were too feeble to be appreciated. Dr Wollaston and MM. Ritter and Beckman, undertook to examine particularly the opposite extremity of the spectrum, which produces the sensation of extreme violet; and they found that this part of the spectrum, whose calorific property was insensible, possessed other peculiar properties, which might be termed chemical, since they cause by their influence combinations which are not produced by the red extremity of the spectrum; and that, as in the latter, the maximum of heat takes place a little beyond the extreme red, so in the former the maximum of chemical action takes place just beyond the sensible limits of extreme violet. Hence they were led to the conclusion that the solar light was a mixture of three sorts of rays, which might be termed colorific, calorific, and chemical, according to the particular effects which they produced.

We shall soon inquire how far this conclusion is rigorous or even probable; but it should first be remarked, that the facts of which we have been speaking, have recently been verified with great care and put beyond a doubt by M. Berard. In order to render the experiments more certain and the effects more sensible, this philosopher perceived the necessity of prolonging their duration; he therefore made use of a solar beam reflected by a heliostat; and having refracted it through a prism, he thus obtained a spectrum very much dispersed and fixed exactly in one place. In order to determine the calorific properties, he placed very sensible thermometers in the seven spaces occupied by the different colours; and in order to determine the chemical properties, he placed in these same spaces, either openly exposed, or in transparent phials, various chemical combinations easily acted upon.

Proceeding in this manner M. Berard obtained the same results as Sir W. Herschel, with respect to the augmentation of the calorific power from the violet to the red, but he found the maximum of heat at the extremity of the spectrum, and not without it. He fixed it at the point where the ball of the thermometer was just covered by the extreme red rays; he perceived the temperature decrease progressively as the ball of the thermometer reached the shade; and finally placing the thermometer entirely without the visible spectrum, where Sir W. Herschel fixed the maximum of heat, the elevation of the temperature above the surrounding air was only a fifth part of what it was in the extreme red rays. The absolute intensity of the heat produced was also less in the experiments of M. Berard, than in those of Sir W. Herschel. It is impossible to determine whether these differences depended upon the substance of the prisms and the diversity in the apparatus, or upon some other physical circumstance belonging to the phenomenon itself.

M. Berard was desirous of ascertaining whether these properties would manifest themselves separately in each of the portions into which light is divided when it traverses a crystal endued with the property of double refraction. He therefore caused the solar beam to traverse a prism of Iceland spar. The division of the ray formed two spectrums, which exhibited the same properties; in both, the calorific property existed in less intensity within the limits of the spectrum, and was retained beyond the last sensible red rays. Thus, when the ray is divided by traversing the crystal, the calorific power is also divided between the two luminous portions.

In this operation the luminous particles are polarised by the crystal. In order to ascertain whether the *obscure calorific* particles experienced the same effect, M. Berard received the solar beam upon a piece of polished and transparent glass, forming with it an angle of 35° 25′, in order that the reflected portion might be completely polarised. The reflected portion was then received upon another glass forming with it the same angle of 35° 25′, and disposed in such a manner that it could turn conically under this constant incidence. This was precisely the apparatus of two glasses used by Malus, to which we have before had occasion to refer. It is evident that by turning the second glass, two positions would be found where no light was reflected; it only remained then to ascertain whether any heat was here reflected. For this purpose M. Berard placed a concave metallic mirror in such a manner as to collect the rays reflected by this glass and concentrate them upon the bulb of a thermometer placed at its focus. But in order to observe easily the different periods of the phenomenon, he connected the thermometer closely with the mirror, and the mirror with the glass; so that when the glass turned, the two other pieces turned with it, preserving always the same position with respect to it. Things being thus disposed, M. Berard brought the second glass successively into all possible azimuths about the ray, and he found that in the positions where no light was reflected, no heat was any longer reflected; for the thermometer placed at the focus of the mirror, did not rise when in these positions, although it rose by a very sensibly quantity, when the glass was placed in the azimuths where the reflection of light could take place upon its surface. In this experiment, therefore, as in the preceding one with Iceland spar, the obscure calorific principle accompanies the luminous particles and conforms to their action.

Although in the case in question, it must be admitted that the calorific principle is very little separated from the light itself, since it is free only for a very small space below the extreme red of the spectrum; still, there is a variety of other circumstances, in which we find it in great abundance, almost entirely separated from light. This is the case whenever we observe bodies which give out strong heat, without any sensible appearance of light; such, for example, as a ball of iron heated below the point where it becomes red, and still more vessels of tin or glass filled with boiling water. The sensation of heat which these bodies produce at a distance on our organs, and the effects they produce upon the thermometer cannot be ascribed in general to a transmission by contact, through the medium of the particles of air or vapour which intervene between us and them; for the calorific impression produced by them, is felt as well in a horizontal or downward direction as in any other, which is altogether contrary to the direction in which the particles of air, or vapour, or any other fluid whatever, move when their temperature is raised; the dilatation which they experience causes them to ascend, as has been abundantly proved both from reason and experiment. We must, therefore, conclude that in these and all analogous cases, there is an immediate transmission of obscure caloric, though we cannot absolutely determine whether the caloric is a material principle, radiating like light, or whether it consists only of vibrations propagated through an imponderable medium; two modes, which, if the medium be very elastic, exhibit nearly the same effects. Now, by substituting dark heated bodies in the place of the spectrum before used, M. Berard found that the effects produced upon the thermometer, still took place according to similar laws.

If one of the dark heated bodies of which we have been speaking, be placed before a concave metallic mirror, we find that there is a focus of heat produced by reflection; and this focus is formed at precisely the same point with that of luminous rays proceeding from the same body. The calorific emanation, therefore, is reflected specularly like light, making the angle of reflection equal to the angle of incidence; and as the specular reflection of light is nothing or nearly nothing, from rough bodies, when it does not fall very obliquely upon their surface; so there are certain surfaces which reflect obscure caloric with different degrees of intensity. It is strongly reflected, for example, from the surface of polished metals, but much less from the surface of glass, however perfectly polished. It was for this reason that M. Berard made use of a metallic mirror in the experiment above mentioned.

We have considered only the colorific and calorific properties of light. We shall now speak of its chemical effects. This point also has been accurately investigated by M. Berard. Chemists had long observed that when the muriate of silver and other white salts are exposed to the influence of light, they soon become black. Gum guaiacum, exposed thus to light, passes from yellow to green, as has been observed by Dr Wollaston. MM. Gay-Lussac and Thénard have made known an action of this kind still more immediate and energetic; for, upon exposing to a solar beam a mixture of hydrogen gas and chlorine in equal volumes, a detonation immediately took place, the product of which was hydrochloric acid, heretofore called muriatic acid. M. Berard made use of these different substances as reagents, in examining and ascertaining the chemical properties of the different rays of the spectrum. For having placed in the spaces occupied by the different colours, small pieces of paper impregnated with muriate of silver, or small vessels filled with a mixture of the two gases, he was enabled to judge of the energy of the action of the different rays by the intensity and rapidity with which chemical changes took place in the substances thus exposed. He ascertained in this manner, that the chemical action was really most intense towards the violet extremity of the spectrum, and that they extended as had been maintained by M. Ritter and Dr Wollaston, a little beyond this extremity. Moreover, when these substances were exposed for a certain time to the action of each ray, which could be easily done on account of the immobility of the spectrum, he succeeded in observing sensible effects, though continually decreasing in intensity, in the indigo and blue rays; whence he considered it probable that if reagents still more sensible were employed, analogous effects might be observed, though more feeble, in the other rays. In order to render evident the great disproportion which exists, in this respect, between the energies of the different rays, M. Berard concentrated, by means of a lens, all that part of the spectrum which is comprised between the green and extreme violet, and he concentrated in a similar manner, by another lens, all that portion which extends from the green to the other extremity. This last portion was collected into one point sensibly white, and so dazzling that the eye could hardly endure it. Nevertheless the muriate of silver remained exposed for more than two hours to this vivid light, without experiencing any sensible alteration. On the contrary, when exposed to the other portion which was much less white and dazzling, in less than ten minutes the muriate was found to become black. M. Berard concluded from this experiment that the chemical effects produced by light are not owing solely to the heat which it developes in bodies, by combining with their substance; since, on this supposition, the power of producing chemical combinations might be expected to be most intense in the rays, which possessed, in the highest degree, the power of producing heat. But perhaps we shall find less opposition between these two considerations, if we bear in mind, that, according to the experiments of De Laroche, there may be essential differences between the obscure caloric employed by chemists, to alter certain combinations, particularly the vegetable colours, and the caloric of the spectrum in the part which does not produce these effects. For example, the difficulty would not exist, if the obscure caloric, obtained by artificial heat, were wholly or partly analogous to the equally obscure emanations which take place at the violet extremity of the spectrum, and there does not seem to be any impossibility in such a supposition.

These experiments of M. Berard leave no doubt that the different portions of a solar ray, dispersed by the prism, have very different properties as to the power which they possess of producing vision, heat, and chemical combination. Are we then to attribute these three powers to three distinct kinds of rays, existing independently of each other, and capable each of producing only one single effect. If this be the case, it will also be necessary that each of these species should be capable of being separated by the prism into an infinity of different modifications, like light itself, since we find by experiment that each of the three properties, chemical, illuminating, and calorific, is distributed, though in very unequal proportions, over a certain extent of the spectrum. Thus, according to this hypothesis, we must conceive three spectrums, one calorific, one chemical, and one luminous, superposed upon each other. It must likewise be admitted that each of the substances which compose the spectrums, and even each of the particles of unequal refrangibility which compose these substances, is endued, like the particles of visible light, with the property of being polarised by reflection, and of escaping the influence of the reflecting force, as is observed with respect to the luminous particles, and in the same cases. A like analogy must exist also in the other properties. But instead of this complicated system, we suppose, conformably to the phenomena, that the solar light is composed of an assemblage of rays unequally refrangible, and consequently capable of being differently modified by the action of bodies, which supposition requires that there should be original differences, either in their masses, their velocities, or their affinities. Why should these rays which differ in so many respects, all produce upon thermometers and upon our organs, the same effects as to heat and light? Why should they have the same power in forming and separating chemical combinations? Is it not very natural to suppose that vision may take place in our eyes only between certain limits of refrangibility, and that too great or too small a refrangibility may render the rays equally unfit to produce this effect. It is possible that these rays are capable of exciting a vision in other eyes; perhaps they are capable of exciting it in certain animals; on this supposition all that is marvellous in the above phenomena disappears, or rather it becomes a part of the general action of light. In a word, we may suppose the calorific and chemical properties to vary throughout the whole extent of the spectrum with the refrangibility, but according to different functions, in such a manner that the calorific property shall be at its *minimum* at the violet extremity of the spectrum, and at its *maximum* at the red extremity; while, on the contrary the chemical faculty, expressed by another function, shall have its minimum at the red extremity, and its maximum at the violet extremity or a little beyond it. This single supposition, which is the simplest representation of the phenomena, perfectly satisfies all the facts stated above, and even enables us to predict a great number of them from analogy alone. Indeed, if all the rays which produce vision, heat, and chemical combinations, are equally rays of light, they must all necessarily be reflected from polished bodies, and reflected according to the same law, making the angle of reflection equal to the angle of incidence; whence it follows that they will be, in like manner, concentrated or dispersed by concave or convex mirrors. They must, moreover, be all polarised in traversing a crystal endued with the property of double refraction, or in being reflected from glass, ice, &c., under a determinate incidence; and when they have received these modifications, they must be reflected from another surface of the same kind, if it be placed in such a manner as to render its reflecting force efficacious upon the luminous particles. On the contrary, if this force produces no effect upon the visible luminous particles, invisible light will no longer be reflected; for the cause which determines or prevents reflection, appears to act equally upon all the particles, whatever be their refrangibility; and must therefore operate upon invisible light, since the condition of visibility or invisibility has reference only to the constitution of our eyes, and not to the nature of the particles themselves which produce the sensation in us. Finally, since, according to the observations of De Laroche, obscure caloric, emanating from bodies gradually heated, approximates gradually to the conditions and properties belonging to luminous caloric, we might suppose that when the emanation begins to become visible, it would be immediately analogous to the least calorific part of the spectrum, which is the extremity of the violet. Accordingly we observe that flame of whatever kind, in its incipient state, is violet or blue, and only attains to whiteness when it has reached a high degree of intensity. Still this view of the subject from the very circumstance that it supposes a progressive state, is not inconsistent with the idea of particular properties, belonging exclusively to a particular stage of the progression. Thus the calorific emanations of different temperatures, and the luminous emanations of different colours, may differ from each other in the property of producing vision, heat, and chemical action, in the power of being transmitted through transparent substances, and perhaps in many other particulars which we have not yet discovered.

IV.

Measure of the Intensities of Light.

It often happens in optical researches, that we have occasion to compare the intensities of two lights presented at the same time or successively. In the first case, which is the most simple, we illuminate separately with these two lights, equal discs of very white paper, or some other unpolished body which is a good reflector; then, viewing at the same time the two discs, we remove the most intense of the two lights, until they appear equally bright. Hence the intensities will be as the squares of the distances of the lights from the discs. This partial illumination may be obtained by illuminating a white space with these two lights, and interposing before them a small opaque disc; the shades of which will indicate the points separately illuminated. It is sufficient, then, to render these shades equally deep, by varying the distance of the luminous body from the screen. We may also admit the two lights separately through Fig. 180. two conical tubes united at their vertices, and terminated at this place by two equal discs of white paper. Then viewing both discs at once, having the head covered for the purpose of excluding all foreign light, we give notice to an assistant which of the two luminous objects must be removed or brought nearer, until the two discs appear equally bright. When we have arrived at this result, the intensities of the two lights are proportional to the squares of the distances from their respective discs.

But if we wish to compare two lights which are not visible at the same time, we have only to select a third, whose brightness is of such a nature as to sustain itself without variation, and compare it successively with each of the other two. By employing these processes and various others of a similar kind, Bouguer obtained a variety of curious results from which the following are selected.

Table of the quantities of light reflected by the surface of water under different obliquities.

1000 expresses the number of incident rays.							
Obliquities reckoned from the surface.	Number of reflected rays.	Ob.	No.	Ob.	No.	Ob.	No.
0° 30′	721	5° 0	501	17° 30′	178	50°	22
1 00	692	7 30	409	20	145	60	19
1 30	669	10	333	25	97	70	18
2 00	639	12 30	271	30	65	80	18
2 30	614	15	211	40	34	90	18

Table of the quantities of light reflected by the first surface of polished glass.

Obliquities of incidence reckoned from the surface.	Number of rays reflected.	Obliq.	No.	Obliq.	No.
2° 30′	584	15°	299	50°	34
5	543	20	222	60	27
7 30	474	25	157	70	25
10	412	30	112	80	25
12 30	356	40	57	90	25

Table of the quantities of light reflected by black marble polished.

Angle of the incident rays.	Number of rays reflected.
3° 35′	600
15 0	156
30 0	51
80 0	23

The first reflection under the angle of 3° 35′ is nearly as intense from marble as from mercury. The same may be said with respect to all plane bodies of whatever substance. They all become very good reflectors, when the incident rays make very small angles with the surface. But the reflecting force diminishes very rapidly as the direction of incidence approaches to a perpendicular. In this they differ from bodies whose reflecting force is powerful; for in these, the intensity of the reflected light undergoes only slight variations, under different incidences. In the case of mercury and telescope metal, for example, the whole extent of this variation scarcely amounts to $\frac{1}{8}$ or $\frac{1}{9}$ from 0 to 90°. According to the experiments of Bouguer, for an incidence of 21° reckoned from the surface, mercury reflects about 637 rays out of 1000. Consequently, reflection under all other angles may vary from about 700 to about 600. This metal, which is perhaps the best of all reflecting substances, absorbs then more than one quarter of the light which falls upon it; and this absorption is much greater in bodies which reflect more imperfectly.

V.

Description of the Kaleidoscope.

'This instrument, in its simplest form, consists of two reflecting Fig. 18
planes, made of glass or metal, from 5 to 10 or 12 inches long, and about an inch broad. These reflectors being put together with two of their edges in contact, and their reflecting faces inclined, for instance, at an angle of 60°, or the sixth part of a circle, when they are put in a tube, and the eye is placed at *E*, as near the angular point as possible, it will observe the opening *AOB* multiplied six times, and arranged round the centre *O*.

'If any object, however shapeless, is placed before the opening *AOB*, and near the ends of the reflectors, the eye at *E* will observe in the two adjoining sectors an inverted image of this irregular object, apparently facing the direct image in *AOB*, and the direct and inverted image will form an object perfectly symmetrical; and these symmetrical images being multiplied by successive reflections, the whole circular space, composed of six sectors, will present to the eye a most perfect picture.

'If the object placed in *AOB* consist of pieces of coloured glass, lying in a cell bounded by discs of glass, the pictures increase in beauty, and as the glass fragments change their position by the motion of the cell, a succession of the most perfect pictures will be displayed, which are literally infinite both in number and variety.

'The objects which give the finest outlines by inversion are those which have a curvilineal form, such as circles, ellipses, looped curves like the letter *S*, spirals, and other forms. Glass, both spun and twisted, and of all colours and shades of colours, should be formed into the preceding shapes, and when these are mixed with pieces of flat coloured glass, blue vitriol, native sulphur, yellow orpiment, differently-coloured fluids moving in small enclosed vessels of glass, &c., they will make the finest transparent objects for the kaleidoscope. A very fine effect is produced when only two colours are used, viz. those that harmonize with each other, such as red and green, blue, and gold yellow. Pieces of glass of these two colours may be mixed with twisted pieces of colourless glass with great effect.

'In the simple kaleidoscope the two reflectors may be fixed at a constant angle, so as to be an aliquot part of a circle, or 360°, or they may be made to vary their inclination by various simple contrivances.

'If the eye is raised above *E*, or if the objects are placed at any distance from the ends of the reflectors, the symmetry of the picture and the uniformity of the light vanish, so that it is essentially necessary to the production of forms perfectly beautiful and symmetrical that the eye be placed as near as possible to the angular point, and the objects as near as possible to the ends of the reflectors.

Fig. 183.

'The utility of the kaleidoscope may be greatly extended by the addition of a lens. "In considering how this change might be effected, it occured to me," says Dr Brewster, "that if *MN* were a distant object, either opaque or transparent, it might be introduced into the picture by placing a lens *LL*, at such a distance before the aperture *AOB*, that its image may be distinctly formed upon the plane passing through *AOB*. By submitting this idea to experiment, I found it to answer my most sangine expectations. The image formed by the lens at *AOB* became a new object, as it were, and was multiplied and arranged by successive reflections, in the very same manner as if the object *MN* had been reduced in the ratio of *ML* to *LA*, and placed close to the aperture.

'"In the compound kaleidoscope, thus constructed, the furniture of a room, books, and papers lying on a table, pictures on the wall, a blazing fire, the moving branches and foliage of trees and shrubs, bunches of flowers, horses and cattle in a park, carriages in motion, the currents of a river, moving insects, and in short every object in nature may be introduced by the aid of the lens into the figures created by the instrument. When the flames of a blazing fire constitute the object, the kaleidoscope creates from it the most magical fire-works, in which the currents of a flame which compose the picture may be turned into every possible direction.

'"The theory of this instrument; the various forms of annular, parallel, and polycentric kaleidoscopes; its application to the magic lanthorn and the solar microscope; and the mode of employing it in the fine and useful arts, have been explained at great length in my *Treatise on the Kaleidoscope*."'—*Brewster's Edition of Ferguson's Mechanics.*

VI.

New Optical Experiments, and the Improvement of Glass.

'M. Frauenhofer has been long known on the continent of Europe as a very distinguished practical optician. He has succeeded beyond any other artist in producing flint glass for optical purposes, of the most complete transparency and freedom from flaws and defects. This superiority in his glass has enabled him to prosecute some very important researches. His primary object was to determine with great exactness, for the formation of achromatic object-glasses, the dispersive powers of different species of glass. He first tried the effect of correcting the colour by opposing prisms, viewed through a telescope, which is in fact the same method as that originally proposed by Dr Brewster. But it became an object of attention to examine the dispersion of each coloured ray separately. To do this is a problem which has always been attended with the essential difficulty of not being able to fix upon rays in the spectrum which are strictly homogeneous, and which can at all times be identified with certainty. In order to get over this difficulty, M. Frauenhofer tried, without success, different coloured media and flames; to trials of this kind we shall soon have occasion to allude, as leading to some important discoveries. Our artist, however, next adopted a plan which he considered as successful; this was to place six lamps in a row behind a small aperture, close before which was a prism. The separate spectrums of each lamp were thus thrown, so that the prism under trial, which was placed at nearly seven hundred feet distance, received only the red rays, for example, from one lamp, and the blue from another, &c., by which means the colours appeared in the form of distinct spaces, separated entirely from each other. We cannot help feeling some difficulty as to the application of this method, but perhaps the description itself is not the clearest that might be given. We do not feel sure that the rays were strictly homogeneous; however, they were capable of exact identification from this further contrivance; a narrow aperture was made in the screen above the six lamps, through which the light of another lamp passed, and was received on the second prism; in viewing this, a bright line was seen at the limits of the red and yellow spaces; this was exactly defined, and by means of its invariable position, in comparison with the coloured spaces below, the observer could always be assured that the same identical ray fell on his prism. A number of measurements were thus made with great exactness, from which the great differences in the ratios of refraction for the same ray in different media, are clearly ascertained.

'But the most important point was the appearance of the bright line above mentioned. This M. Frauenhofer next proceeded to study; he found it exhibited alike by the light from all flames, &c., when received through a narrow aperture. He next tried the light of the sun; this was received into a dark room through a narrow crevice, at the distance of twenty-four feet, by a prism of excellent flint glass; in looking at the spectrum thus formed through a small telescope, he observed not only the bright line before spoken of, but an infinity of lines, some dark and some bright, crossing every part of the spectrum at right angles to the direction of its elongation, and not forming the boundaries of the different coloured spaces, but existing in the middle of them, and in fact, distributed in some places more plentifully than in others along the whole length, in some parts more conspicuous, and in others more faint. Of all these lines the observer has given an accurate delineation; he counted upon the whole 574 of them. If the aperture be so wide as to subtend an angle of more than 15″ to the eye, the lines disappear. Some of the fainter ones also are not seen, unless the eye be shaded from the glare of the brighter parts. With English flint glass, M. Frauenhofer could only see the brightest lines; but with every sort of glass of his own manufacture, and with prisms formed of liquids, they were all distinctly seen. He then proceeded, by an extended series of measurements, with a repeating circle, to determine the angles of deviation which these lines formed when viewed through different media. These lines in fact supply the great desideratum in researches of this nature, and enabled him to determine the deviations belonging to points in the spectrum strictly definite, with any degree of accuracy.

'From observing the great number of lines crossing the spectrum, we might be led to suppose that the inflection of light at the edges of the aperture had some connexion with the phenomenon; in order to examine this point, M. Frauenhofer varied the experiment in the following manner; he received the rays through a small circular hole nearly 15″ in diameter; the spectrum thus formed had almost no breadth, but in order to widen it, M. Frauenhofer made the rays pass through a semi-cylinder of glass. By this means the length and order of colours remained unaltered, but the breadth being magnified, he saw as before all the lines. By means of the same contrivance he detected similar lines in the light of the planet Venus, without employing any aperture; the brightest lines only were visible, but they coincided in position with the corresponding ones in the solar spectrum. The light of some of the principal fixed stars was subjected to the same examination; in some of these lines were observed in positions different from those before observed. The electric light was tried in the same way; the points of two conductors were connected by a fine fibre of glass, along which the succession of sparks was so rapid as to produce the appearance of a fine line of light. In the spectrum formed by this light, (without passing any aperture,) lines different from any of the former were observed. The light of several was similarly examined, and several curious results obtained.

'Such is a brief outline of the most important parts of M. Frauenhofer's experiments; they indicate a very remarkable property of light, and present appearances which we believe have not yet been accounted for on any known principles. We must here take occasion to remind our readers, that the discovery of the fact itself, (though evidently unknown to M. Frauenhofer,) was made some years ago by Dr Wollaston. His experiments was, however, somewhat different; and owing to the great superiority of his glass, M. Frauenhofer has the merit of having ascertained the almost infinite number of those lines, which in Dr Wollaston's experiments appeared only a few.

'M. Frauenhofer must also have the credit of being the first to apply these lines to the purpose of accurate determination of the dispersive power, although Dr. Wollaston made a few observations of this kind. It may be satisfactory to many to mention here, that with an ordinary prism of English glass, the principal lines may be very well seen by looking through the prism at a narrow aperture in a shutter or screen placed against a window so as to receive the light of the clouds; this was Dr Wollaston's method; his experiments are given in the Philosophical Transactions, for 1802; he examined also the light from flame. If the blue part of a candle flame be received through a narrow slit, the separation of the colours is very wide and complete.

The mere inspection of the prismatic colours is sufficient to show that the different parts of the spectrum, independently of their colour, possess very different degrees of brightness or illuminating intensity. The late Sir W. Herschel was, we believe, the first who attempted any accurate determination of these relative intensities; he found the greatest illumination in the yellowish green space, and a gradual decrease from thence towards each extremity. M. Frauenhofer tried similar experiments by a different method, and his determinations were made with greater attention to exactness than perhaps any former; but there appear to us two essential difficulties in his method. In the first place, the intensity of each coloured ray was to be equalised with the white or yellowish light reflected by a plane mirror from a lamp; M. Frauenhofer considers it easy, with a little practice, for the eye to judge of this equalisation with the requisite accuracy. This, we must confess, appears to us very doubtful; though the sensation of colour and of intensity may possibly depend on modifications of the same cause, yet the two sensations follow such very different laws, and that difference is dependent upon principles so wholly unknown to us, that we can hardly conceive the possibility of abstracting so entirely from the idea of colour that of intensity, as to enable the mind to decide in any thing like a certain and satisfactory manner upon the equality of illuminating effect in lights of two different colours simultaneously presented to the eye.

'Another and more serious difficulty appears to us to arise from the following considerations; supposing the illuminating intensities to be really equal, it is well established, that if two rays of light, one of a colour approaching more to whiteness than the other, be presented in juxta-position to the eye, the deeper colour of the one will be diluted by the proximity of the lighter colour of the other; that is to say, though not actually combined or blended together, the sensation which the one produces in the eye tends to diminish that which arises from the other. If this, as is highly probable, is owing to the different convergency required for the two it will obviously take place in a greater degree, in proportion as the coloured ray differs in refrangibility from the white.

'Whatever weight may be attributed to the objections against this particular method, it is certain that the illuminating intensity sustains a regular decrease from the central yellowish green to the violet on one side, and the red on the other. The series of numbers given by M. Frauenhofer decrease in a more rapid ratio than those found by any other observers, and the tendency of the causes just considered as influencing his results, would be precisely that of producing this rapid diminution. But the decrease of illuminating power towards the red boundary, will become a point of considerable interest in the sequel.

'M. Frauenhofer's observations on the illuminating powers of the prismatic rays, led him to several suggestions of practical importance in the construction of telescopes. He attends particularly to the distinction between diminishing the aberration of colour, and producing greater distinctness in the image; as also to the aberration from the want of achromatism in the human eye. When different specimens of glass were examined by the accurate test of the spectral lines, the difference in their dispersive powers was shown, when not otherwise capable of detection. M. Frauenhofer found differences of this kind in specimens taken not only from the same crucible, but from the opposite parts of the same piece of glass. By unwearied diligence and laborious trials, he has, however, at length succeeded in the manufacture of flint glass, to such a degree, that in a crucible containing four hundred pounds, two pieces, one taken from the bottom and the other from the top of the same mass, exhibited absolutely the same power.

'This becomes the place for noticing the results obtained by a fellow-labourer in the same work. M. Guinand was the son of a joiner at Neufchatel; as a youth he worked at that trade; subsequently made watch cases; and thus acquiring some idea of casting metals, undertook, on examining a reflecting telescope, to make one; in which he soon succeeded, without any knowledge of optics, and left entirely to his own resources for every part of the work. His next attempt was to make a pair of spectacles. He learned the art of grinding and polishing the lenses by having once witnessed the process. He hence proceeded to make lenses of telescopes, and constructed several small refracting ones. He now accidentally became acquainted with the principle of the achromatic object glass; and all his energies and labours seemed concentrated upon the means of endeavouring to procure glass free from imperfections for this purpose. This is in fact one of the most difficult problems with which the practical optician is concerned; and the patience, the sagacity, the perseverence, which M. Guinand displayed, in a long series of attempts under the most discouraging circumstances, to obtain his object, were truly surprising. At every failure he seemed to be occupied solely in studying the cause which had occasioned it. And thus, step by step, he contrived to approach at length towards the wished for object, and produced glass more free from striæ and imperfections than any before made. Every disappointment taught him some further improvement, and it was thus that he acquired, what is perhaps the distinguishing characteristic of his method, the mode of joining together into one large disk separate pieces of glass, selected as the most perfectly homogeneous. These he contrived to soften and unite together again, after which they were formed into the required lens, without any perceptible joining, or imperfection; in this way he has formed lenses of twelve or eighteen inches diameter. In 1805, his fame had reached M. Frauenhofer, who invited him to Batavia, to give his important services to the establishment at Benedictbauern, where glass for optical purposes is largely manufactured under M. Frauenhofer's direction. The glass made by M. Guinand has since become known over Europe; specimens have been tried by the opticians and astronomers of France and England. The report of that eminent artist, Mr. Tulley, as to its great superiority over any made in England, is couched in the strongest terms; and there can be little doubt that, owing to the very perfect transparency which it possesses, we may expect a great increase in the power of refracting telescopes, hitherto so much limited in their degree of improvement. M. Guinand returned to his native place, and continued the construction of telescopes with uncommon ingenuity and success, himself not only having melted, formed, and polished the glasses, and calculated the adjustments, but also constructed every part of the apparatus, and put it together. This remarkable example of untaught genius died 1823, aged seventy-six. His secret is confined to his son, who undertakes to continue the manufacture so important to the scientific world, upon the same principles as the father.

'We before mentioned that M. Frauenhofer's first attempts were directed to obtaining homogeneous light by means of flames and coloured media; in this he was unsuccessful. Dr Brewster, however, and Mr Herschel have been more fortunate.

'Dr Brewster was in want of homogeneous light, to illuminate objects under microscopic examination. Mr Herschel wished to obtain it for the prosecution of certain optical researches. Dr Brewster, after numerous trials, ascertained the remarkable fact, that almost all bodies in which the combustion is imperfect, such as paper, linen, &c., gave a light in which strictly homogeneous yellow rays predominated; that the yellow light increased with the *humidity* of these bodies; and that a great proportion of the same light was generated when various flames were urged mechanically with a blow-pipe, or a pair of bellows. He thence concludes, that the yellow rays are the produce of an imperfect combustion. However, the most important circumstance was that the presence of aqueous vapour increased the quantity of yellow light; this was a new fact, and supplied Dr Brewster with a lamp whose light was truly homogeneous. Diluted alcohol is the pabulum he employs, and he has suggested a convenient form for a lamp for the purpose wanted.

'Various media, such as coloured glasses, were also tried. Dr. Brewster investigated the effect of heat in changing the tints of these glasses; in some, the power of absorbing particular colours is altered transiently, in others permanently. He tried the effect of different media in absorbing the different rays of the spectrum, and has given dilineations of the spectrum, as seen through different coloured glasses.

'In Mr Herschel's experiments the object was nearly the same in the first instance, but he has pursued it in a somewhat different manner from Dr Brewster, and has arrived at some other results of considerable consequence.

'He first examined, as also Dr Brewster did, the effects of certain coloured glasses in almost obliterating certain coloured spaces in the spectrum, whilst others were transmitted in all their brilliancy. This fact was noticed by Dr Young. Mr Herschel, in applying to the examination of it the uncommon powers of his analytical skill, has resolved the phenomena into their most general expression, and thus traced the causes of many interesting consequences which otherwise would not have been deduced.

'For example, one of the glasses he tried was of a ruby red colour; this permitted to pass almost the whole red, and a considerable portion of the orange; and even in strong lights a portion of yellow or a trace of green, but the rest were obliterated. He represents the effect by conceiving a straight line divided according to the proportions of the coloured spaces, to be taken as the abscissa, and at each point ordinates erected, representing the proportion of rays transmitted by any medium; the extremities of these ordinates give a curve, which he calls *the type* of this medium. The nature of this curve is determined by observation for each medium; but Mr Herschel has given an analytical expression, showing the law by which the nature of the curve is altered, according to an increase of thickness in the medium; this is in fact one of the most curious parts of the subject.

'"It would appear at first sight," Mr Herschel observes, "that the effect of doubling or tripling the thickness of any coloured medium, would simply be to increase the depth and intensity of the tint, but not to alter its character. If a white object appear blue through a blue glass, we should expect it to appear still bluer through two, and yet more so through three such glasses. The above formula shows, however, that this is so far from being the case, that the tint of the emergent pencil is essentially dependent on the thickness of the medium; and that it is only from a knowledge of the relative values of the ratios of the intensity after traversing a thickness equal to unity, for the various parts of the spectrum, that we can say *à priori*, whether the tint of a thick glass will retain any similarity to that of a thin one of the same kind."

'The fact is, the quantity of any coloured ray, transmitted by an homogeneous medium, decreases in *geometrical* progression, as the thickness increases in *arithmetical*. Thus, however trifling the difference may be at first in the effect of two media, it is always possible to render it sensible by taking a sufficiently great thickness; thus the water of the lake of Geneva is indigo-blue, that of the lake of Como, emerald-green, when viewed through a considerable thickness, though colourless in small quantities. Of this, numerous other instances will occur; such as the difference in the colour of the sea according to its depth, so well known to pilots, as often enabling them to perceive their approach to shoals, &c.

'"In some instances the curve has two unequal maxima in different parts of the spectrum; and if at the same time the greater of these should happen to correspond to a ray of feebler illuminating power than the less, the tint, in small thicknesses of the medium, will, generally speaking, be that of the lesser maximum; the greater vividness of these rays giving them a predominance over the others, though more numerous; but as this inequality of number increases with the increase of thickness, the feebler rays will at length begin to influence the tint, and finally obtain the predominance; thus producing in several cases, a complete change of colour, not a little surprising to those who are ignorant of its cause. Dr Thomson's muriated liquor (chloride of sulphur,) which is yellowish green in very small thicknesses, and bright red in considerable ones, is a case in point; a solution of sap-green presents the same phenomenon yet more strikingly. If enclosed between glass plates, slightly inclined, so as form a thin wedge, its colour towards the edge will appear emerald green, and towards the back, blood red, passing in the intermediate thicknesses through a kind of livid neutral tint."

'The existence of any real homogeneous yellow in the solar spectrum has been denied by Dr Wollaston. The researches both of Mr Herschel and of Dr Brewster tend to show, that though the insulation of these rays in perfect purity may not be practicable, yet they may be so far separated, as to place their existence beyond all doubt. Dr Brewster considers these rays as encroaching on the limits both of the red and green. Mr Herschel attributes to them a breadth not less than one-fourth of the interval between red and blue. Dr Brewster draws the conclusion, that both the orange and green are really composite colours.

'Of the numerous subsequent experiments of Mr Herschel, we shall not give any details; they are all of a very interesting nature, and exhibit several very singular changes in the aspect of the spectrum, by a mere addition of thickness in the coloured glasses; the curves which represent them, are all delineated.

'From the solar light Mr Herschel afterwards turned his attention to that from flames, &c.; these lights differ extremely in their *types* when examined by the prism, and that in an apparently most capricious manner. Among other results, he found that sulphur, at a stage of inflammation which is extremely violent, as when thrown into a white hot crucible, emits a perfectly homogeneous and brilliant yellow light. In examining the light of a spirit lamp, it was found to become perfectly homogeneous when viewed through a glass consisting of a pale orange and a pale green one, cemented together. A lantern formed of such glass, would afford a monochromatic lamp for microscopical purposes.

'Some beautiful appearances are detailed, as produced from the tinge given to flames by various substances held in solution with alcohol, &c.

'One of the first glasses which Mr Herschel tried, and one which gave the most important results, was of that blue kind with a purplish tint which is employed for finger-glasses, &c. When the spectrum is viewed through a thickness of 0,04 inch of this glass, the red space was divided into two by a dark line; other changes took place in the rest of the spectrum, which we need not here detail. With a double thickness of this glass further alterations were observed; among which was that the outer red alone remained visible, the inner being totally obliterated. When a great many thicknesses were laid together, the extreme red and violet only were transmitted. Mr Herschel directed his attention particularly to these outer red rays, and we extract the following important observations upon them:

'"The species of light alluded to is remarkable; first, for its perfect homogeneity, and secondly, for its position in the spectrum. When the solar spectrum, received on a white paper in a darkened room, is viewed through a moderate thickness (0,08 inch) of that glass, cemented to any red glass of a tolerably pure colour, it will be seen reduced to a perfectly circular and well defined image of a deep red colour. If a pin be now stuck in the centre of the red circle it will be found on removing the glass from the eye, to have been fixed in what an ordinary observer would call the very *furthest termination* of the red rays; and a mark similarly made at its circumference, will appear to lie wholly without the spectrum, among the dispersed light which usually hangs about its edges; in other words, the red, thus insulated, is of too feeble an illuminating power to affect the sight in the immediate vicinity of the other more brilliant rays, and only becomes visible when they are extinguished, or greatly enfeebled. To an eye defended by such a glass, vision through a prism with the largest refracting angles, is as sharp, and the outlines of minute objects as free from nebulosity and indistinctness, as if their rays had suffered no refraction. These characters,—the absolute homogeneity of the rays,—their situation precisely at the least refracted limit of the spectrum, and the facility with which they may be insulated, render them of peculiar importance as standards of comparison in optical experiments."

'In this simple and unpretending manner does Mr Herschel announce, what we must consider one of the greatest accessions to the catalogue of optical facts, which has been made since Newton first pointed out the unequal refrangibility of the primary rays. To their number Mr Herschel has added another, whose existence had not previously been suspected; in the analysis of light he has detected a new ingredient, and has thus found a new and exact means of measuring the dispersive powers of different media. To this purpose he has, at the conclusion of his paper, applied the insulation of these extreme red rays, and of the extreme violet; the deviation thus obtained, being of course greater for every sort of glass than any obtained by former methods, and the measurement extremely exact, from the circumstance of the rays being precisely defined and truly homogeneous. The method of operating is, we believe, new, and very simple.

'The utility of the extreme red rays for this purpose is unquestionably very great; but the fact will be interesting to philosophers in a variety of other points of view. We have already made some remarks on the decrease of illuminating intensity in the different spaces of the spectrum, from the centre to the extremities; this is closely connected with the existence of invisibly rays. It has been ascertained that the eye is somewhat deficient in its power of converging red light; from this cause alone, if the red rays were presented to it in an insulated state, the outer part of the red would be indistinct, and it would be very probable that certain extreme rays might exist which would be altogether invisible; but when the rays are presented in juxta-position, the influence of the central rays which converge at a shorter distance will tend to increase the deficiency in the perception of the extreme red; and this would be the case on the supposition that all the rays possessed an intrinsic equal illuminating power, and were all of equal density; but if in this respect they differ, (as we have seen they do,) the diminution will be still more considerable. It would thus be evident, that at whatever distance from the central point the real termination of the spectrum were situated, the apparent illuminating powers must decrease by a much more rapid law, than the absolute and intrinsic intensities would do; so that the apparent limit of the spectrum would be at a much shorter distance from the point of maximum illumination.

'The discovery of the new red rays has, as might be expected, excited great interest; they have been recently examined by Mr Powell, who has measured their deviation, and observed them also, in the moon's light. In forming the spectrum as in Dr Wollaston's experiment above described, their appearance is remarkably distinct; in the spectrum of the blue part of a flame they do not exist, although there is much of the more refrangible red."—*British Critic.*

VII.

On the Magnetizing Power of the more refrangible rays of Light.

'Dr Morichini, a respectable physician in Rome, discovered this remarkable property of the violet rays. Professor Playfair saw the experiment by Dr Carpi, in the absence of Morichini, before a party of English and Italian gentlemen. The following account of the experiment was drawn up from a conversation which the writer of this notice had with that distinguished philosopher, and was afterwards submitted to him for his approbation.

"The violet light was obtained in the usual manner, by means of a common prism, and was collected into a focus by a lens of a sufficient size. The needle was made of soft wire, and was found, upon trial, to possess neither polarity nor any power of attracting iron filings. It was fixed horizontally upon a support, by means of wax, and in such a direction as to cut the magnetic meridian at right angles. The focus of violet rays was carried slowly along the needle, proceeding from the centre towards one of the extremities, care being taken never to go back in the same direction, and never to touch the other half of the needle. At the end of half an hour, after the needle was exposed to the action of the violet rays, it was carefully examined, and it had acquired neither polarity, nor any force of attraction; but after continuing the operation twenty-five minutes longer, when it was taken off and placed on its point, it traversed with great alacrity, and settled in the direction of the magnetical meridian, with the end over which the rays had passed turned towards the north. It also attracted and suspended a fringe of iron filings. The extremity of the needle that was exposed to the action of the violet rays, repelled the north pole of a compass needle. This effect was so distinctly marked, as to leave no doubt in the minds of any who were present, that the needle had received its magnetism from the action of the violet rays."

'Such was the state of this subject when Mrs Somerville directed to it her attention; and it is no slight praise to say, that she has set to rest a question on which the scientific world was divided, and that by the sagacity and ingenuity with which she has conducted her experiments, she has rendered visible, even in our northern climate, one of the most delicate of the magnetic influences, which, it was agreed on all hands, required for its developement the serene sky of an Italian climate.

'The following is a general outline of these interesting experiments.

'Having obtained the prismatic spectrum by means of an equiangular prism of flint glass placed in a hole in the window-shutter, Mrs Somerville took a sewing needle, about an inch long, and entirely devoid of magnetism.[*] Conceiving that no polarity would be superinduced if the whole needle were exposed to its action, she covered one half of it with paper, and exposed the other half to the violet rays of the spectrum cast upon a pannel at the distance of five feet. *In about two hours, the needle had acquired magnetism, the ex-*

---

[*] 'This was ascertained by its attracting indifferently either pole of a sewing needle magnetized in the usual way. This magnetized needle was pushed through a piece of cork, in which was inserted a glass cap, and it was in that state made to revolve freely on the point of another sewing needle.'

posed end exhibiting north polarity. This experiment was often repeated, and always with the same result.

'By a similar process, Mrs Somerville ascertained that the *indigo* rays had nearly as great an effect as the violet, and that the *blue* and *green* rays likewise produced the same effect, though in a less degree.

'Mrs Somerville next tried the *yellow*, *orange*, and *red* rays, but neither in them nor in the calorific rays, was the slightest effect produced, even when the experiments were continued for three successive days.

'Mrs Somerville now applied the same method to pieces of clock and watch springs, about 1½ inches long, and from ⅛ to ¼ of an inch broad,* and they were found to receive a stronger degree of magnetism from the violet rays, an effect which was attributed to their blue colour, and their greater extent of surface. Bodkins were not affected. When the violet ray was concentrated by a lens, the magnetic influence was imparted to the needles in a shorter time.

'In order to give additional confirmation to these results, Mrs Somerville exposed unmagnetized needles, half covered as formerly, to the sun's rays transmitted through glass coloured blue by cobalt, and they were distinctly magnetized as before. Needles exposed under green glass received the same property.

'Mrs Somerville now enclosed unmagnetized needles in pieces of *blue* and *green* ribband, one half of each being covered with paper, and after they had hung a day in the sun's rays behind a pane of glass, they had acquired magnetic polarity, the exposed ends being north poles, as in the former experiments. When *red*, *orange*, or *yellow* ribband was used, no magnetic influence was imparted.

'In performing these experiments, Mrs Somerville found that the most favourable time of the day was from ten to one o'clock; and that, as the season advanced, the magnetism acquired was less permanent, as the needle required a longer exposure to acquire the same degree of magnetic virtue.'—*Edinburgh Journal of Science for* 1826.

---

* 'When these possessed any magnetism, it was removed by heating.'

VIII.

Of the Colours of Striated Surfaces.

It is well known that beautiful colours are produced upon polished surfaces by means of fine lines or scratches. 'Boyle appears to have been the first that observed these phenomena. Newton has not noticed them. Mazéas and Mr Brougham have made some experiments on the subject, yet without deriving any satisfactory conclusion. But all the varieties of these colours are very easily deduced from the theory undulations.

'Let there be, in a given plane, two reflecting points very near each other, and let the plane be so situated that the reflected image of a luminous object seen in it may appear to coincide with the points; then it is obvious that the length of the incident and reflected ray, taken together, is equal with respect to both points, considering them as capable of reflecting in all directions. Let one of the points be now depressed below the given plane; then the whole path of the light, reflected from it, will be lengthened by a line which is to the depression of the point as twice the cosine of incidence to the Fig. 181. radius.

'If, therefore, equal undulations of given dimensions be reflected from two points situated near enough to appear to the eye but as one, wherever this line is equal to half the breadth of a whole undulation, the reflection from the depressed point will so interfere with the reflection from the fixed point, that the progressive motion of the one will coincide with the retrograde motion of the other, and they will both be destroyed; but, when this line is equal to the whole breadth of an undulation, the effect will be doubled; and when to a breadth and a half, again destroyed; and thus for a considerable number of alternations; and if the reflected undulations be of different kinds, they will be variously affected, according to their proportions to the various lengths of the line which is the difference between the lengths of their two paths, and which may be denominated the interval of retardation.

'In order that the effect may be the more perceptible, a number of pairs of points must be united into two parallel lines; and, if several such pairs of lines be placed near each other, they will facilitate the observation. If one of the lines be made to revolve round the other, as an axis, the depression below the given plane will be as the sine of the inclination; and while the eye and luminous object remain fixed, the difference of the lengths of the paths will vary as this sine.

‘ The best subjects for the experiment are Mr Coventry’s exquisite micrometers; such of them as consist of parallel lines drawn on glass, at the distance of one five hundredth of an inch, are the most convenient. Each of these lines appears under a microscope to consist of two or more finer lines, exactly parallel, and at the distance of somewhat more than a twentieth of that of the adjacent lines. “ I placed one of these,” says Dr Young, “ so as to reflect the sun’s light at an angle of 45°, and fixed it in such a manner, that while it revolved round one of the lines as an axis, I could measure its angular motion ; and I found that the brightest red colour occured at the inclinations 10¼°, 20¾°, 32°, and 45° ; of which the sines are as the numbers 1, 2, 3, and 4. At all other angles also, when the sun’s light was reflected from the surface, the colour vanished with the inclination, and was equal at equal inclinations on either side.

‘ “ It is not improbable that the colours of the integuments of some insects, and of some other natural bodies, exhibiting in different lights the most beautiful versatility, may be found to be of this description, and not to be derived from thin plates. In some cases, a single scratch or furrow may produce similar effects by the reflections of its opposite edges.” ’—*Young’s Nat. Phil.* vol. ii, p. 625.

TABLE

Of the Refractive and Dispersive Powers of different Substances, with their Densities compared with that of Water, which is taken as the Unit.

The substances marked (*) are combustible.

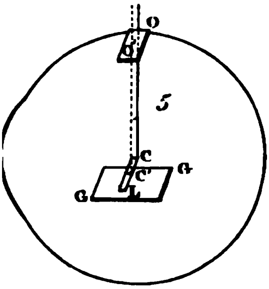
The refraction is supposed to take place between the given substance and a vacuum.

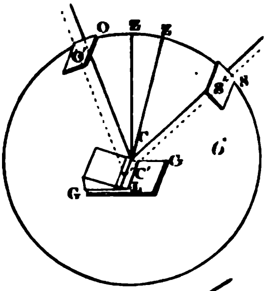
Substance.	Ratio of refraction.	Dispersive power.	Density.
Chromate of lead (strongest)	2,974	0,4	5,8
Realgar	2,549	0,267	3,4
Chromate of lead (weakest).	2,503	0,262	5,8
*Diamond	2,45	0,038	3,521
*Sulphur (native)	2,115		2,033
Carbonate of lead (strongest)	2,084	0,091	6,071
—— weakest.	1,813		4,000
Garnet	1,815	0,033	3,213
Axinite	1,735	0,030	
Calcareous Spar (strongest) . .	1,665	0,04	2,715
—— weakest.	1,519		
*Oil of Cassia	1,641	0,139	
Flint glass	1,616	0,048	3,329
—— another kind	1,590		
Rock crystal	1,562	0,026	2,653
Rock salt	1,557	0,053	2,130
Canada balsam	1,549	0,045	
Crown glass	1,544	0,036	2,642
Selenite	1,536	0,037	2,322
Plate glass	1,527	0,032	2,488
Gum arabic	1,512	0,036	1,452
*Oil of almonds	1,483		0,917
*Oil of turpentine	1,475	0,042	0,869
Borax	1,475	0,030	1,718
Sulphuric acid	1,440	0,031	1,850
Fluor spar	1,436	0,022	3,168
Nitric acid	1,406	0,045	1,217
Muriatic acid	1,374	0,043	1,194
*Alcohol	1,374	0,029	0,825
White of egg	1,361	0,037	1,090
Salt water	1,343		1,026
Water	1,336	0,035	1,000
Ice	1,307		0,930
Air	1,00029		0,0013
Oxygen	1,00028		0,0014
*Hydrogen	1,00014		0,0001
Nitrogen	1,00029		0,0012
Carbonic acid gass	1,00045		5,0018

THE END.

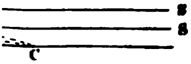
ERRATA.

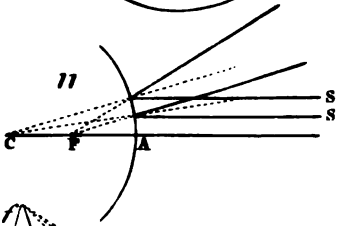
, line	31	from the top,	for	S,	read	I.
, —	33	“	“	s,	“	I.
, —	19	“	“	T'',	“	T.
, —	16	“	“	II,	“	II'.
, —	13	“	“	SOS',	“	SOI'.
, —	30	“	“	$S'C'O'C'$,	“	$S'C', O'C'$.
, —	1	“	“	*not,*	“	*only.*
, —	25	“	“	*interest,*	“	*intersect.*
, —	18	“	“	*extraordinary,*	“	*ordinary.*
, —	34, 35		“	*one the divisions,*		*one of the* &c.
, —	36	“	“	$I'O'i''$,	“	$I'Oi''$.
, —	13	“	“	$\frac{z^2}{h}$,	“	$\frac{z_2}{h}$.
, —	39	“	“	$\frac{1}{7747,84}$,	“	$\frac{1}{17747,84}$.

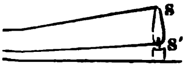

O
O'
5
C
C'
G
G
L

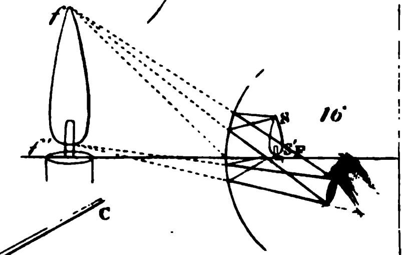

O
Z
Z
O'
S'
S
r
6
C'
G
G
L

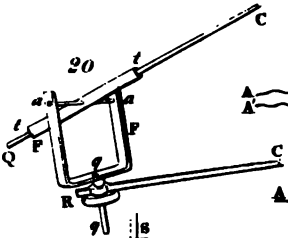
10

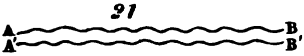

S
S
C

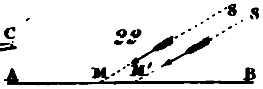

11
S
S
C
F
A

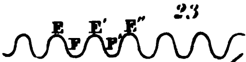

S
S'


f
f'
S
16
S'
F

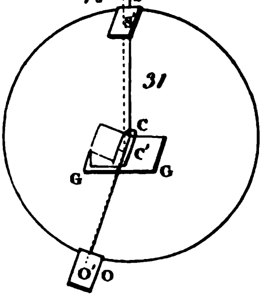

C
20
t
a
a
t
F
F
Q
C
q
R
q

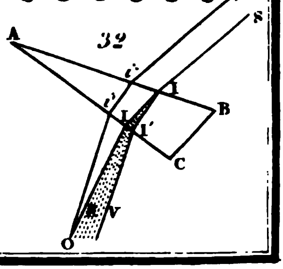

21
A
B
A'
B'


S
S
22
A
M
M'
B

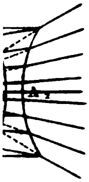

23
E
E'
E''
F
F'

2

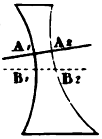

S'
31
C
C'
G
G
O'
O


S
S
A
32
i
i'
I
L
I'
B
C
R
V
O

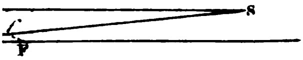
Optics Pl. II.



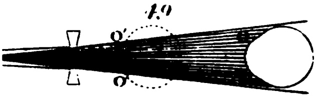
44



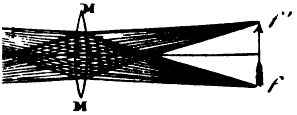
47



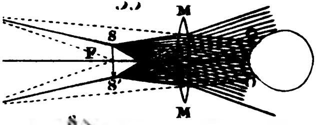
49

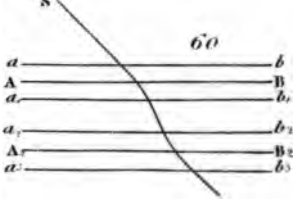


51

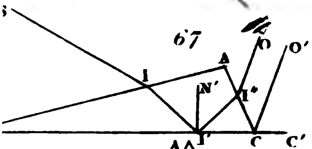


53

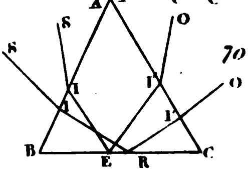




67

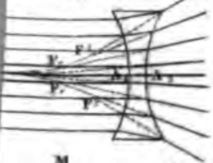


70

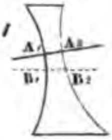


Optics Pl. II.

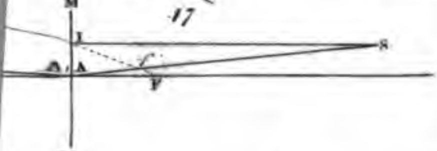
45



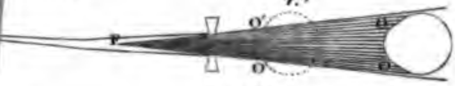
44



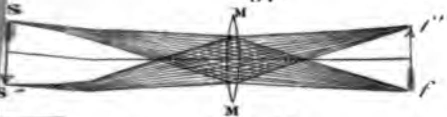
47



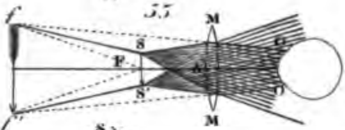
49



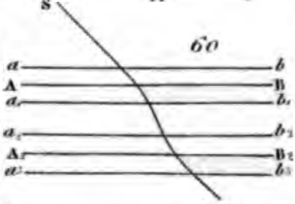
51

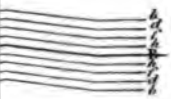


53

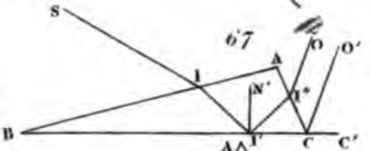


60

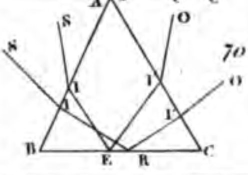


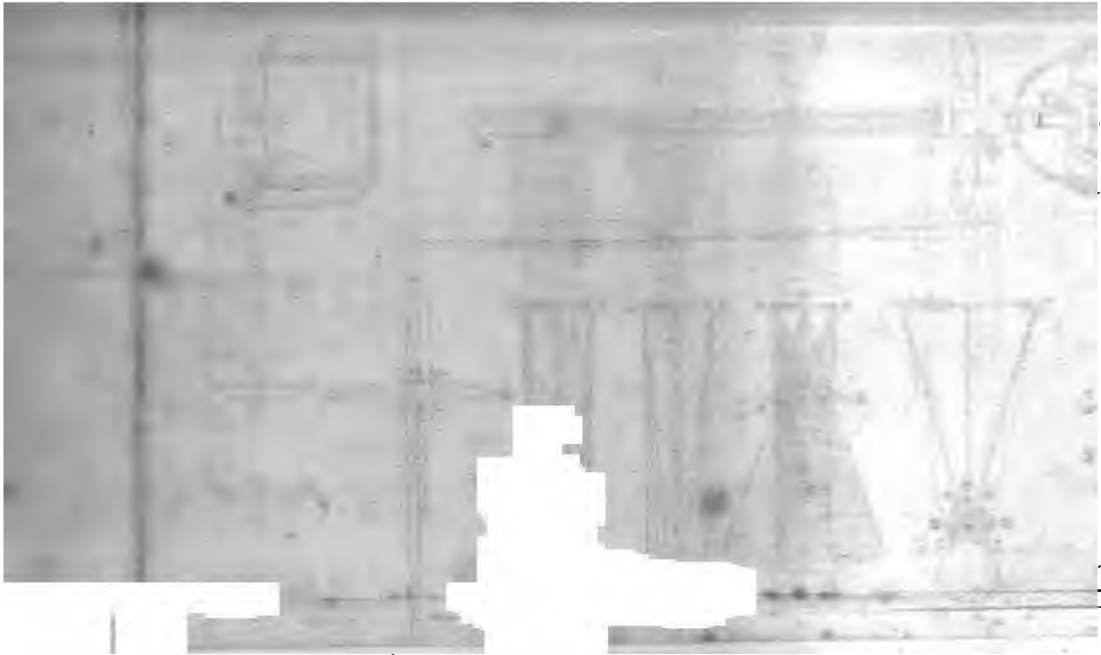


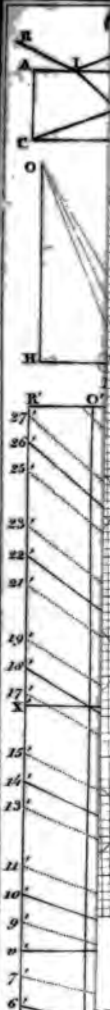
67

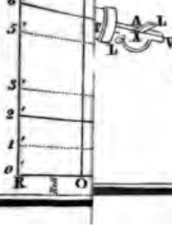


70

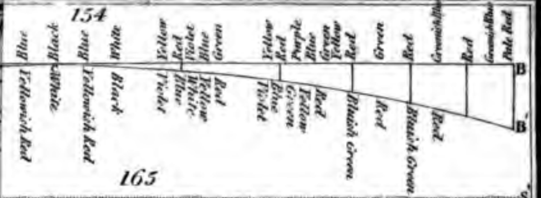




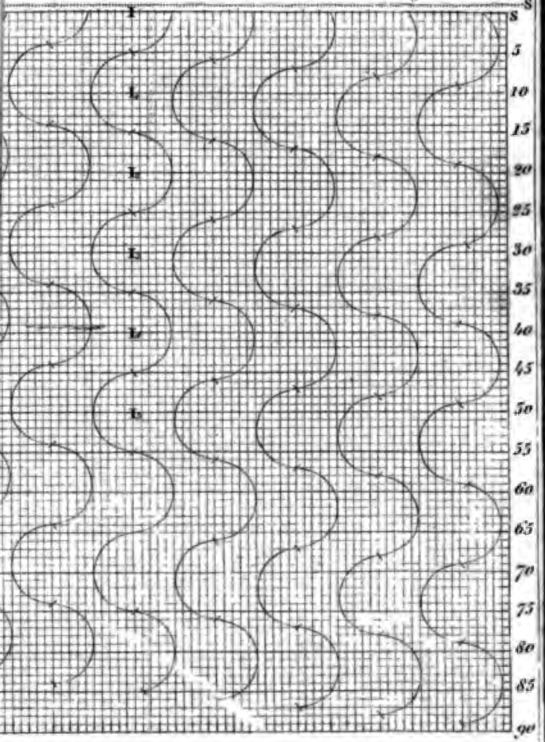




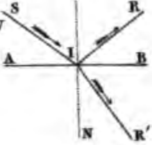
154



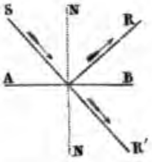
165



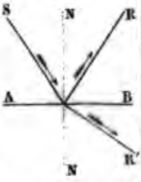
168

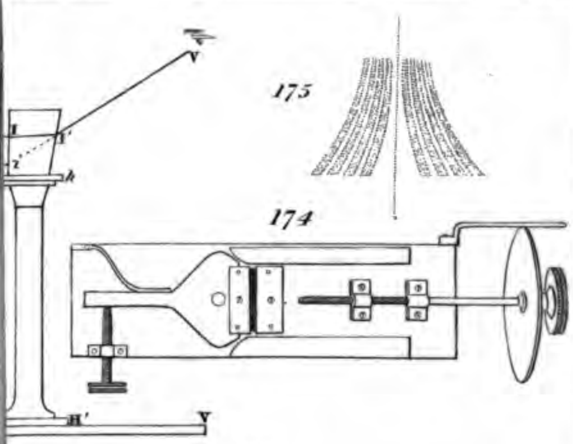


169



170

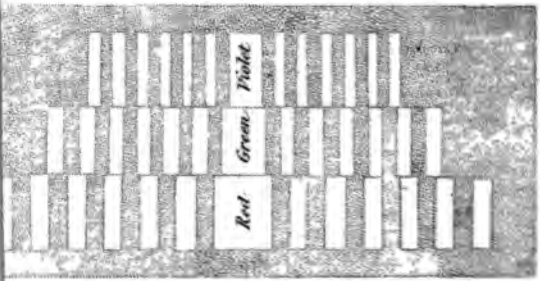




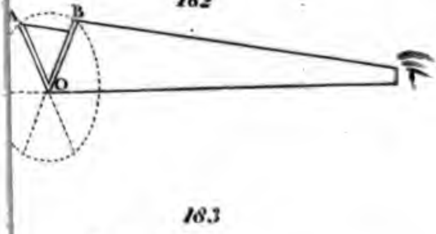
175

174

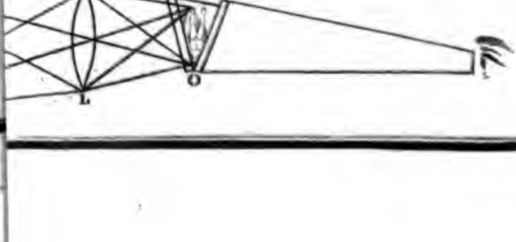
178

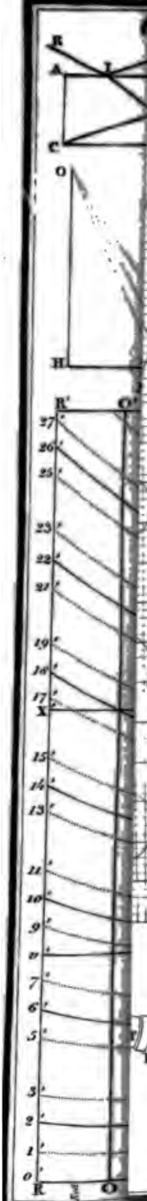


182

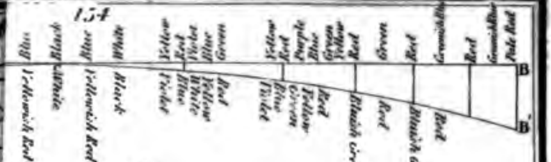


183

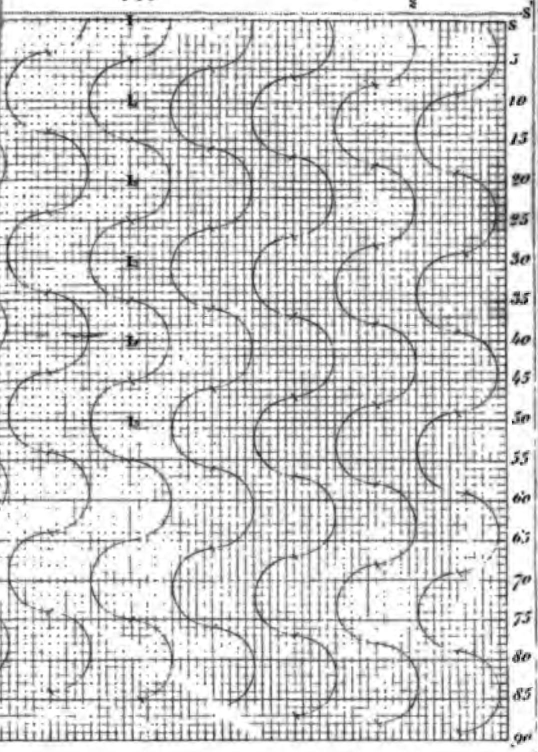




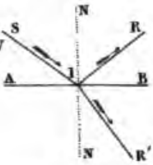
154



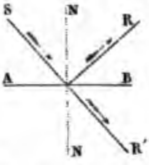
163



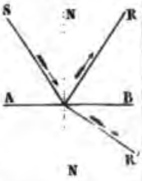
168



169



170



3 2044 004 356 234

The borrower must return this item on or before the last date stamped below. If another user places a recall for this item, the borrower will be notified of the need for an earlier return.

*Non-receipt of overdue notices does **not** exempt the borrower from overdue fines.*

Harvard College Widener Library
Cambridge, MA 02138 617-495-2413

WIDENER
JAN 21 2002
CANCELLED

WIDENER
SEP 10 2002
BOOK DUE

Please handle with care.
Thank you for helping to preserve
library collections at Harvard.